A Characterization of 2-vertex Switching of Two Cyclic Graphs

C. Jayasekaran [1], M. S. Kalaiselvi [2]

[1] Associate Professor, Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil - 629003, Tamil Nadu, India.

[2] Research Scholar,Reg. No: 23113132092003, Department of Mathematics, Pioneer Kumaraswamy College, Nagercoil - 629003, Tamil Nadu, India. Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627012,Tamil Nadu, India.
email : jayacpkc@gmail.com[1],
kalaims.1995@gmail.com[2]

Abstract

In the context of a finite undirected graph $G(V,E)$ and a non-empty subset $\sigma \subseteq V$, the graph generated by switching G by σ is known as $G^{\sigma}(V,E^{'})$. This graph is obtained from G by summing up all non-edges between σ and $V-\sigma$ and terminating all edges between σ and its complement $V-\sigma$. We record G^{v} for $\sigma=\{v\}$, and the associated switching is known as vertex switching. Nevertheless, we refer to this as $|\sigma|$-vertex switching. It is reported as 2 -vertex switching when $|\sigma|=2$. The process of G being isomorphic to G^{σ} is referred to as self vertex switching. An acyclic graph is one that lacks cycles. A graph that is unicyclic only has one cycle in it. A two cyclic graphs contains exactly two cycles in it. Any two vertices in a connected graph are linked together by a path. A disconnected graph has more than one component. Christabel Sudha introduced the idea of 2 -vertex self switching in 2018. The notion of self switching of two cyclic and bicyclic graphs was defined by sumathy in 2014. The two vertex switching properties of connected and disconnected unicyclic graphs are described by Vinoth Kumar. In this article, we come up with neccessary and sufficient requirements for G^{σ}, the switching of G at $\sigma=\{u,v\}$ to be connected and two cyclic graph when $uv \notin E(G)$.

Keywords :Switching, Connected two cyclic graphs, 2-vertex self switching.
Subject Classification Number: 05C60, 05C40.

1 Introduction

For any graph $G(V,E)$ with $|V(G)|=p, G^{\sigma}(V,E^{'})$ is described as the graph formed from G by terminating all edges between σ and its counterpart, $V-\sigma$, and any non-edges between σ and $V-\sigma$ are added as edges where $\sigma \subseteq V$. Seidel defined switching also known as $|\sigma|$ vertex switching[1]. When $|\sigma| = 2$, it is called as 2 -vertex switching [9,10]. Hage deliberate about switching of a vertices in a graph [2,3]. A graph which contains exactly two cycles is called an two cyclic graph. In the paper[5], the concept of self vertex switchings were studied. Vilfred V. et al., established the theory of branches and joints in graphs[11]. A joint at σ in G is a subgraph B of G that includes $G[\sigma]$ if $B - \sigma$ is connected and maximum. If B is connected, we refer it as a c-joint or else a d-joint. In the paper[7], graphs were distinguished for self vertex switching of connected two cyclic graphs. In the paper[6,8], C. Jayasekaran et al., examined the graphs for 2 -vertex switching of joints and connected unicyclic graphs. For standard symbols and definitions, we make reference to F. Harary[4]. In this paper, we intiated the concept of 2-vertex switching of connected two cyclic graphs.

2 Preliminaries

Theorem 2.1. [6]
Let G be a graph of order p and let $\sigma = \{u, v\}$ be a subset of $V(G)$ such that uv $\notin E(G)$. Let B be a c-joint at σ in G. Then B^σ is a c-joint at σ in G^σ if and only if $B - \sigma$ is connected, $|V(B)| \geq 4$,$0 < d_B(u) \leq |V(B)| - 3$ and $0 < d_B(v) \leq |V(B)| - 3$.

3 Main Results: 2-vertex Switching of Connected Two cyclic Graphs

Theorem 3.1. For a graph G of order $p \geq 5$ and let $\sigma = \{u, v\} \subseteq V(G)$ be such that uv $\notin E(G)$. Let M be the set of non-adjacent vertices of u and N be the set of non-adjacent vertices of v in G. Let B be a c-joint at σ in G. Then B^σ is a c-joint and two-cyclic at σ in G^σ if and only if $|V(B)| \geq 5$ and one of the following holds:

1. $B - \sigma$ is connected, acyclic, $d_B(u) = d_B(v) = |V(B)| - 4$ and the path formed by elements of M and by elements of N have either at most one vertex in common for $M \cap N = \varphi$ or the vertex a in common for $M \cap N = \{a\}$.
2. $B - \sigma$ is connected, unicyclic, $\{d_B(u), d_B(v)\} = \{|V(B)| - 4, |V(B)| - 3\}$ and for $|M - \{v\}| = 2$ and $|N - \{u\}| = 1(|M - \{v\}| = 1$ and $|N - \{u\}| = 2)$ either the elements of $M(N)$ do not lie on the cycle of $B - \sigma$ and the unique path connecting them contains at most one vertex of the cycle or one of the elements of $M(N)$, say a, lies on the cycle and the unique path connecting them contains no vertex of the cycle other than a.
3. $B - \sigma$ is connected, two-cyclic and $d_B(u) = d_B(v) = |V(B)| - 3$.

Proof. Let M and N be the set of non-adjacent vertices of u and v respectively in G. Let B be a c-joint at σ in G such that B^σ is a c-joint and two-cyclic. By Theorem 2.1, $B - \sigma$ is connected, $|\ VB)| \geq 4$,$0 < d_B(u) \leq |V(B)| - 3$ and $0 < d_B(v) \leq |V(B)| - 3$. Since B^σ is two-cyclic, $B - \sigma$ is either acyclic or unicyclic or two-cyclic.

Case 1. $B - \sigma$ is acyclic
If $d_B(u) < |V(B)| - 4$, then there exist at least three elements of M, say u_1, u_2, u_3, in $V(B) - \sigma$ such that u_1, u_2 and u_3 are adjacent to u in B^σ. Since $B - \sigma$ is connected, there exist $u_1 - u_2, u_2 - u_3$ and $u_1 - u_3$ paths in B and hence in B^σ. Now the

edges u_1u, u_2u and u_3u and the paths $u_1 - u_2, u_2 - u_3$ and $u_1 - u_3$ form at least three cycles in B^σ which is a contradiction to B^σ is two-cyclic. Hence, either $d_B(u) = |V(B)| - 4$ or $d_B(u) = |V(B)| - 3$.

Subcase 1.a. $d_B(u) = |V(B)| - 4$
Since $uv \notin E(G)$, there exist two vertices of M, say u_1 and u_2, in $V(B) - \sigma$ which are adjacent to u in B^σ. Clearly, $M = \{u_1, u_2\}$. Since $B - \sigma$ is connected, there exists an $u_1 - u_2$ path in B^σ. Here, the edge uu_1, the path $u_1 - u_2$ and the edge u_2u form a cycle $C1$ in B^σ.

If $d_B(v) = |V(B)| - 3$, then there exists exactly one vertex of N, say w, in $V(B) - \sigma$. Implying that vw is an edge in B^σ. Since $B - \sigma$ is acyclic, the addition of the edge wv in B^σ and the edges uu_1, the path $u_1 - u_2$ and the edge u_2u form the unique cycle $C1$ in B^σ, which is a contradiction to B^σ is two-cyclic.

If $d_B(v) = |V(B)| - 4$, then there exist two vertices of N, say v_1 and v_2, in $V(B) - \sigma$ such that v_1 and v_2 adjacent to v in B^σ. Clearly, $N = \{v_1, v_2\}$.

We consider the following three subcases
Subcase 1.a.1. $M = N$
Then the edges $vv_1 = vu_1, vu_2 = vv_2$ and the path $u_1 - u_2$ form a cycle $C2$ and the edges uu_1, u_1v, vu_2 and u_2u form another cycle $C3$ in B^σ which is different from $C1$ and $C2$ giving a contradiction to B^σ is two-cyclic.

Subcase 1.a.2. $M \neq N$
Then $M \cap N$ is either φ or has exactly one element.
Subcase 1.a.2.A. $M \cap N = \varphi$

Then $u_1 \neq v_1, v_2$ and $u_2 \neq v_1, v_2$. We consider the following three possibilities.

Subcase 1.a.2.A.1. Both v_1 and v_2 lie on $u_1 - u_2$ path
Then there exists an $v_1 - v_2$ path, the edges v_2v and vv_1 form a cycle $C2$ and the edges uu_1, the path $u_1 - v_1$, the edges v_1v and vv_2, the path $v_2 - u_2$, the edge u_2u form cycle in B^σ which is different from $C1$ and $C2$, which contradicts B^σ is two-cyclic.

Subcase 1.a.2.A.2. Either v_1 or v_2 lies on $u_1 - u_2$ path
The edge uu_1, the path $u_1 - v_1$, the edges v_1v and vv_2, the path $v_2 - u_2$ and the edge u_2u form a cycle $C4$ in B^σ different from $C1$ and $C2$ giving a contradiction to B^σ is two-cyclic.

Subcase 1.a.2.A.3. Both v_1 and v_2 do not lie on $u_1 - u_2$ path
Since $B - \sigma$ is acyclic, the $v_1 - v_2$ path has either no vertex or at least one vertex of $u_1 - u_2$ path.

Subcase 1.a.2.A.3.a. The $v_1 - v_2$ path has no vertex or exactly one vertex of $u_1 - u_2$ path

If the path $v_1 - v_2$ has no vertex of $u_1 - u_2$ path, then the edges v_1v, vv_2 and the path $v_2 - v_1$ form a cycle $C2$ in B^σ different from $C1$. If the path $v_1 - v_2$ has exactly one vertex of $u_1 - u_2$ path, that is, the two paths are edge disjoint, then no other cycles are formed other than $C2$ in B^σ, which is different from $C1$. Hence, B^σ is a two-cyclic graph.

Subcase 1.a.2.A.3.b. The $v_1 - v_2$ path has at least two vertices of $u_1 - u_2$ path

Then the path $v_1 - v_2$, the edges v_2v and vv_1 forms a cycle $C2$ in B^σ and the edges u_1u, uu_2, the path $u_2 - v_2$, the edges v_2v, vv_1, the path $v_1 - u_1$ form another cycle in B^σ which is different from $C2$ and $C1$ giving a contradiction to B^σ is two-cyclic.

Subcase 1.a.2.B. $M \cap N$ has exactly one element
Then $\{u_1, u_2\} \cap \{v_1, v_2\}$ has exactly one element. Without loss of generality, let $u_1 = v_1$ and $u_2 \neq v_2$. Then we have two possibilities according as $u_1 - u_2$ path contains v_2 or does not contain v_2.

Subcase 1.a.2.B.1. The $u_1 - u_2$ path contains v_2
Then the $v_1 - v_2$ path in B^σ and the edges v_2v and $vu_1 = vv_1$ form a cycle $C2$ and the edges u_1u, uu_2, the path $u_2 - v_2$, the edges v_2v and $vv_1 = vu_1$ form a cycle in B^σ different from $C1$ and $C2$ giving a contradiction to B^σ is two-cyclic.

Subcase 1.a.2.B.2. The $u_1 - u_2$ path does not contain v_2
The $v_1 - v_2$ path has either no vertex or at least one vertex of $u_1 - u_2$ path other than v_1.

Subcase 1.a.2.B.2.a. The $v_1 - v_2$ path has no vertex of $u_1 - u_2$ path other than v_1

Then the edges $v_1v = u_1v, vv_2$ and the path $v_2 - v_1$ form a cycle $C2$ in B^σ, different from $C1$. Hence, B^σ is a two-cyclic graph.

Subcase 1.a.2.B.2.b. The $v_1 - v_2$ path has at least one vertex of $u_1 - u_2$ path other than v_1

Then the $v_1 - v_2$ path and the edges v_2v and $vv_1 = vu_1$ form a cycle $C2$ in B^σ and the edges $uu_1, u_1v = v_1v, vv_2$, the path $v_2 - u_2$ and the edge u_2u form another cycle in B^σ different from $C1$ and $C2$ giving a contradiction to B^σ is two-cyclic.

Thus the path formed by elements of M and by elements of N have either at most one vertex in common for $M \cap N = \varphi$ or the vertex say a in common for $M \cap N = \{a\}$.

Subcase 1.b. $d_B(u) = |V(B)| - 3$
Since $uv \notin G$, there exists exactly one vertex of M, say u_1, in $V(B) - \sigma$, which is adjacent to u in B^σ. Hence, uu_1 is an edge in B^σ.

If $d_B(v) = |V(B)| - 3$, then there exists exactly one vertex of N, say v_1, in $V(B) - \sigma$, which is adjacent to v in B^σ. Hence, vv_1 is an edge in B^σ. This shows that the edges vv_1

and uu_1 do not form any cycle in B^σ and hence B^σ is acyclic, which is a contradiction to B^σ is two-cyclic.

If $d_B(v) = |V(B)| - 4$, then there exist two vertices of N, say v_1 and v_2 in $V(B) - \sigma$ which are adjacent to v in B^σ. Since $B - \sigma$ is connected, there exists an $v_1 - v_2$ path in B^σ. If $u_1 = v_1$, then the edges $v_1v = u_1v, vv_2$ and the path $v_2 - v_1$ form a cycle $C1$ in B^σ. If $u_1 \neq v_1$, then the edges v_1v, vv_2 and the path $v_2 - v_1$ form a cycle $C2$ in B^σ. In both cases, we get a unique cycle in B^σ, which is a contradiction to B^σ is two-cyclic.

Case 2. $B - \sigma$ is unicyclic
Let C be the unique cycle of $B - \sigma$ in G. Then C is also a cycle of $B - \sigma$ in G^σ. If $d_B(u) < |V(B)| - 4$, then there exist at least three vertices of M, say u_1, u_2 and u_3, in $V(B) - \sigma$ which are adjacent to u in B^σ. Clearly, $M \supseteq \{u_1, u_2, u_3\}$. Since $B - \sigma$ is connected, there exist $u_1 - u_2, u_2 - u_3$ and $u_1 - u_3$ paths in $B - \sigma$ and hence in B^σ. Now the edges u_1u, u_2u and u_3u and the paths $u_1 - u_2, u_2 - u_3$ and $u_1 - u_3$ form at least three cycles in B^σ in addition to C which is a contradiction to B^σ is two-cyclic. Hence either $d_B(u) = |V(B)| - 4$ or $d_B(u) = |V(B)| - 3$.

Subcase 2.a. $d_B(u) = |V(B)| - 4$

Since $uv \notin E(G)$, there exist two vertices of M, say u_1, u_2 in $V(B) - \sigma$. Clearly, $M = \{u_1, u_2\}$. We consider the following three subcases.

Subcase 2.a.1. u_1 and u_2 do not lie on cycle C
Since $B - \sigma$ is connected, there exists either one or two $u_1 - u_2$ paths in $B - \sigma$ according as at most one vertex or at least two vertices of C lie on the path and hence in B^σ also.

If $B - \sigma$ contains only one $u_1 - u_2$ path, say P_1, then the edge uu_1, path P_1 and the edge u_2u form another cycle C_1 in B^σ. Hence, B^σ is two-cyclic.

If $B - \sigma$ contains two $u_1 - u_2$ paths, say P_2 and P_3, then the edge uu_1, path P_2 and the edge u_2u form a cycle $C2$ and the edge uu_1, path P_3 and the edge u_2u form a cycle $C3$ different from $C2$ in B^σ. Hence, B^σ has at least 3 cycles, which is a contradiction to B^σ is two-cyclic.

Subcase 2.a.2. u_1 and u_2 lie on the cycle C
Then the cycle C gives two $u_1 - u_2$ paths in $B - \sigma$ and hence by subcase 2.a.1, B^σ has at least 3 cycles, which is a contradiction to B^σ is two-cyclic.

Subcase 2.a.3. Either u_1 or u_2 lies on the cycle C
Let us assume that u_1 lies on C. Since B^σ is connected, there exists either one $u_1 - u_2$ path or two $u_1 - u_2$ paths according as either the $u_1 - u_2$ path has no vertex or a vertex of C other than u_1.

If $B - \sigma$ contains only one $u_1 - u_2$ path, then the path contains no vertex of the cycle C. Now the edge uu_1, the $u_1 - u_2$ path and the edge u_2u form another cycle $C1$ in B^σ different from C. Hence, B^σ is two-cyclic.

If $B-\sigma$ contains two u_1-u_2 paths, then as in subcase 2.a.1, B^σ has at least 3 cycles, which is a contradiction.

If $d_B(v)<|V(B)|-3$, then there exist at least two vertices of N, say v_1 and v_2, in $V(B)-\sigma$ which are adjacent to v in B^σ. If $\{u_1,u_2\}=\{v_1,v_2\}$, then the edge $vv_1=vu_1$, the u_1-u_2 path and the edge $v_2v=u_2v$ form a cycle $C2$ different from $C1$. And if $\{u_1,u_2\}\neq\{v_1,v_2\}$, then $\{u_1,u_2\}\cap\{v_1,v_2\}$ is either φ or has exactly one element. If $\{u_1,u_2\}\cap\{v_1,v_2\}=\varphi$, then $u_1\neq v_1,v_2$ and $u_2\neq v_1,v_2$. Clearly, we have the u_1-u_2 path contains no vertex or one vertex or both vertices of $\{v_1,v_2\}$. In all possibilities, the edges v_2v,vv_1 and the v_1-v_2 path form a cycle $C2$ in B^σ. If $\{u_1,u_2\}\cap\{v_1,v_2\}\neq\varphi$, then without loss of generality, let $u_1=v_1,u_2\neq v_2$. Then we have two possibilities according as u_1-u_2 path contains v_2 or does not contain v_2. In both cases, the edges v_2v,vv_1 and the v_1-v_2 path form a cycle $C2$ in B^σ.

Hence in all the possibilities, we get a cycle $C2$, which is different from C and $C1$ in B^σ which is a contradiction to B^σ is two-cyclic. Hence, $d_B(v)=|V(B)|-3$.

Hence either the elements of M do not lie on the cycle C and at most one vertex of the cycle lies on the unique u_1-u_2 path or one of the elements of M, say a, lies on C and the unique u_1-u_2 path does not contain any vertex of C other than a for which $|M-\{v\}|=2$ and $|N-\{u\}|=1$.

Subcase 2.b. $d_B(u)=|V(B)|-3$
Since $uv\notin E(G)$, there is only one vertex of M, say u_1, in $V(B)-\sigma$ and hence uu_1 is an edge in B^σ.

If $d_B(v)=|V(B)|-3$, then there exist only one vertex of N, say v_1, in $V(B)-\sigma$ and hence vv_1 is an edge in B^σ. Since $B-\sigma$ is unicyclic and $uv\notin E(G),vv_1$ and u_1u do not form a cycle in B^σ and hence we have B^σ is unicyclic which is contradiction to B^σ is two-cyclic.

If $d_B(v)=|V(B)|-4$, then from subcases 2.a.1 to 2.a. 3 of subcase 2.a, we get either the elements of N, say a, do not lie on the cycle C and at most one vertex of the cycle lies on the unique v_1-v_2 path or one of the element of N lies on C and the unique v_1-v_2 path does not contain any vertex of C other than a for which $|M-\{v\}|=1$ and $|N-\{u\}|=2$.

Case 3. $B-\sigma$ is two-cyclic
Let $C1$ and $C2$ be the cycles in $B-\sigma$ in G. Then $C1$ and $C2$ are also the cycles of B^σ in G^σ. We have $0<d_B(u)\leq|V(B)|-3$ and $0<d_B(v)\leq|V(B)|-3$ in G. If $d_B(u)<|V(B)|-3$, then there exist at least two vertices of M, say u_1 and u_2, in $V(B)-\sigma$. Now, $uv\notin E(G)$ shows that u is adjacent to u_1 and u_1 in B^σ. Since $B-\sigma$ is connected, there is an u_1-u_2 path in B and hence in B^σ. Now the edges u_1u,u_2u and the u_1-u_2 path form a cycle $C3$ in B^σ different from $C1$ and $C2$, which is a contradiction to B^σ is two-cyclic. Hence, $d_B(u)=|V(B)|-3$. Similarly, $d_B(v)=|V(B)|-3$.

Conversely, let either (1) or (2) or (3) in the statement hold.
Case A. $B-\sigma$ is connected, acyclic, $d_B(u)=d_B(v)=|V(B)|-4$ and the path formed by elements of M and by elements of N have either at most one vertex in common for $M\cap N=\varphi$ or the vertex a in common for $M\cap N=\{a\}$

By Theorem 2.1, B^σ is connected since $d_B(u)=d_B(v)=|V(B)|-4$. Since $uv\notin E(G)$ and $d_B(u)=d_B(v)=|V(B)|-4$, let $M=\{u_1,u_2\}$ and $N=\{v_1,v_2\}$. Now the edges u_1u, uu_2 and u_2-u_1 path form a cycle C in B^σ and the edges vv_1, vv_2 and v_2-v_1 path form another cycle $C1$ in B^σ.

For $M\cap N=\varphi$, the paths u_1-u_2 and v_1-v_2 have at most one vertex in common and hence B^σ has only two cycles C and $C1$. The branches B^σ with minimum number of vertices is given in figure 1.1.

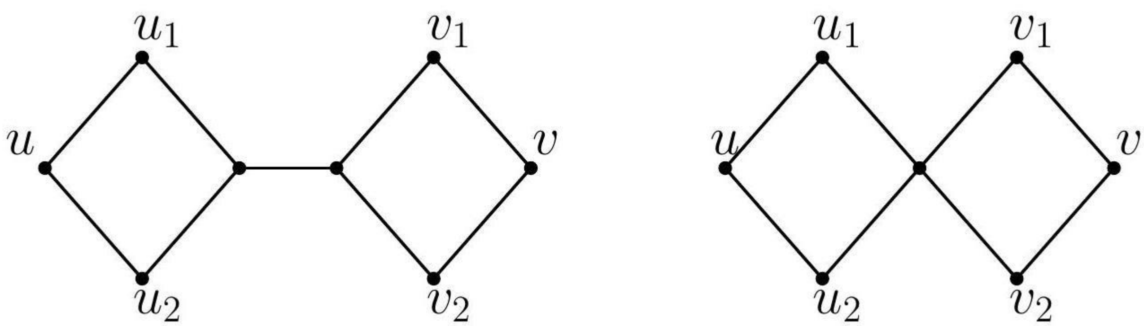

Fig 1.1: B^σ

For $M\cap N=\{a\}$, the paths u_1-u_2 and v_1-v_2 have the vertex a in common. Let a be $u_1=v_1$. In this case, B^σ has only two cycles C and $C1$. The branch B^σ with minimum number of vertices is given in figure 1.2.

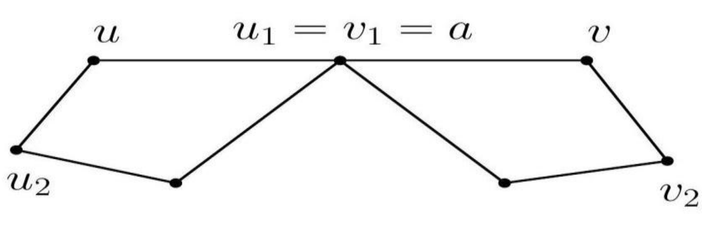

Fig 1.2 : B^σ

Case B. $B-\sigma$ is connected, unicyclic, $\{d_B(u),d_B(v)\}=\{|V(B)|-4,|V(B)|-3\}$ and for $|M-\{v\}|=2$ and $|N-\{u\}|=1(|M-\{v\}|=1$ and $|N-\{u\}|=2)$ either the elements of $M(N)$ do not lie on the cycle of $B-\sigma$ and the unique path connecting them contains at most one vertex of the cycle or one of the elements of $M(N)$, say a, lies on the cycle and the unique path connecting them contains no vertex of the cycle other than a

Without loss of generality, let $d_B(u) = |V(B)| - 4$ and $d_B(v) = |V(B)| - 3$. By Theorem 2.1, B^σ is connected. Let C be the unique cycle in $B - \sigma$. Since $uv \notin E(G)$, let $M = \{u_1, u_2\}$ and $N = \{v_1\}$. Clearly, uu_1, uu_2 and vv_1 are edges in B^σ. Since B^σ is connected, there exists an $u_1 - u_2$ path in B^σ. Now the edges uu_1, uu_2 and the path $u_1 - u_2$ form a cycle $C1$ in B^σ.

If the elements u_1 and u_2 of M do not lie on the cycle C of $B - \sigma$ and the unique path connecting them contains at most one vertex of the cycle C, then we cannot have any cycle other than C and $C1$ in B^σ implies that B^σ is two-cyclic. The branches B^σ with minimum number of vertices is given in figure 1.3.

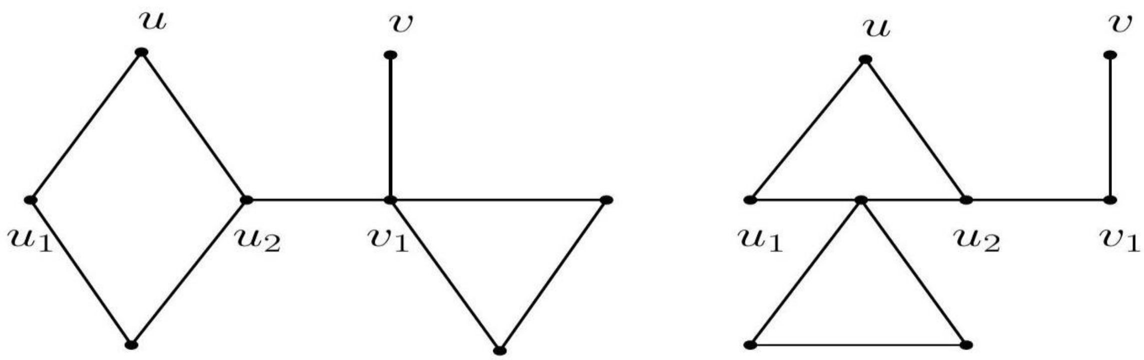

Fig 1.3 : B^σ

If one of the elements of M, say u_2, lies on the cycle C and the unique path connecting u_1 and u_2 contains no vertex of the cycle C other than u_2, then we cannot have any cycle other than C and $C1$ in B^σ implies that B^σ is two-cyclic. The branch B^σ with minimum number of vertices is given in figure 1.4

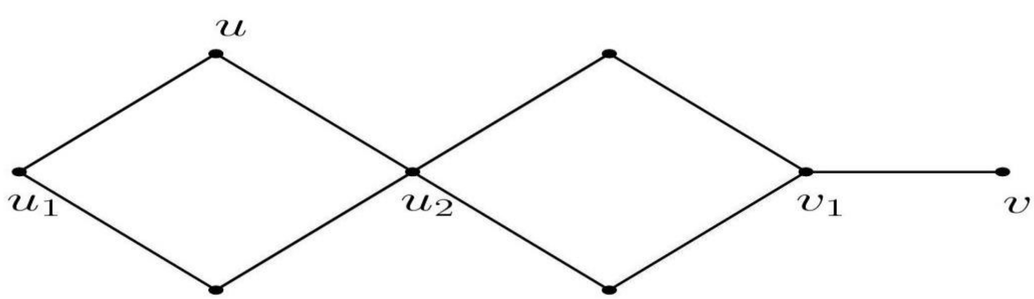

Fig 1.4: B^σ

Case C. $B - \sigma$ is connected, two-cyclic and $d_B(u) = d_B(v) = |V(B)| - 3$

Since $B - \sigma$ is two-cyclic, $|V(B)| \geq 5$ and hence by Theorem 2.1, B^σ is connected. Since $d_B(u) = |V(B)| - 3$, there exist exactly one vertex of M, say u_1, of $V(B) - \sigma$ in B such that uu_1 is an edge in B^σ. In the same way, $d_B(v) = |V(B)| - 3$ implies that there exists exactly one vertex, say v_1, of $V(B) - \sigma$ in B such that vv_1 is an edge in B^σ. Now $B - \sigma$ is two-cyclic and $uv \notin E(G)$, the addition of the edges uu_1 and vv_1 for both $u_1 = v_1$ and $u_1 \neq v_1$ do not form any other cycle in B^σ. Hence, B^σ is two-cyclic.

4 Conclusion

In this article, we came up with necessary and sufficient requirements for G^σ, the switching of G at $\sigma = \{u, v\}$ to be connected and two cyclic graph when $uv \notin E(G)$.

5 Application

2-Vertex Switching is a technique used in molecular biology to simulate and examine molecular structures, including networks of interactions between proteins. In social network analysis, 2-Vertex Switching can be applied to study the evolution of social networks, identifying key players and community structures. By identifying important people and community structures, 2-Vertex Switching can be used in social network analysis to examine how social networks have changed over time.

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