J. COMPUTATIONAL ANALYSIS AND APPLICATIONS, VOL. 29, NO.4, 2021, COPYRIGHT 2021 EUDOXUS PRESS, LLC

Algebras and Smarandache Types

Jung Mi Ko[1] **and Sun Shin Ahn**[2,*]

[1] ***Department of Mathematics, Gangneung-Wonju National University, Gangneung 25457, Korea***

[2] ***Department of Mathematics Education, Dongguk University, Seoul 04620, Korea***

Abstract. In this paper we introduce the notion of a BQ-algebra and show that is is equivalent to an abelian group. For deep investigations of several algebraic structures, we introduce the notions of a Smarandache V-algebra-type U-algebra and a Smarandache V-algebra-trans-type U-algebra, and apply the notions to several algebras.

1. Introduction

W. B. Vasantha Kandasamy ([8]) studied the concept of Smarandache groupoids, ideals of groupoids, Smarandache Bol groupoids and strong Bol groupoids, and obtained many interesting results about them. Smarandace semigroups are very important for the study of congruences, and it was studied by R. Padilla ([18]). It will be very interesting to study the Smarandache structure in general algebraic structures. Kim et al. ([11]) defined the concept of a Smarndache d-algebra and investigated some related properties of it. Seo et al. ([19]) introduced the concept of a Smarndache fuzzy BCI-algebra and investigated some related properties of it. Neggers et al. ([17]) defined the notion of a B-algebra and investigated some related properties of it. Some properties of B-algebra are studied in ([3, 12, 13]).

In this paper, we introduce the notion of a BQ-algebras and show that it is equivalent to an abelian group. Moreover, we introduce the notions of a Smarandache V-algebra-type U-algebra and a Smarandache V-algebra-trans-type U-algebra, and apply the notions to several algebras.

2. Preliminaries

A *B-algebra* ([17]) is a non-empty set X with a selected point 0 and a binary operation "$*$" satisfying the following axioms: (i) $x*x=0$, (ii) $x*0=x$, (iii) $(x*y)*z=x*(z*(0*y))$ for any $x,y,z\in X$. A B-algebra $(X,*,0)$ is said to be *0-commutative* ([2]) if $x*(0*y)=y*(0*x)$ for any $x,y\in X$. Let $(X,*,0)$ be a B-algebra and let $g\in X$. We define $g^{[0]}:=0, g^{[1]}:=g^{[0]}*(0*g)=0*(0*g)=g$ and $g^{[n]}:=g^{[n-1]}*(0*g)$ where $n\geq 1$.

---

[0] **2010 Mathematics Subject Classification**: 06F35; 03G25.

[0] **Keywords**: Smarandache algebra, point algebra, p-derived algebra.

* The corresponding author. Tel: +82 2 2260 3410, Fax: +82 2 2266 3409

[0] **E-mail**: jmko@gwnu.ac.kr (J. M. Ko); sunshine@dongguk.edu (S. S. Ahn)

Theorem 2.1. *Let $(X, *, 0)$ be a B-algebra and let $g \in X$. Then*

$$g^{[m]} * g^{[n]} = \begin{cases} g^{[m-n]} & \text{if } m \geq n, \\ 0 * g^{[n-m]} & \text{otherwise.} \end{cases}$$

Theorem 2.2. ([10]) *Every 0-commutative B-algebra is a BCI-algebra.*

Theorem 2.3. ([10]) *The following are equivalent:*

(i) *X is an abelian group,*
(ii) *X is a p-semisimple BCI-algebra,*
(iii) *X is a 0-commutative B-algebra.*

Let $(X, *, 0)$ be a B-algebra. Given $x, y \in X$, we define $x *^{\langle 1 \rangle} y := x * y, x *^{\langle 2 \rangle} y := (x * y) * y, x *^{\langle n \rangle} y := (x *^{\langle n-1 \rangle} y) * y$ where $n \geq 3$. For general references for BCK/BCI-algebras, we refer to [5, 6, 14].

3. Several algebras

Let $(X, *)$ be a groupoid (or a binary system, an algebra), i.e., X is a set and "$*$" is a binary operation on X. If we take an element p in X which plays an important role in $(X, *)$, then we say that p is a *selected point* and we write it by $(X, *, p)$. Such an algebra $(X, *, p)$ is said to be a *pointed algebra*.

Example 3.1. *Let $(X, *)$ be a group with identity e. The identity element e plays an important role in $(X, *)$ and hence we may write it by $(X, *, e)$ and e becomes a selected point in $(X, *)$.*

We regard all algebras below as pointed algebras without loss of generality. For simplicity's sake, we shall write $p = 0$, not intending 0 to have the usual meaning. Thus, in Example 3.1, $(X, *, e)$ becomes $(X, *, 0)$ unless it is important to distinguish the algebra $(X, *, 0)$ which contains the subalgebra (not necessary a subgroup) $(Y, *)$ with its selected point p to produce $(Y, *, p)$.

Example 3.2. Consider $X := \{a, b, c, d\}$ with the following table:

*	a	b	c	d
a	a	a	a	a
b	a	a	a	b
c	a	b	c	c
d	a	b	c	d

Then $(X, *, d)$ is an pointed algebra and the selected point d is the right identity. Consider $Y := \{a, c\}$ and $Z := \{a, d\}$ with the following tables:

$*$	a	c
a	a	a
c	a	c

$*$	a	d
a	a	a
d	a	d

Then $(Y, *, c)$ is a pointed algebra with a selected point c is the right identity, and $(Z, *, d)$ is also a pointed algebra with a special point d is the left identity.

Definition 3.3. Let $(X, *, p)$ be a pointed algebra. Define a binary operation "$\bullet$" on X by

$$x \bullet y := x * (p * y)$$

for any $x, y \in X$. Then the algebra $(X, \bullet, p)$ is called a *p-derived algebra from* $(X, *, p)$.

Example 3.4. (i) Let $(X, *, e)$ be a group with identity e. If $(X, \bullet, e)$ is an e-derived algebra of $(X, *, e)$, then $(X, \bullet) = (X, *)$, since e is the identity, we have $x \bullet y = x * (e * y) = x * y$ for all $x, y \in X$.
(ii) Let $(X, *, p)$ be a left-zero-semigroup with a selected point p. If $(X, \bullet, p)$ is a p-derived algebra of $(X, *, p)$, then $(X, *) = (X, \bullet)$.

Let X be a d-algebra and $x \in X$. Define $x * X := \{x * a | a \in X\}$. X is said to be *edge* ([16]) if for any $x \in X$, $x * X = \{x, 0\}$.

Lemma 3.5. ([16]) *Let X be an edge d-algebra. Then*

(i) $x * 0 = x$ *for all* $x \in X$.
(ii) $(x * (x * y)) * y = 0$ *for all* $x, y \in X$.

Example 3.6. (i) Let $(X, *, 0)$ be an edge d-algebra. If $(X, \bullet, 0)$ is an e-derived algebra of $(X, *, 0)$, then $(X, \bullet)$ is a left-zero-semigroup.
(ii) Let $(X, *, 0)$ be a BCK-algebra. If $(X, \bullet, 0)$ is an e-derived algebra of $(X, *, 0)$, then $(X, \bullet)$ is a left-zero-semigroup. In fact, $x \bullet y = x * (0 * y) = x * 0 = x$ for all $x, y \in X$.

In terms of list of axioms to be used to describe the various algebra types we note the following section of axioms:

(1) $x * x = 0$ for all $x \in X$.
(2) $x * 0 = x$ for all $x \in X$.
(3) $0 * x = x$ for all $x \in X$.
(4) $x * y = y * x$ for all $x, y \in X$.
(5) $x * y = y * x = 0 \Leftrightarrow x = y$ for all $x, y \in X$.
(6) $x * y = y * x = 0 \Rightarrow x = y$ for all $x, y \in X$.
(7) $x * y = y * x \Rightarrow x = y$ for all $x, y \in X$.
(8) $0 * x = 0$ for all $x \in X$.

(9) $(x * y) * z = (x * z) * y$ for all $x, y, z \in X$.
(10) $(x * y) * z = x * (z * y)$ for all $x, y, z \in X$.
(11) $(x * y) * z = x * (z * (0 * y))$ for all $x, y, z \in X$.
(12) $(x * y) * z = (x * z) * (y * z)$ for all $x, y, z \in X$.
(13) $(x * y) * (0 * y) = x$ for all $x, y \in X$.
(14) $x * (y * z) = (x * y) * z$ for all $x, y, z \in X$.
(15) $(x * (x * y)) * y = 0$ for all $x, y \in X$.
(16) $((x * y) * (x * z)) * (x * y) = 0$ for all $x, y, z \in X$.
(17) for any $x \in X$, there exists $y \in X$ with $x * y = 0$.
(18) for any $x \in X$, there exists $y \in X$ with $y * x = 0$.

An algebra $(X, *, 0)$ is called a *group* if it satisfies (2),(3), (14), (17), and (18). An algebra $(X, *)$ is called a *semigroup* if it satisfies (14). An algebra $(X, *, 0)$ is called a *semigroup with identity* if it satisfies (2),(3), and (14). An algebra $(X, *, 0)$ is called a *B-algebra* ([17]) if it satisfies (1),(2), and (11). An algebra $(X, *, 0)$ is called a *BG-algebra* ([9]) if it satisfies (1),(2), and (13). An algebra $(X, *, 0)$ is called a *BH-algebra* ([7]) if it satisfies (1), (2), and (6). An algebra $(X, *, 0)$ is called a *Q-algebra* ([15]) if it satisfies (1), (2), and (9). An algebra $(X, *, 0)$ is called a *d-algebra* ([16]) if it satisfies (1), (5), and (8). An algebra $(X, *, 0)$ is called a *BCK-algebra* ([14]) if it satisfies (1), (5), (8), (15), and (16). An algebra $(X, *, 0)$ is called a *$gBCK$-algebra* ([4]) if it satisfies (1), (2), (9) and (12). An algebra $(X, *, 0)$ is called an *abelian group* if it satisfies (2), (3), (4), (14), (17), and (18). An algebra $(X, *, 0)$ is called a *commutative semigroup* if it satisfies (4) and (14).

4. BQ-algebras

In this section, we introduce the notion of a BQ-algebra and we show that it is equivalent to an abelian group. An algebra $(X, *, 0)$ is said to be a *BQ-algebra* if it satisfies the conditions (1), (2), (9) and (11).

Theorem 4.1. *Let $(X, *, 0)$ be a BQ-algebra. If we define $x \bullet y := x * (0 * y)$ for any $x, y \in X$, then $(X, \bullet, 0)$ is an abelian group.*

Proof. Since $(X, *, 0)$ is a BQ-algebra, it is both a B-algebra and a Q-algebra. It was proved that if $(X, *, 0)$ is a B-algebra, then $(X, \bullet, 0)$ is a group ([1]). By (9) we obtain $(x * (0 * y)) * (0 * z) = (x * (0 * z)) * (0 * y)$ for any $x, y, z \in X$. It follows that $(x \bullet y) \bullet z = (x \bullet z) \bullet y$ for any $x, y, z \in X$. If we take $x := 0$, then $(0 \bullet y) \bullet z = (0 \bullet z) \bullet y$. Since $(X, *, 0)$ is a B-algebra, we have $0 \bullet y = 0 * (0 * y) = y$ for any $y \in X$. Hence we obtain $y \bullet z = z \bullet y$ for any $y, z \in X$. This proves that $(X, \bullet, 0)$ is an abelian group. □

Theorem 4.2. *Let $(X, \bullet, 0)$ be an abelian group. If we define $x * y := x \bullet y^{-1}$ for any $x, y \in X$, then $(X, *, 0)$ is a BQ-algebra.*

Proof. (1) For any $x \in X$, we have $x*x = x \bullet x^{-1} = 0$. (2) For any $x \in X$, $x*0 = x \bullet 0^{-1} = x \bullet 0 = x$. (9) Given $x, y, z \in X$, since $(X, \bullet, 0)$ is a group, we obtain

$$\begin{aligned}(x * y) * z &= (x \bullet y^{-1}) \bullet z^{-1} \\ &= (x \bullet z^{-1}) \bullet y^{-1} \\ &= (x * z) * y.\end{aligned}$$

(11) Given $x, y, z \in X$, since $(X, \bullet, 0)$ is a group, we have

$$\begin{aligned}x * (z * (0 * y)) &= x \bullet [z \bullet [(y^{-1})^{-1} \bullet 0^{-1}]]^{-1} \\ &= x \bullet [z \bullet (y \bullet 0)]^{-1} \\ &= x \bullet (z \bullet y)^{-1} \\ &= x \bullet (y^{-1} \bullet z^{-1}).\end{aligned}$$

Similarly, we prove that $(x * y) * z = (x \bullet y^{-1}) \bullet z^{-1}$. Since $(X, \bullet, 0)$ is a group, we obtain $(x * y) * z = x * (z * (0 * y))$. Hence $(X, *, 0)$ is a BQ-algebra. □

By Theorems 4.1 and 4.2, we conclude that *the class of all BQ-algebras is equivalent to the class of all abelian groups.*

The interesting fact to note is that we are able to take advantage of the relationship $x \bullet y = x * (0 * y)$ to understand better what the meaning of the class of BQ-algebra is. Other such questions around in this setting as well as others. E.q., what class of B-algebras corresponds to the class of solvable groups ? Can it be considered to be of the form: B"V"-algebras corresponds to solvable groups where "V"-algebras is some nicely identifiable class, as the same as the class for BQ-algebras ?

5. Smarandache types

Let $(X, *)$ be an U-algebra. Then $(X, *)$ is said to be a *Smarandache V-algebra-type U-algebra* if there exists $Y \subseteq X$ such that $(Y, *)$ is a non-trivial subalgebra of $(X, *)$ and $|Y| \geq 2$, and $(Y, *)$ is a V-algebra. For example, a B-algebra $(X, *, 0)$ is said to be a *Smarandache Q-algebra-type B-algebra* it it contains a non-trivial sub-B-algebra $(Y, *, 0)$ of $(X, *, 0)$ and $|Y| \geq 2$, and $(Y, *, 0)$ is a Q-algebra. Similarly, a Q-algebra $(X, *, 0)$ is called a *Smarandache group-type Q-algebra* if it contains a non-trivial sub-Q-algebra $(Y, *, 0)$ of $(X, *, 0)$, and $(Y, *, 0)$ is a group where $|Y| \geq 2$.

Theorem 5.1. *There is no Smarandache d-algebra-type commutative groupoid.*

Proof. Assume that there is a Smarandache d-algebra-type commutative groupoid $(X, *, 0)$. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial subgroupoid of a commutative groupoid $(X, *, 0)$, $|Y| \geq 2$ and $(Y, *, 0)$ is a d-algebra. It follows that $0 * y = 0$ for all $y \in Y$. Since $(X, *, 0)$

is a commutative groupoid and $Y \subseteq X$, we obtain $0 * y = y * 0 = 0$ for all $y \in Y$. Since $(X, *, 0)$ is a d-algebra and $Y \subseteq X$, we obtain $y = 0$, i.e., $|Y| = 1$, a contradiction. $\square$

Theorem 5.2. *There is no Smarandache semigroup-type d-algebra.*

Proof. Assume that there is a Smarandache semigroup-type d-algebra $(X, *, 0)$. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial subalgebra of a d-algebra $(X, *, 0)$, $|Y| \geq 2$ and $(Y, *, 0)$ is a semigroup. It follows that $0 * (y * 0) = 0$ for any $y \in Y$, since $Y \subset X$ and $(X, *, 0)$ is a d-algebra. Hence

$$\begin{aligned} y * 0 &= y * (0 * (y * 0)) \\ &= (y * 0) * (y * 0) \\ &= 0. \end{aligned}$$

Since $Y \subseteq X$ and $(X, *, 0)$ is a d-algebra, we obtain $0 * y = 0$ for all $y \in Y$. By (6), we have $y = 0$, i.e., $|Y| = 1$, a contradiction. $\square$

Theorem 5.3. *A Smarandache group-type B-algebra is equal to a Smarandache Boolean-group-type B-algebra.*

Proof. Since every Boolean group is a group, it is enough to show that every Smarandache group-type B-algebra is a Smarandache Boolean-group-type B-algebra. Assume $(X, *, 0)$ is a Smarandache group-type B-algebra. Then there exists $Y \subseteq X$ such that $|Y| \geq 2$, $(Y, *, 0)$ is a non-trivial subalgebra of a B-algebra and $(Y, *, 0)$ is a group. For any $y \in Y$, since $Y \subseteq X$ and $(X, *, 0)$ is a B-algebra, we obtain $y * y = 0$. Since $(Y, *)$ is a group, the order of y is 2 in the group $(Y, *)$ for any $y \neq 0$ in Y and hence $(Y, *, 0)$ is a Boolean group, proving the theorem. $\square$

Corollary 5.4. *A Smarandache group-type Q-algebra is equal to a Smarandache Boolean-group-type Q-algebra.*

Proof. Every Q-algebra has also the condition (1), and the proof is similar to the proof of Theorem 5.3. $\square$

Theorem 5.5. *Every Smarandache B-algebra-type group is a Smarandache Boolean-group-type group.*

Proof. Let $(X, *, 0)$ be a Smarandache B-algebra-type group. Then there exists $Y \subseteq X$ such that $|Y| \geq 2$, $(Y, *, 0)$ is a non-trivial subgroup of a group $(X, *, 0)$ and $(Y, *, 0)$ is a B-algebra. It follows that $y * y = 0$ for all $y \in Y$. Since $Y \subseteq X$ and $(X, *, 0)$ is a group, we obtain $y = y^{-1}$ in the group. Hence $x * y^{-1} = x * y \in Y$, which shows that $(Y, *)$ is a subgroup of $(X, *)$ and the order of y is 2. Thus $(Y, *)$ is a Boolean group. This proves that $(X, *, 0)$ is a Smarandache Boolean-group-type group. $\square$

J. COMPUTATIONAL ANALYSIS AND APPLICATIONS, VOL. 29, NO.4, 2021, COPYRIGHT 2021 EUDOXUS PRESS, LLC

Theorem 5.6. *Let $(X, *, 0)$ be a Smarandache L-algebra-type M-algebra. If every L-algebra is an N-algebra, then $(X, *, 0)$ is a Smarandache N-algebra-type M-algebra.*

Proof. It is easy and omit the proof. □

Theorem 5.7. *Let $(X, *, 0)$ be a Smarandache 0-commutative-B-algebra-type M-algebra. Then $(X, *, 0)$ is a Smarandache BCI-algebra-type M-algebra, where M-algebra is any algebra.*

Proof. By applying Theorems 2.2 and 5.6, we prove the theorem. □

Theorem 5.8. *Let $(X, *, 0)$ be an M-algebra. Then the following are equivalent:*

(i) *X is a Smarandache abelian-group-type M-algebra*
(ii) *X is a Smarandache p-semisimple BCI-algebra-type M-algebra,*
(iii) *X is a Smarandache 0-commutative B-algebra-type M-algebra.*

Proof. It follows immediately from Theorems 2.3 and 5.6. □

Proposition 5.9. *If $(X, *, 0)$ is a Smarandache Q-algebra-type group, then it is a Smarandache Boolean-group-type group.*

Proof. Let $(X, *, 0)$ be a Smarandache Q-algebra-type group. Then there exists $Y \subseteq X$ such that $|Y| \geq 2$, $(Y, *, 0)$ is a non-trivial subgroup of a group $(X, *, 0)$ and $(Y, *, 0)$ is a Q-algebra. Since Y is a Q-algebra, we have $y * y = 0$ for any $y \in Y$. This means the order of y is 2 in the group $(Y, *)$, i.e., $y = y^{-1}$, which shows that $(Y, *, 0)$ is a Boolean-group. Hence $(X, *, 0)$ is a Smarandache Boolean-group-type group. □

Theorem 5.10. *Any non-trivial d-algebra cannot be a Smarandache group-type d-algebra.*

Proof. Assume there exists a Smarandache group-type d-algebra $(X, *, 0)$. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial sub-d-algebra of $(X, *, 0)$ and $(Y, *, 0)$ is a group where $|Y| \geq 2$. Since $(Y, *, 0)$ is a group and $(X, *, 0)$ is a d-algebra, we have $y = 0 * y = 0$ for all $y \in Y$. It follows that $|Y| = 1$, a contradiction. □

Theorem 5.11. *Any non-trivial group cannot be a Smarandache d-algebra-type group.*

Proof. Assume that there exists a Smarandache d-algebra-type group $(X, *, 0)$. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial subgroup of a group $(X, *, 0)$, and $(Y, *, 0)$ is a d-algebra and $|Y| \geq 2$. Then $0 * x = 0$ for all $x \in Y$. Since $(Y, *, 0)$ is a group, we obtain $x = 0$ for all $x \in Y$, proving that $|Y| = 1$, a contradiction. □

Theorem 5.12. *Any non-trivial $gBCK$-algebra cannot be a Smarandache group-type $gBCK$-algebra.*

Proof. Let $(X, *, 0)$ be a Smarandache group-type $gBCK$-algebra. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial sub-$gBCK$-algebra of $(X, *, 0)$, and $(Y, *, 0)$ is a group and $|Y| \geq 2$. Since $Y \subseteq X$ and $(X, *, 0)$ is a $gBCK$-algebra, we obtain $y * y = 0$ for all $y \in Y$. It follows from

$(Y, *, 0)$ is a group that the order of y is 2, i.e., $(Y, *, 0)$ is a Boolean group. Now, since $(Y, *, 0)$ is a $gBCK$-algebra, we have $(x * y) * z = (x * z) * (y * z)$ for all $x, y, z \in X$. It follows that $(x * x) * x = (x * x) * (x * x)$ for all $x \in X$. Since $(X, *, 0)$ is a group, we obtain $x = 0$ for all $x \in X$, proving that $|X| = 1$, a contradiction. □

Corollary 5.13. *Any non-trivial group cannot be a Smarandache $gBCK$-algebra-type group.*

Proof. The proof is similar to Theorem 5.12, and we omit it. □

Definition 5.14. Let $(X, *, p)$ be an L-algebra and let $(Y, *, p)$ be both a sub-L-algebra of $(X, *, p)$ and an M-algebra. $(X, *, p)$ is said to be a *Smarandache N-algebra-trans-type L-algebra* if $(Y, *, p)$ is isomorphic with an N-algebra $(Y, \odot, q)$.

$$\begin{tikzcd}
(X, *, p) \arrow[d, dash] & \\
(Y, *, p) \arrow[r, "\cong"] & (Y, \odot, q)
\end{tikzcd}$$

where $L-, M-, N-$ algebras are arbitrary algebras.

Theorem 5.15. *If $(X, *, 0)$ is a Smarandache B-algebra-type Q-algebra, then it is a Smarandache abelian-group-trans-type Q-algebra.*

Proof. Let $(X, *, 0)$ be a Smarandache B-algebra-type Q-algebra. Then there exists $Y \subseteq X$ such that $(Y, *, 0)$ is a non-trivial sub-Q-algebra of a Q-algebra $(X, *, 0)$, $|Y| \geq 2$ and $(Y, *, 0)$ is a B-algebra. Define $x \bullet y := x * (0 * y)$ for any $x, y \in Y$. Then $(Y, \bullet, 0)$ is an abelian group. In fact, since Y is both a Q-algebra and B-algebra, $(Y, *, 0)$ is a BQ-algebra. By Theorem 4.1, $(Y, \bullet, 0)$ is an abelian group. By Theorems 4.1 and 4.2, $(Y, *, 0) \cong (Y, \bullet, 0)$. This shows that $(X, *, 0)$ is a Smarandache abelian-group-trans-type Q-algebra. □

Corollary 5.16. *If $(X, *, 0)$ is a Smaradache Q-algebra-type B-algebra, then it is a Smarandache abelian-group-trans-type B-algebra.*

Proof. It is similar to Theorem 5.15. □

Proposition 5.17. *Every B-algebra is a Smarandache BQ-algebra-trans-type B-algebra.*

Proof. Let $(X, *, 0)$ be a B-algebra. Define $x \bullet y := x * (0 * y)$ for all $x, y \in X$. Then $(X, \bullet, 0)$ is a group. Let $x \in X$ such that $x \neq 0$. Let $\langle x \rangle$ be a cyclic group generated by x. Then $\langle x \rangle$ is a non-trivial abelian subgroup of $(X, \bullet, 0)$. If we let $Y_x := \{x *^{\langle n \rangle} (0 * x) \mid n \in \mathbf{Z}\}$, then $Y_x \cong \langle x \rangle$. By Theorems 4.1 and 4.2, Y_x is a non-trivial BQ-algebra. This shows that X is a Smarandache BQ-algebra-trans-type B-algebra. □

6. Conclusion

We introduced the notion of a BQ-algebra and proved that it is equivalent to an abelian group. For detailed investigations among several algebraic structures, we introduced the notions of a Smarandache V-type U-algebra and a Smarandache V-trans-type U-algebra, and applied this notions to several algebras. For further investigations, we will apply the notions of a hyper structure theory and several fuzzy related algebras to the notions of a Smarandache V-type U-algebra and a Smarandache V-trans-type U-algebra.

References

[1] P. J. Allen, J. Neggers and H. S. Kim, *B-algebras and groups*, Sci. Math. Japo. **59**(2004), 23-29.

[2] J. R. Cho and H. S. Kim, *On B-algebras and quasigroups*, Quasigroups and Related Systems **8**(2001), 1-6.

[3] J. S. Han and S. S. Ahn, *Quotient B-algebras induced by an int-soft normal subalgebras*, J. Comput. Anal. Appl. **26**(2019), no. 5, 791-801.

[4] S. M. Hong, Y. B. Jun and M. A. Ozturk, *Generalizations of BCK-algebras*, Sci. Math. Japo. **58**(2003), 603-611.

[5] Y. Huang, BCI-algebras, Science Press, Beijing, 2006.

[6] A. Iorgulescu, Algebras of logic as BCK-algebras, Editura ASE, Bucharest, 2008.

[7] Y. B. Jun, E. H. Roh and H. S. Kim, *On BH-algebras*, Sci. Mathematicae **1**(1998), 347-354.

[8] W. B. V. Kandasamy, *Smarandache groupoids*, http://www.gallwp.unm.edu/∼smarandache/Groupoids.pdf.

[9] C. B. Kim and H. S. Kim, *On BG-algebras*, Demonstratio Math. **41**(2008), 497-505.

[10] H. S. Kim and H. G. Park, *On 0-commutative B-algebras*, Sci. Math. Japo. **62**(2005), 31-36.

[11] Y. H. Kim, Y. H. Kim and S. S. Ahn, *Smarandache d-algebras*, Honam Math. J. **40**(2018), no.3, 539-548.

[12] J. M. Ko and S. S. Ahn, *On fuzzy B-algebras over t-norm*, J. Comput. Anal. Appl. **19**(2015), no. 6, 975-983.

[13] J. M. Ko and S. S. Ahn, *Hesitant fuzzy normal subalgebras in B-algebras*, J. Comput. Anal. Appl. **26**(2019), no. 6, 1084-1094.

[14] J. Meng and Y. B. Jun, BCK-algebras, Kyungmoon Sa, Seoul, 1994.

[15] J. Neggers, S. S. Ahn and H. S. Kim, *On Q-algebras*, Int. J. Math. & Math. Sci. **27**(2001), 749-757.

[16] J. Neggers, and H. S. Kim, *On d-algebras*, Math. Slovaca **49**(1999), 19-26.

[17] J. Neggers and H. S. Kim, *On B-algebras*, Mate. Vesnik **54**(2002), 21-29.

[18] R. Padilla, Smarandache algebric structures, Bull. Pure Appl. Sci., **1998**, 17E, 119-121.

[19] Y. J. Seo and S. S. Ahn, *Smarndache fuzzy BCI-algebras*, J. Comput. Anal. Appl. **24**(2018), no. 4, 619-627.