Exact Solitary Wave Solutions for Wick-type Stochastic (2+1)-dimensional Coupled KdV equations

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Abstract

Variable coefficients and Wick-type stochastic (2+1)-dimensional coupled KdV equations are investigated. By using the F-expansion method , Hermite transform and white noise theory, the white noise functional solutions for Wick-type stochastic (2+1)-dimensional coupled KdV equations are obtained. The exact travelling wave solutions are expressed in terms of Jacobi elliptic (JEF), trigonometric and hyperbolic functions.

Keywords: KdV equations; F-expansion method; Hermite transform; Wick product.
PACS No. : 05.40. ± a, 02.30.Jr.

1 Introduction

In this paper, we shall explore exact solutions for the following variable coefficients (2+1)-dimensional coupled KdV equations.

$$\begin{cases} u_t + \phi_1(t)uv_x + \phi_2(t)vu_x + \phi_3(t)u_{xxx} = 0, \\ u_x + v_y = 0, \end{cases} \tag{1.1}$$

where $(t, x) \in \mathbb{R}_+ \times \mathbb{R}$ and $\phi_1(t)$, $\phi_2(t)$ and $\phi_3(t)$ are bounded measurable or integrable functions on $\mathbb{R}_+$. Random wave is an important subject of stochastic partial differential equations (PDEs). Many authors have studied this subject. Wadati first introduced and studied the stochastic KdV equations and gave the diffusion of soliton of the KdV equation under Gaussian noise in [30, 32] and others [3, 4, 5, 25] also researched stochastic KdV-type equations. Xie first introduced Wick-type stochastic KdV equations on white noise space and showed the auto- Backlund transformation and the exact white noise functional solutions in [37]. Furthermore, Xie [38, 39, 40, 41], Ghany et al. [11, 12, 13, 15, 16, 17, 18, 19, 20] researched some Wick-type stochastic wave equations using white noise analysis.

In this paper we use F-expansion method for finding new periodic wave solutions of nonlinear evolution equations in mathematical physics, and we obtain some new periodic wave solutions for (2+1)-dimensional coupled KdV equations. This method is more powerful and will be used in further works to establish more entirely new solutions for other kinds of nonlinear partial differential equations arising in mathematical physics. The effort in finding exact solutions to nonlinear equations is important for the understanding of most nonlinear physical phenomena. For instance, the nonlinear wave phenomena observed in fluid dynamics, plasma, and optical fibers[24]. Many effective methods have been presented, such as tanh-function method [34, 42, 8], variational iteration method [6, 7], exp-function method [22, 23, 36, 43, 44] , homotopy perturbation method [10, 29, 35], homotopy analysis method [1], tanh-coth method [33, 34, 31], Jacobi elliptic function expansion method [27, 28, 9, 26] and F-expansion method [45, 46, 47, 48]. The main objective of this paper is using the F-expansion method to construct white noise functional solutions for wick-type stochastic (2+1)-dimensional coupled KdV equations via hermite transform, wick-type product and white noise analysis. If equation (1.1) is considered in a random environment, we can get stochastic (2+1)-dimensional coupled KdV equations. In order to give the exact solutions of stochastic (2+1)-dimensional coupled KdV equations, we only consider this problem in white noise environment. We shall study the following Wick-type stochastic (2+1)-dimensional coupled KdV equations.

$$\begin{cases} U_t + \Phi_1(t) \diamond U \diamond V_x + \Phi_2(t) \diamond V \diamond U_x + \Phi_3(t) \diamond U_{xxx} = 0, \\ U_x + V_y = 0, \end{cases} \tag{1.2}$$

where " $\diamond$ " is the Wick product on the Kondratiev distribution space $(\mathcal{S})_{-1}$ which was defined in [21] and $\Phi_1(t), \Phi_2(t)$ and $\Phi_3(t)$ are $(\mathcal{S})_{-1}$ -valued functions.

2 Description of the F-expansion Method

In order to at the same time obtain more periodic wave solutions expressed by various Jacobi elliptic functions to nonlinear wave equations, we introduce an F-expansion method which can be thought of as a succinctly over-all generalization of Jacobi elliptic function expansion. We briefly show what is F-expansion method and how to use it to obtain various periodic wave solutions to nonlinear wave equations. Suppose a nonlinear wave equation for $u(t,x)$ is given by

$$\theta_1(u, u_t, u_x, u_y, u_{xx}, u_{xxx}, ...) = 0, \tag{2.1}$$

where $u = u(t,x)$ is an unknown function, θ_1 is a polynomial in u and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of a deformation F-expansion method.

Step 1. Look for traveling wave solution of Eq.(2.1) by taking

$$u(t,x,y) = u(\xi) \ ,\xi(t,x,y) = kx + ly + \mu \int_0^t \omega(\tau)d\tau + c, \tag{2.2}$$

Hence, under the transformation (2.2). Eq.(2.1) can be transformed into the following ordinary differential equation (ODE) as following

$$\theta_2(u, \mu\omega u', ku', lu', k^2u'', k^3u''', ...) = 0, \tag{2.3}$$

Step 2. Suppose that $u(\xi)$ can be expressed by a finite power series of $F(\xi)$ of the form

$$u(t,x,y) = u(\xi) = \sum_{i=1}^{N} a_i F^i(\xi), \tag{2.4}$$

where $a_0, a_1, ..., a_N$ are constants to be determined later, while $F'(\xi)$ in(2.4) satisfy

$$[F'(\xi)]^2 = PF^4(\xi) + QF^2(\xi) + R, \tag{2.5}$$

and hence holds for $F(\xi)$

$$\begin{cases} F'F'' = 2PF^3F' + QFF', \\ F'' = 2PF^3 + QF, \\ F''' = 6PF^2F' + QF', \\ \cdots \end{cases} \tag{2.6}$$

where $P, Q,$ and R are constants.

Step 3. The positive integer N can be determined by considering the homogeneous balance between the highest derivative term and the nonlinear terms appearing in (2.3). Therefore, we can get the value of N in (2.4).

Step 4. Substituting (2.4) into (2.3) with the condition (2.5), we obtain polynomial in $F^i(\xi)[F'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. Setting each coefficient of this polynomial to be zero yields a set of algebraic equations for $a_0, a_1, ..., a_N, \mu$ and ω .

Step 5. Solving the algebraic equations with the aid of Maple we have $a_0, a_1, ..., a_N, \mu$ and ω can be expressed by (P, Q, R) . Substituting these results into F-expansion (2.4), then a general form of traveling wave solution of Eq. (2.1) can be obtained.

Step 6. Since the general solutions of (2.4) have been well known for us Choose properly (P, Q and R .) in ODE (2.5) such that the corresponding solution $F(\xi)$ of it is one of Jacobi elliptic functions. (See Appendices A, B and C .)[45, 46, 47]

3 New Exact Wave Solutions of Eq. (1.2)

Taking the Hermite transform, white noise theory, and F-expansion method to explore new exact wave solutions for Eq.(1.2). Applying Hermite transform to Eq.(1.2), we get the deterministic equation.

$$\begin{cases} \widetilde{U}_t(t,x,y,z) + \widetilde{\Phi_1}(t,z)\widetilde{U}(t,x,y,z)\widetilde{V}_x(t,x,y,z) + \widetilde{\Phi_2}(t,z)\widetilde{V}(t,x,y,z)\widetilde{U}_x(t,x,y,z) \\ +\widetilde{\Phi_3}(t,z)\widetilde{U}_{xxx}(t,x,y,z) = 0, \\ \\ \widetilde{U}_x(t,x,y,z) + \widetilde{V}_y(t,x,y,z) = 0, \end{cases} \tag{3.1}$$

where $z = (z_1, z_2, ...) \in (\mathbb{C}^{\mathbb{N}})$ is a vector parameter. To look for the travelling wave solution of Eq.(3.1), we make the transformations $\widetilde{\Phi_1}(t,z) := \phi_1(t,z)$, $\widetilde{\Phi_2}(t,z) := \phi_2(t,z)$, $\widetilde{\Phi_3}(t,z) := \phi_3(t,z)$, $\widetilde{U}(t,x,y,z) := u(t,x,y,z) = u(\xi(t,x,y,z))$ and $\widetilde{V}(t,x,y,z) := v(t,x,y,z) = v(\xi(t,x,y,z))$ with

$$\xi(t,x,y,z) = kx + ly + \mu\int_0^t \omega(\tau,z)d\tau + c,$$

where k, μ and c are arbitrary constants which satisfy $k\mu \neq 0$, $\omega(\tau,z)$ is a nonzero function of the indicated variables to be determined later. Hence, Eq.(3.1) can be transformed into the following (ODE).

$$\begin{cases} \mu\omega u' + k\phi_1 uv' + k\phi_2 vu' + k^3\phi_3 u''' = 0, \\ ku' + lv' = 0, \end{cases} \tag{3.2}$$

where the prime denote to the differential with respect to ξ . In view of F-expansion method, the solution of Eq. (3.1), can be expressed in the form.

$$\begin{cases} u(t,x,y,z) = u(\xi) = \Sigma_{i=1}^{N} a_i F^i(\xi), \\ v(t,x,y,z) = v(\xi) = \Sigma_{i=1}^{M} b_i F^i(\xi), \end{cases} \tag{3.3}$$

where a_i and b_i are constants to be determined later. considering homogeneous balance between the highest order nonlinear terms and the highest order partial derivative of u in (3.2), then we can obtain $N = M = 2$ so (3.3) can be rewritten as following

$$\begin{cases} u(t,x,y,z) = a_0 + a_1F(\xi) + a_2F^2(\xi), \\ v(t,x,y,z) = b_0 + b_1F(\xi) + b_2F^2(\xi), \end{cases} \tag{3.4}$$

where a_0, a_1, a_2, b_0, b_1 and b_2 are constants to be determined later. Substituting (3.4) with the conditions (2.5),(2.6) into (3.2) and collecting all terms with the same power of

$F^i(\xi)[F'(\xi)]^j$, $(i = 0 \pm 1, \pm 2, ..., j = 0, 1)$. as following

$$\begin{cases} [\mu\omega a_1 + ka_0b_1\phi_1 + ka_1b_0\phi_2 + k^3a_1\phi_3Q]F' \\ +[2\mu\omega a_2 + 2ka_0b_2\phi_1 + ka_1b_1\phi_1 + 2ka_2b_0\phi_2 + ka_1b_1\phi_2 + 8k^3a_2\phi_3Q]FF' \\ +k[2a_1b_2\phi_1 + a_2b_1\phi_1 + 2a_2b_1\phi_2 + a_1b_1\phi_2 + 6k^2a_1\phi_3P]F^2F' \\ +2ka_2[b_2\phi_1 + b_2\phi_2 + 12k^2\phi_3P]F^2F' = 0, \\ (ka_1 + lb_1)F' + 2[ka_2 + lb_2]FF' = 0. \end{cases} \tag{3.5}$$

Setting each coefficients of $F^i(\xi)[F'(\xi)]^j$ to be zero, we get a system of algebraic equations which can be expressed by.

$$\begin{cases} \mu\omega a_1 + ka_0b_1\phi_1 + ka_1b_0\phi_2 + k^3a_1\phi_3Q = 0, \\ 2\mu\omega a_2 + 2ka_0b_2\phi_1 + ka_1b_1\phi_1 + 2ka_2b_0\phi_2 + ka_1b_1\phi_2 + 8k^3a_2\phi_3Q = 0, \\ k[2a_1b_2\phi_1 + a_2b_1\phi_1 + 2a_2b_1\phi_2 + a_1b_1\phi_2 + 6k^2a_1\phi_3P] = 0, \\ 2ka_2[b_2\phi_1 + b_2\phi_2 + 12k^2\phi_3P] = 0, \\ ka_1 + lb_1 = 0, \\ 2[ka_2 + lb_2] = 0. \end{cases} \tag{3.6}$$

with solving by *Maple* to get the following coefficients

$$\begin{cases} a_2 = b_2 = 0, \ \ a_0 \ , b_0 = \text{arbitrary constant}, \\ a_1 = \frac{6lk\phi_3(t,z)P}{\phi_2(t,z)}, \\ b_1 = -\frac{6k^2\phi_3(t,z)P}{\phi_2(t,z)}, \\ \omega = \frac{k^2a_0\phi_1(t,z) - lk[b_0\phi_2(t,z) + k^2\phi_3(t,z)Q]}{l\mu}. \end{cases} \tag{3.7}$$

Substituting by coefficient (3.7) into (3.4) yields general form solutions of Eq. (1.2).

$$u(t, x, y, z) = a_0 + \frac{6lk\phi_3(t, z)P}{\phi_2(t, z)} \ F(\xi), \tag{3.8}$$

$$v(t, x, y, z) = b_0 - \frac{6k^2\phi_3(t, z)P}{\phi_2(t, z)} \ F(\xi), \tag{3.9}$$

with

$$\xi(t, x, y, z) = kx + ly + \int_0^t \frac{k^2a_0\phi_1(\tau, z) - lk[b_0\phi_2(\tau, z) + k^2\phi_3(\tau, z)Q]}{l} \ d\tau.$$

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From Appendix A, we give the special cases as following.

Case I:
If we take $P=\frac{1}{4}, Q=\frac{m^2-2}{2}$ and $R=\frac{m^2}{4}$, we have $F(\xi)\to ns(\xi)\pm ds(\xi)$,

$$u_1(t,x,y,z)=a_0+\frac{3lk\phi_3(t,z)}{2\phi_2(t,z)}\left[ns\left(\xi_1(t,x,y,z)\right)\pm ds\left(\xi_1(t,x,y,z)\right)\right], \tag{3.10}$$

$$v_1(t,x,y,z)=b_0-\frac{3k^2\phi_3(t,z)}{2\phi_2(t,z)}\left[ns\left(\xi_1(t,x,y,z)\right)\pm ds\left(\xi_1(t,x,y,z)\right)\right], \tag{3.11}$$

with

$$\xi_1(t,x,y,z)=kx+ly+\int_0^t\left\{\frac{2k^2a_0\phi_1(\tau,z)-lk[2b_0\phi_2(\tau,z)+k^2\phi_3(\tau,z)(m^2-2)]}{2l}\right\}d\tau.$$

In the limit case when $m\to o$, we have $ns(\xi)\pm ds(\xi)\to 2\csc(\xi)$, thus (3.10),(3.11) become.

$$u_2(t,x,y,z)=a_0+\frac{3lk\phi_3(t,z)}{\phi_2(t,z)}\csc\left(\xi_2(t,x,y,z)\right), \tag{3.12}$$

$$v_2(t,x,y,z)=b_0-\frac{3k^2\phi_3(t,z)}{\phi_2(t,z)}\csc\left(\xi_2(t,x,y,z)\right), \tag{3.13}$$

with

$$\xi_2(t,x,y,z)=kx+ly+\int_0^t\left\{\frac{k^2a_0\phi_1(\tau,z)-lk[b_0\phi_2(\tau,z)-k^2\phi_3(\tau,z)]}{l}\right\}d\tau.$$

In the limit case when $m\to 1$ we have $ns(\xi)\pm ds(\xi\to\coth(\xi)\pm(\xi)$, thus (3.10).(3.11) become.

$$u_3(t,x,y,z)=a_0+\frac{3lk\phi_3(t,z)}{2\phi_2(t,z)}\left[\coth\,\xi_3(t,x,y,z)\pm\,(\xi_3(t,x,y,z))\right], \tag{3.14}$$

$$v_3(t,x,y,z)=b_0-\frac{3k^2\phi_3(t,z)}{2\phi_2(t,z)}\left\{[\coth\,\xi_3(t,x,y,z)\pm\,(\xi_3(t,x,y,z))\right], \tag{3.15}$$

with

$$\xi_3(t,x,y,z)=kx+ly+\int_0^t\left\{\frac{2k^2a_0\phi_1(\tau,z)-lk[2b_0\phi_2(\tau,z)-k^2\phi_3(\tau,z)]}{2l}\right\}d\tau.$$

Case II:
If we take $P=1, Q=-(1+m^2)$ and $R=m^2$, then $F(\xi)\to ns(\xi)$,

$$u_4(t,x,y,z)=a_0+\frac{6lk\phi_3(t,z)}{\phi_2(t,z)}\ ns\left(\xi_4(t,x,y,z)\right), \tag{3.16}$$

$$v_4(t,x,y,z)=b_0-\frac{6k^2\phi_3(t,z)}{\phi_2(t,z)}\ ns\left(\xi_4(t,x,y,z)\right), \tag{3.17}$$

with

$$\xi_4(t,x,y,z)=kx+ly+\int_0^t\left\{\frac{2k^2a_0\phi_1(\tau,z)-lk[2b_0\phi_2(\tau,z)+k^2\phi_3(\tau,z)(m^2-2)]}{l}\right\}d\tau.$$

In the limit case when $m\to o$ we have $ns(\xi)\pm ds(\xi)\to\csc(\xi)$, thus (3.10),(3.11) become.

$$u_5(t,x,y,z)=a_0+\frac{6lk\phi_3(t,z)}{\phi_2(t,z)}\ \csc\left(\xi_2(t,x,y,z)\right), \tag{3.18}$$

$$v_5(t,x,y,z)=b_0-\frac{6k^2\phi_3(t,z)}{\phi_2(t,z)}\ \csc\left(\xi_2(t,x,y,z)\right). \tag{3.19}$$

In the limit case when $m\to 1$ we have $ns(\xi)\to\coth(\xi)$, thus (3.10).(3.11) become.

$$u_6(t,x,y,z)=a_0+\frac{6lk\phi_3(t,z)}{2\phi_2(t,z)}\ \coth\left(\xi_5(t,x,y,z)\right), \tag{3.20}$$

$$v_6(t,x,y,z)=b_0-\frac{6k^2\phi_3(t,z)}{2\phi_2(t,z)}\ \coth\left(\xi_5(t,x,y,z)\right), \tag{3.21}$$

with

$$\xi_5(t,x,y,z)=kx+ly+\int_0^t\left\{\frac{k^2a_0\phi_1(\tau,z)-lk[b_0\phi_2(\tau,z)-2k^2\phi_3(\tau,z)]}{l}\right\}d\tau.$$

Case III:
If we take $P=1, Q=(2-m^2)$ and $R=1-m^2$, then $F(\xi)\to cs(\xi)$,

$$u_7(t,x,y,z)=a_0+\frac{6lk\phi_3(t,z)}{\phi_2(t,z)}\ cs\left(\xi_6(t,x,y,z)\right), \tag{3.22}$$

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$$v_7(t,x,y,z) = b_0 - \frac{6k^2\phi_3(t,z)}{\phi_2(t,z)}\; cs\,(\xi_6(t,x,y,z)), \tag{3.23}$$

with

$$\xi_6(t,x,y,z) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0\phi_1(\tau,z) - lk[2b_0\phi_2(\tau,z) + k^2\phi_3(\tau,z)(2-m^2)]}{l} \right\} d\tau.$$

In the limit case when $m \to o$ we have $cs(\xi) \to \cot(\xi)$, thus (3.10),(3.11) become.

$$u_8(t,x,y,z) = a_0 + \frac{6lk\phi_3(t,z)}{\phi_2(t,z)}\; \cot\,(\xi_7(t,x,y,z)), \tag{3.24}$$

$$v_8(t,x,y,z) = b_0 - \frac{6k^2\phi_3(t,z)}{\phi_2(t,z)}\; \cot\,(\xi_7(t,x,y,z)), \tag{3.25}$$

$$\xi_7(t,x,y,z) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0\phi_1(\tau,z) - lk[b_0\phi_2(\tau,z) + 2k^2\phi_3(\tau,z)]}{l} \right\} d\tau.$$

In the limit case when $m \to 1$ we have $cs(\xi) \to (\xi)$, thus (3.10).(3.11) become.

$$u_9(t,x,y,z) = a_0 + \frac{6lk\phi_3(t,z)}{\phi_2(t,z)}\; (\xi_8(t,x,y,z)), \tag{3.26}$$

$$v_9(t,x,y,z) = b_0 - \frac{6k^2\phi_3(t,z)}{\phi_2(t,z)}\; (\xi_8(t,x,y,z)), \tag{3.27}$$

with

$$\xi_8(t,x,y,z) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0\phi_1(\tau,z) - lk[b_0\phi_2(\tau,z) + k^2\phi_3(\tau,z)]}{l} \right\} d\tau.$$

Obviously, there are another solutions for Eq.(1.2). These solutions come from setting different values for the coefficients P, Q and R. (see Appendix A, B and C.)[46, 47]. The above mentioned cases are just to clarify how far our technique is applicable.

4 White Noise Functional Solutions of Eq.(1.2)

In this section, we employ the results of the Section 3 by using Hermite transform to obtain exact white noise functional solutions for Wick-type stochastic (2+1)-dimensional coupled KdV equations (1.2). The properties of exponential and trigonometric functions yield that there exists a bounded open set $\mathbf{G} \subset \mathbb{R}_+ \times \mathbb{R}^2$, $\rho < \infty$, $\lambda > 0$ such that the so-

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lution $u(t,x,y,z)$ of Eq. (3.1) and all its partial derivatives which are involved in Eq. (3.1) are uniformly bounded for $(t,x,y,z) \in \mathbf{G} \times K_\rho(\lambda)$, continuous with respect to $(t,x,y) \in \mathbf{G}$ for all $z \in K_\rho(\lambda)$ and analytic with respect to $z \in K_\rho(\lambda)$, for all $(t,x,y) \in \mathbf{G}$. From Theorem 4.1.1 in [21], there exists $U(t,x,y,z) \in (\mathcal{S})_{-1}$ such that $u(t,x,y,z) = \widetilde{U}(t,x,y)(z)$ for all $(t,x,y,z) \in \mathbf{G} \times K_\rho(\lambda)$ and $U(t,x,y)$ solves Eq.(1.2) in $(\mathcal{S})_{-1}$. Hence, by applying the inverse Hermite transform to the results of Section 3, we get exact white noise functional solutions of Eq. (1.2) as follows.

- **White noise functional solutions of JEF type**:

$$U_1(t,x,y) = a_0 + \frac{3lk\Phi_3(t)}{2\Phi_2(t)} \diamond \Big[ns^\diamond\left(\Xi_1(t,x,y)\right) \pm ds^\diamond\left(\Xi_1(t,x,y)\right) \Big], \tag{4.1}$$

$$V_1(t,x,y) = b_0 - \frac{3k^2\Phi_3(t)}{2\Phi_2(t)} \diamond \Big[ns^\diamond\left(\Xi_1(t,x,y)\right) \pm ds^\diamond\left(\Xi_1(t,x,y)\right) \Big], \tag{4.2}$$

$$U_2(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{\Phi_2(t)} \diamond ns^\diamond\left(\Xi_2(t,x,y)\right), \tag{4.3}$$

$$V_2(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{\Phi_2(t)} \diamond ns^\diamond\left(\Xi_2(t,x,y)\right), \tag{4.4}$$

$$U_3(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{\Phi_2(t)} \diamond cs^\diamond\left(\Xi_3(t,x,y)\right), \tag{4.5}$$

$$V_3(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{\Phi_2(t)} \diamond cs^\diamond\left(\Xi_3(t,x,y)\right), \tag{4.6}$$

with

$$\Xi_1(t,x,y) = kx + ly + \int_0^t \left\{ \frac{2k^2a_0\Phi_1(\tau) - lk[2b_0\Phi_2(\tau) + k^2\phi_3(\tau)(m^2-2)]}{2l} \right\} d\tau,$$

$$\Xi_2(t,x,y) = kx + ly + \int_0^t \left\{ \frac{2k^2a_0\Phi_1(\tau) - lk[2b_0\Phi_2(\tau) + k^2\Phi_3(\tau)(m^2-2)]}{l} \right\} d\tau,$$

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$$\Xi_3(t,x,y) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[2b_0\Phi_2(\tau) + k^2\Phi_3(\tau)(2-m^2)]}{l} \right\} d\tau.$$

- **White noise functional solutions of trigonometric type**:

$$U_4(t,x,y) = a_0 + \frac{3lk\Phi_3(t)}{\Phi_2(t)} \diamond \csc^{\diamond}(\Xi_4(t,x,y)), \tag{4.7}$$

$$V_4(t,x,y) = b_0 - \frac{3k^2\Phi_3(t)}{\Phi_2(t)} \diamond \csc^{\diamond}(\Xi_4(t,x,y)), \tag{4.8}$$

$$U_5(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{\Phi_2(t)} \diamond \csc^{\diamond}(\Xi_4(t,x,y)), \tag{4.9}$$

$$V_5(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{\Phi_2(t)} \diamond \csc^{\diamond}(\Xi_4(t,x,y)), \tag{4.10}$$

$$U_6(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{\Phi_2(t)} \diamond \cot^{\diamond}(\Xi_5(t,x,y)), \tag{4.11}$$

$$V_6(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{\Phi_2(t)} \diamond \cot^{\diamond}(\Xi_5(t,x,y)), \tag{4.12}$$

with

$$\Xi_4(t,x,y) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[b_0\Phi_2(\tau) - k^2\Phi_3(\tau)]}{l} \right\} d\tau,$$

$$\Xi_5(t,x,y) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[b_0\Phi_2(\tau) + 2k^2\Phi_3(\tau)]}{l} \right\} d\tau.$$

- **White noise functional solutions of hyperbolic type**:

$$U_7(t,x,y) = a_0 + \frac{3lk\Phi_3(t)}{2\Phi_2(t)} \diamond \Big[\coth^{\diamond}(\Xi_6(t,x,y)) \pm^{\diamond} (\Xi_6(t,x,y)) \Big], \tag{4.13}$$

$$V_7(t,x,y) = b_0 - \frac{3k^2\Phi_3(t)}{2\Phi_2(t)} \diamond \Big[\coth^{\diamond}(\Xi_6(t,x,y)) \pm^{\diamond} (\Xi_6(t,x,y)) \Big], \tag{4.14}$$

$$U_8(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{2\Phi_2(t)} \diamond \coth^{\diamond}(\Xi_7(t,x,y)), \tag{4.15}$$

$$V_8(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{2\Phi_2(t)} \diamond \coth^{\diamond}(\Xi_7(t,x,y)), \tag{4.16}$$

$$U_9(t,x,y) = a_0 + \frac{6lk\Phi_3(t)}{\Phi_2(t)} \diamond {}^{\diamond}(\Xi_8(t,x,y)), \tag{4.17}$$

$$V_9(t,x,y) = b_0 - \frac{6k^2\Phi_3(t)}{\Phi_2(t)} \diamond {}^{\diamond}(\Xi_8(t,x,y)), \tag{4.18}$$

with

$$\Xi_6(t,x,y) = kx + ly + \int_0^t \left\{ \frac{2k^2 a_0 \Phi_1(\tau) - lk[2b_0\Phi_2(\tau) - k^2\Phi_3(\tau)]}{2l} \right\} d\tau,$$

$$\Xi_7(t,x,y) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[b_0\Phi_2(\tau) - 2k^2\Phi_3(\tau)]}{l} \right\} d\tau,$$

$$\Xi_8(t,x,y) = kx + ly + \int_0^t \left\{ \frac{k^2 a_0 \Phi_1(\tau) - lk[b_0\Phi_2(\tau) + k^2\Phi_3(\tau)]}{l} \right\} d\tau.$$

We observe that, for different forms of Φ_1, Φ_2 and Φ_3, we can get different exact white noise functional solutions of Eq. (1.2) from Eqs. (4.1)-(4.18).

5 Example

It is well known that Wick version of function is usually difficult to evaluate. So, in this section, we give non-Wick version of solutions of Eq. (1.2). Let $W_t = \dot{B}_t$ be the Gaussian white noise, where B_t is the Brownian motion. We have the Hermite transform $\widetilde{W_t}(z) = \sum_{i=1}^{\infty} z_i \int_0^t \eta_i(s)ds$ [21]. Since $\exp^{\diamond}(B_t) = \exp(B_t - \frac{t^2}{2})$, we have $\cot^{\diamond}(B_t) = \cot(B_t - \frac{t^2}{2})$, $\csc^{\diamond}(B_t) = \csc(B_t - \frac{t^2}{2})$, $\coth^{\diamond}(B_t) = \coth(B_t - \frac{t^2}{2})$ and ${}^{\diamond}(B_t) = (B_t - \frac{t^2}{2})$. Suppose that. $\Phi_1(t) = \psi_1\Phi_3(t), \Phi_2(t) = \psi_2\Phi_3(t)$ and $\Phi_3(t) = \Gamma(t) + \psi_3 W_t$ where ψ_1, ψ_2 and ψ_3 are arbitrary constants and $\Gamma(t)$ is integrable or bounded measurable function on $\mathbb{R}_+$. Therefore, for $\Phi_1(t)\Phi_2(t)\Phi_3(t) \neq 0$. thus exact white noise functional solutions of Eq. (1.2) are as follows.

$$U_{10}(t,x,y) = a_0 + \frac{3lk}{\psi_2}\ \csc\left(\Omega_1(t,x,y)\right), \tag{5.1}$$

$$V_{10}(t,x,y) = b_0 - \frac{3k^2}{\psi_2}\ \csc\left(\Omega_1(t,x,y)\right), \tag{5.2}$$

$$U_{11}(t,x,y) = a_0 + \frac{6lk}{\psi_2}\ \csc \Omega_1(t,x,y), \tag{5.3}$$

$$V_{11}(t,x,y) = b_0 - \frac{6k^2}{\psi_2}\ \csc\left(\Omega_1(t,x,y)\right), \tag{5.4}$$

$$U_{12}(t,x,y) = a_0 + \frac{6lk}{\psi_2}\ \cot\left(\Omega_2(t,x,y)\right), \tag{5.5}$$

$$V_{12}(t,x,y) = b_0 - \frac{6k^2}{\psi_2}\ \cot\left(\Omega_2(t,x,y)\right), \tag{5.6}$$

with

$$\Omega_1(t,x,y) = kx + ly + \big(\frac{k^2 a_0 \psi_1 - lk[b_0\psi_2 - k^2]}{l}\big)\left\{\int_0^t \Gamma(\tau)d\tau + \psi_3[B_t - \frac{t^2}{2}]\right\},$$

$$\Omega_2(t,x,y) = kx + ly + \big(\frac{k^2 a_0 \psi_1 - lk[b_0\psi_2 + 2k^2]}{l}\big)\left\{\int_0^t \Gamma(\tau)d\tau + \psi_3[B_t - \frac{t^2}{2}]\right\},$$

and

$$U_{13}(t,x,y) = a_0 + \frac{3lk}{2\psi_2}\ \Big[\coth\left(\Omega_3(t,x,y)\right) \pm\ \left(\Omega_3(t,x,y)\right)\Big], \tag{5.7}$$

$$V_{13}(t,x,y) = b_0 - \frac{3k^2}{2\psi_2}\ \Big[\coth\left(\Omega_2(t,x,y)\right) \pm\ \left(\Omega_3(t,x,y)\right)\Big], \tag{5.8}$$

$$U_{14}(t,x,y) = a_0 + \frac{6lk}{2\psi_2}\ \coth\left(\Omega_4(t,x,y)\right), \tag{5.9}$$

$$V_{14}(t,x,y) = b_0 - \frac{6k^2}{2\psi_2}\ \coth\left(\Omega_4(t,x,y)\right), \tag{5.10}$$

$$U_{15}(t,x,y) = a_0 + \frac{6lk}{\psi_2}\ \left(\Omega_5(t,x,y)\right), \tag{5.11}$$

$$V_{15}(t,x,y) = b_0 - \frac{6k^2}{\psi_2}\ \left(\Omega_5(t,x,y)\right), \tag{5.12}$$

with

$$\Omega_3(t,x,y) = kx + ly + \big(\frac{2k^2a_0\psi_1 - lk[2b_0\psi_2 - k^2]}{2l}\big)\left\{\int_0^t \Gamma(\tau)d\tau + \psi_3[B_t - \frac{t^2}{2}]\right\},$$

$$\Omega_4(t,x,y) = kx + ly + \big(\frac{k^2a_0\psi_1 - lk[b_0\psi_2 - 2k^2]}{l}\big)\left\{\int_0^t \Gamma(\tau)d\tau + \psi_3[B_t - \frac{t^2}{2}]\right\},$$

$$\Omega_5(t,x,y) = kx + ly + \big(\frac{k^2a_0\psi_1 - lk[b_0\psi_2 + k^2]}{l}\big)\left\{\int_0^t \Gamma(\tau)d\tau + \psi_3[B_t - \frac{t^2}{2}]\right\}.$$

6 Conclusion

We have discussed the solutions of (SPDEs) driven by Gaussian white noise. There is a unitary mapping between the Gaussian white noise space and the Poisson white noise space. This connection was given by Benth and Gjerde [2]. By the aid of this connection, we can derive some stochastic exact soliton solutionsfor our problem. In this paper, using Hermite transformation, white noise theory and F-expansion method, we study the white noise functional solutions of the Wick-type stochastic (2+1)-dimensional coupled KdV equations. This paper shows that the F-expansion method is sufficient to solve many stochastic nonlinear equations in mathematical physics. The method which we have proposed in this paper is standard, direct and computerized method, which allows us to do complicated and tedious algebraic calculation. It is shown that the algorithm can be also applied to other nonlinear (PDEs) in mathematical physics such as modified Hirota-Satsuma coupled KdV, KdV-Burgers, modified KdV Burgers, Sawada-Kotera, Zhiber-Shabat equations and Benjamin-Bona-Mahony equations. Since the equation (1.2) has other solutions if select other values of P, Q and R (see Appendices A, B, C), and there are many other of exact solutions for wick-type stochastic (2+1)-dimensional coupled KdV equations.

Appendix A. The ODE and Jacobi Elliptic Functions
Relation between values of (P , Q , R) and corresponding $F(\xi)$ in ODE.

$$(F')^2(\xi) = PF^4(\xi) + QF^2(\xi) + R,$$

P	Q	R	$F(\xi)$
m^2	$-1-m^2$	1	$\mathrm{sn}\xi$, $\mathrm{cd}\xi = \frac{cn\xi}{dn\xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$\mathrm{cn}\xi$
-1	$2-m^2$	m^2-1	$\mathrm{dn}\xi$
1	$-1-m^2$	m^2	$\mathrm{ns}\xi = \frac{1}{\mathrm{sn}\xi}$, $\mathrm{dc}\xi = \frac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$\mathrm{nc}\xi = \frac{1}{\mathrm{cn}\xi}$
m^2-1	$2-m^2$	-1	$\mathrm{nd}\xi = \frac{1}{\mathrm{dn}\xi}$
$1-m^2$	$2-m^2$	1	$\mathrm{sc}\xi = \frac{\mathrm{sn}\xi}{\mathrm{cn}\xi}$
$-m^2(1-m^2)$	$2m^2-1$	1	$\mathrm{sd}\xi = \frac{\mathrm{sn}\xi}{\mathrm{dn}\xi}$
1	$2-m^2$	$1-m^2$	$\mathrm{cs}\xi = \frac{\mathrm{cn}\xi}{\mathrm{sn}\xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$\mathrm{ds}\xi = \frac{\mathrm{dn}\xi}{\mathrm{sn}\xi}$
$\frac{m^4}{4}$	$\frac{m^2-2}{2}$	$\frac{1}{4}$	$\frac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi}$, $\frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi}$
$\frac{m^2}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{sn}\xi \pm i\mathrm{cn}\xi$, $\frac{\mathrm{dn}\xi}{i\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{cn}\xi}$, $\frac{\mathrm{m\,sn}\xi}{1\pm\mathrm{dn}\xi}$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$\mathrm{ns}\xi \pm \mathrm{cs}\xi$, $\frac{\mathrm{cn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi\pm\mathrm{dn}\xi}$, $\frac{\mathrm{sn}\xi}{1\pm\mathrm{cn}\xi}$,
$\frac{m^2-1}{4}$	$\frac{m^2+1}{2}$	$\frac{m^2-1}{4}$	$\frac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi}$
$\frac{1-m^2}{4}$	$\frac{m^2+1}{2}$	$\frac{1-m^2}{4}$	$\mathrm{nc}\xi \pm i\mathrm{sc}\xi$ $\frac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi}$
$\frac{-1}{4}$	$\frac{m^2+1}{2}$	$\frac{-(1-m^2)^2}{4}$	$m\mathrm{cn}\xi \pm \mathrm{dn}\xi$
$\frac{1}{4}$	$\frac{m^2+1}{2}$	$\frac{(1-m^2)^2}{4}$	$\frac{\mathrm{sn}\xi}{\mathrm{cn}\xi\pm\mathrm{dn}\xi}$
$\frac{1}{4}$	$\frac{m^2-2}{2}$	$\frac{m^2}{4}$	$\mathrm{ns}\xi \pm \mathrm{ds}\xi$

Appendix B.
the jacobi elliptic functions degenerate into trigonometric functions when $m \to 0$.

$$sn\xi \to \sin\xi, cn\xi \to \cos\xi, dn\xi \to 1, sc\xi \to \tan\xi, sd\xi \to \sin\xi, cd\xi \to \cos\xi,$$
$$ns\xi \to \csc\xi, nc\xi \to \sec\xi, nd\xi \to 1, cs\xi \to \cot\xi, ds\xi \to \csc\xi, dc\xi \to \sec\xi.$$

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Appendix C.
the jacobi elliptic functions degenerate into hyperbolic functions when $m \to 1$.

$$\begin{aligned} &sn\xi \to \tan\xi, cn\xi \to \xi, dn\xi \to \xi, sc\xi \to \sinh\xi, sd\xi \to \sinh\xi, cd\xi \to 1, \\ &ns\xi \to \coth\xi, nc\xi \to \cosh\xi, nd\xi \to \cosh, cs\xi \to \xi, ds\xi \to \xi, dc\xi \to 1. \end{aligned}$$

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