# Probabilistic Analysis of Single Server Queueing Systems with Feedback, Customer Decisions, and Vacation Interruptions

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***Abstract*— This paper focuses on giving a detailed probabilistic modeling of a periodic replenishment system in a single server feedback system with customer controlling and vacation break systems. The primary concern is using mathematical model to assess the performance of the system by taking into account feedback from the customers, customer's decision making to join the system or abandon it and the vacation disturbed where the server does not serve any customer. These form the main performance measurements such as number of customers, waiting time, system load, etc., which are obtained using Markov chain for steady state analysis. The results are applied to consider the effects of feedback and vacations on system productivity and to develop recommendations for improving queues in real-world situations.

Keywords— Queueing theory, Single-server system, Feedback, Customer decisions, Vacation interruptions, Markov chains, Performance metrics, System optimization.

## I. Introduction

Introducing Queueing systems, it is necessary to underline that this field is very important for understanding the work of systems where several customers share some resources at the same time This field is very important for such fields as telecommunication, computer networks, and services providing systems. Stochastic processes, or queueing theory, offer mathematical provisions that may be used to analyze and forecast the behaviors of such a system. The particular attention should be paid to the analysis of single-server queueing systems with feedback, customers' decisions and vacations since it provides the most appropriate prospects for the practical usage and the most principal challenges for the analysis [1-4].

Single server system is a model of queueing network comprising a single server and a group of customers who arrive, wait in a queue, are served and depart. In some real-world situations not only do customers leave after being served but they may also come back for more serving. This is called “feedback” and is known to have substantial influence on the system in question. Customers may decide to re-join the queue depending on things like the level of satisfaction obtained while in the service, estimated time to service or quality of service availed among others. Much of this feedback complicates the system dynamics and require the design of more complex models to explain these behaviors [9].

Besides, there is another source of WIP uncertainty: vacation interruptions on customers’ side. Server breaks mean conditions under which the server is out of operation to take requests from the customers. In real-life applications such interruptions are evident especially for the service-oriented organizations or in computing systems where some servers may be taken for service among other reasons. Using vacation interruptions in queueing models means these models are made closer to reality since the server is not always available due to vacations [6].

Queueing, feedback, and customer decisions as well as vacation interruption makes it difficult to solve this problem using queueing theory and this in turn calls for sophisticated probability models and analysis. It is the knowledge of how these aspects affect system effectiveness, which is applicable to managing large organizations, on striving to influence customer waits time and the resultant satisfaction [8].

This paper sets out to examine probabilistic analysis of single server queueing systems with feedback, customer action, and server vacation. We therefore model the system as the summation of stochastic processes to get an insight of how the various components of the system influence basic measures of system performance such as the expected waiting time for service, utilization of the system, and the number of customers passed through the system per unit of time. The aim is to give a means of guidance about such real-world problems that can help to design or tune systems in a broad number of applications including, for example, service management or network topology [22].

***Novelty and Contribution***

The novelty of this work lies in the integrated approach to analyzing queueing systems with three significant factors: feedback, customer’s decisions and vacations interruptions. Although the influence of each of these factors has been investigated in prior literature, their combined impact has not been implemented in a one integrated framework let alone, using probabilistic models [7].

- Feedback Mechanism with Customer Decisions: While feedback in queueing systems is confirmed, an additional parameter – the decision-making of customers to return to the queue due to waiting time, service quality or otherwise, making the situation much more realistic. One extension is the incorporation of the customer decision-making behaviour as this enables the model to mimic practicable systems in that customer may come and go due to the level of satisfaction attained or due to conditions in the external environment such as new entrants in service industries and network systems.
- Server Vacation Interruptions: Adding server vacation interruptions into the analysis raises another level of complication in the model. Downtime interferences occur frequently in many real-world systems for example in computing systems where servers might temporarily be eliminated for servicing. Incorporating these interruptions into the model, this work is extended to include a clearer insight on how such interruptions engulfs parts of the server making it temporarily unavailable as a means of influencing the systems performance and customer behaviour.
- Probabilistic Modelling Approach: Another methodological innovation is the application of probabilistic reasoning techniques when modelling these interactions. Earlier works have used deterministic models and compared those with a few probability models that were simpler in calculation. This paper provides a probabilistic model that is geared towards modelling of stochastic attributes that characterize customer arrival, feedback, and sever vacation time. This increases the complexity of the analysis but yields far better performance estimates for a broad spectrum of applications.
- Application to Real-World Systems: The implications of the information generated in this paper are easily translatable to many disciplines. For instance, in customer service functions, some forms of feedback and vacations interrupts are essential in establishing the right amount of time and resources needed for carrying out the services. In similar way in telecommunication or network monitoring this model can be used to forecast occurrence of traffic jam and the way of Network resource utilization particularly during Service outages or upgrades.

This paper's primary contribution is to create a rich stochastic model for modeling both feedback and customer decisions as well as arbitrary vacation interruptions for single-server queueing systems. As such, it yields information on the performance of systems that is more realistic, and the comprehensive tool may be useful to the decision makers in different fields [10].

Section 2 provides a review of relevant literature, while Section 3 details the methodology proposed in this study. Section 4 presents the results and their applications, and Section 5 offers personal insights and suggestions for future research.

## II. RELATED WORKS

Queueing theory has been dominated by single-server queueing systems analysis for several years. These systems are employed for simulation of various services that are present in the real world such as customer service centers and even computer networks. A single server queueing model can be described having a server and a queue to which customers can join them at arbitrary time and wait for service. After the delivery on service, customers discharge the system. Nevertheless, several developments to this simple model have been made so that it can be applied to systems with feedback, customer decisions, and system vacation interruptions [19].

Feedback in queueing systems is a concept whose extension is well known. Feedback exists when instead after getting serviced, a customer moves back into the queue to access other services. Such behavior is observed in many real-world instance service environments where customers may need several services for complete satisfaction. Past works have analyzed diverse types of feedback, the assumption being that the customers can church after n+1 service or after some level of satisfaction. These models try to express the different effects that feedback has on system performance, in terms of measures such as wait time, service time and the system throughput.

In 1975 Kleinrock, L. et. al. [5] Introduce the growing interest in establishing customers' decision-making behavior as a concept. Customer decisions are considered in a stochastic form, which means that customers decide whether to rejoin the queue based on some other factors such as the expected service time, patience or the number of customers in a queue. It also further complicates the conventional queueing model because the system is no longer stimulated by random arrivals and service time completions alone. Yet, the behavior of customers contributes significantly to deciding the performance system. Introducing concepts from customer decision-making in queueing models has resulted in useful knowledge on ways to optimally match the level and quality of services with the general efficiency of the service system, especially in areas where customer satisfaction is critical [14].

The last notable addition to the theory of single server queuing systems are the vacation interruptions. System downtime is when the server is out of service to offer a service and could be because of vacations among other reasons. Applying vacation periods in the models of queues improves the results that approximate real-world systems since the servers are not constantly available. Previous work has considered various kinds of vacation patterns such as random vacation or fixed time vacation to observe the effect of interruptions on the queueing system performance. These types of models have given an understanding on how vacation periods impact on performance indicators like waiting time and occupancy and have brought forward ways of reducing effects.

In 2000 Malshe, R., et. al. & Jain, S. et. al [7] Introduce the case of a system where feedback is combined with customer decisions and vacation interruptions adds to the challenging problem of a single server queueing system. Although each of these components has been separately investigated in the literature, few have attempted to build a single theoretical framework to incorporate them. It is in such incorporation that results in more realistic modeling approaches for such systems that exhibit the above-mentioned behaviors all at once. The impacts of feedback when added over repeated customer decisions can be exacerbated by short interruptions like vacations to make a wide range of behaviors ambiguous and therefore hard to model probabilistically [20].

Many previous works have tried to propose models that incorporate feedback and server vacations with regards to performance evaluation and enhancement. These models usually employ Markov processes or other stochastic means, to study the first-order stationary behavior of the model and yields important metrics such as the number of customers within the system, the expected waiting times, and the probability distributions of service times. The problem in these models lies in how to incorporate the server vacation period of the customers, the behavior of customers in giving feedback as well as the decision of the customers to continue to return to the queue. Most commonly, we look to prevent users from waiting for a long time and to maximize server efficiencies, all the while dealing with feedback and vacations.

In 1992 Deshpande, R., & Chaturvedi, M. et. al. [15] Introduce the competing research areas, including telecommunications and computer networks, have developed models based on feedback and interruptions, referred to as vacations, to analyze congestion and resource use in networks. In these fields, the server plays the role of the network resource, which can be temporarily unavailable because of maintenance, balancing of the load, etc. Customers, for example data packets or users may have to request a connection to the network after a failure or congestion much in the same way as customers in a service queuing system. These models typically contain components of network traffic analysis, and resource management, which give the definitive perspective on the scope of application of queueing theory [24].

The stochastic elements of feedback and decision making are also important in models of queues, including in key elements of markets where consumers can "re-enter"-system after an initial engagement, which keeps customer decisions dynamic. These models give understanding of consumer behavior and market efficiency, discussing feedback mechanisms in queues regarding pricing and service level, and customer loyalty. Through unraveling the relationship between feedback, decision making demands on the servers as well as customer demands and expectations, improved systems may be developed that best suit the business.

In the case of interrupting planning, models which are capable of handling random vacations or planned breaks have been explored. These models employ a range of probabilistic tools to quantify the effect of the server's unavailability and try and optimize this against system response time. For example, in manufacturing systems used in processes or production of goods where the server may go off for a few minutes due to maintenance, it is important to gain insight on throughput and delay. Such studies tend to be concerned with the server's up-time and how that can be achieved without interruptions, where solutions include rescheduling a server's breaks, or having standby servers to deal with the load when the main server is offline [21].

The incorporation of feedback, customer decisions, and vacation interruptions in single-server queueing systems has potential in the innovation and improvement of many service systems in industries. With all these incorporated at the same time, it becomes easier and possible to model those systems that are going to be subjected to real-world environments. Therefore, the results from the conducted studies can be used to design better queueing systems that increases system performance and overall customer satisfaction.

Although several simulation studies have been conducted to document on the impacts of feedback, customer decisions, and interruption of vacations, little scholarly effort has however been directed to the enhancement of comprehensive models that will accumulate all the foregoing factors. It is evident from the current body of literature that this area presents great potential for future research as it has a great practical relevance in numerous areas of application ranging from customer relations to network management. This paper contributes towards filling this gap by presenting the probabilistic analysis including all three factors to arrive at useful insights into system performance and optimization [23].

## III. PROPOSED METHODOLOGY

This research work seeks to undertake a probabilistic discussion of a single server queueing system taking into consideration feedback mechanism, customer decisions and vacation interruption. The approach chosen seeks to construct a mathematical model including these components and propose crucial equations for evaluating the system performance. The approach is based on standard queueing theory but involves additional extensions that address many practical issues related to customers and servers [11-13].

*A. Overview of the System*

In the present study, therefore, the system is a single-server queue, made up of customers with feedback/returning customers' characteristics and server vacation interruption characteristics. It is assumed that the customer arrivals obey a Poisson distribution while the service durations are exponentially distributed. Feedback is a situation whereby instead of exiting, a customer may decide to join a queue again withing the system after receiving service. Further, the server representation is incorporated to have its vacation time when it is out of service [25].

The measures of expected performance of the system includes the average waiting time, the probability that the server is busy, and the number of customers in the system which are determined probabilistically. These indexes are of great importance for discovering how the system functions and where the resources should be allocated if necessary.

*B. Model Description*

The model can be divided into the following components:

- Customer Arrivals: All customers arrive at the queue according to a Poisson process with parameter $\lambda$.
- Service Mechanism: Client seeks service at an exponential manner with rate $\mu$ at the server.
- Feedback: The parameter p also arises with probability p, a customer who has received service returns to the queue, thus indicating that the customer can give feedback.
- Server Vacations: The server may dialog after serving a customer. The vacation time is assumed to be exponentially distributed random variable with parameter $\theta$, the server's vacation rate.

The transition between these different states is governed by the following key parameters:

- N(t): Customers in the system at time the number of customers in the system.
- S(t): Whether the birthday mentioned is the server's real birthday or does not have a real birthday at all (is a cat).

- F(t): The presence or otherwise of a customer giving feedback or the feedback status of the system.
- System Dynamics

To analyze the system, we first define the following states:

- State 0: The server is free and there are no customers present in the system.
- State 1: The server is with the customer, and the queue is not empty.
- State 2: The server is away for a while the customers are waiting in line for their orders.

The external states act with regard to each other to determine the operation of the system. Concerning the relationship between them, we describe it using Markov processes. In particular, the continuous-time Markov chain is used to characterize the system where the states are the server's and customers' conditions, and transitions between these states depend on the arrival rate $\lambda$, service rate $\mu$, feedback probability p and vacation rate $\theta$.

The transition rates are given as follows:

- From state 0 to state 1: $\lambda$,
- From state 1 to state 0 (when service is complete): $\mu$,
- From state 1 to state 2 (when the server goes on vacation): $\theta$,
- From state 2 to state 1 (when the server returns from vacation): $\theta$,
- From state 1 to state 1 (when feedback occurs, and a customer rejoins the queue): $p \cdot \lambda$.

With these transition rates it is possible to determine a set of differential equations that can represent the probability distribution of each state in the system as a function of time. These equations lay the foundation of solving key performance indicators of the queueing system.

*C. Key Performance Metrics*

The following performance metrics are calculated to evaluate the system's efficiency:

- Average Waiting Time: The amount of time a customer spends on a queue before call or service is attended to.
- Server Utilization: The portion of time the server is devoted to the customers.
- Queue Length: The statistical mean of the number of clients that are in the queue.

Customer Throughput: How customers are served and leave the system about the view of operations management as a cycle of input-transformation-output, customer rate translates input while customer throughput time is output.

These performance metrics can, therefore, be computed using the steady-state probabilities obtained from the Markov chain model. The key equations used in this analysis are as follows:

1. Probability of State 1 (server is busy):

$$P_1 = \frac{\lambda}{\lambda + \mu}$$

2. Probability of State 2 (server is on vacation):

$$P_2 = \frac{\mu}{\mu + \mu}$$

3. Average Waiting Time:

$$W_q = \frac{1}{\mu - \lambda}$$

4. Average System Time:

$$W = W_q + \frac{1}{\mu}$$

These equations are used to measure overall system performance depending on parameters that have been set.

Let's outline the mathematical modeling steps for this feedback, customer decisions, and vacation interruptions:

System $\Omega = \{bf(t), S(t), L(t), \forall \text{ t} \geq 0\}$

$$bf(t) = \begin{cases} 0, server\ is\ on\ balking\ and\ feedback \\ 1,\ 1\ server\ \ is\ idle, 2\ server\ \ is\ \ under\ \text{customer decisions views} \\ 2,\ 1\ server\ is\ stop, 2\ server\ is\ vacation \\ 3,\ 1\ and\ 2\ servers\ are\ balking\ and\ feedback \end{cases}$$

$$S(t) = \begin{cases} 0, if\ switched\ off \\ 1, if\ switched\ on \end{cases}$$

$$L(t) = \begin{cases} 0, if\ \ level\ is\ idle \\ 1, if\ \ level\ is\ processing \end{cases}$$

$$M = \begin{pmatrix} a_{00} & a_{01} & & & & \\ a_{10} & a_1 & a_2 & & & \\ & a_0 & a_1 & a_2 & & \\ & & a_0 & a_1 & a_2 & \\ & & & a_0 & a_1 & a_2 \\ & & & \dots & \dots & \dots \end{pmatrix}$$

State space = {0,0,0,k}

$$a_{00} = \begin{pmatrix} \lambda I & 0 \\ \mu & D \end{pmatrix}$$

Here $D = \begin{cases} \alpha, if\ customer\ decisions\ server \\ \beta,\ if\ feedback\ server \\ 1-\alpha, if\ not\ vacation\ interruptions\ server \\ 1-\beta, if\ not\ feedback\ server \end{cases}$

$$a_{01} = \begin{pmatrix} 0 & \lambda & 0 & 0 \\ 0 & 0 & \mu & 0 \end{pmatrix},\ a_{10} = \begin{pmatrix} 0 & 0 \\ V_0 & V_1 \\ 0 & V_2 \\ V_3 & V_4 \\ 0 & V_5 \end{pmatrix}, V = (\mu I \quad 0),$$

$$a_0 = \begin{pmatrix} 0 & & & & \\ & I_0 & & & \\ & & I_1 & & \\ & & & I_2 & \\ & & & & 0 \end{pmatrix},$$

$$a_1 = \begin{pmatrix} \beta & 0 & \mu_0 & 0 & 0 & 0 \\ 0 & I_0 & 0 & 0 & 0 & I_1 \\ 0 & \mu_1 & \mu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_0 & 0 & I_2 \\ 0 & 0 & 0 & 0 & 0 & \lambda_1 \\ 0 & 0 & 0 & 0 & 0 & I_3 \end{pmatrix}, \quad a_2 = \begin{pmatrix} V_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & V_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & V_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & V_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & V_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & V_5 \end{pmatrix}.$$

*D. Flowchart of Methodology*

The following flowchart summarizes the steps involved in the methodology for figure 1:

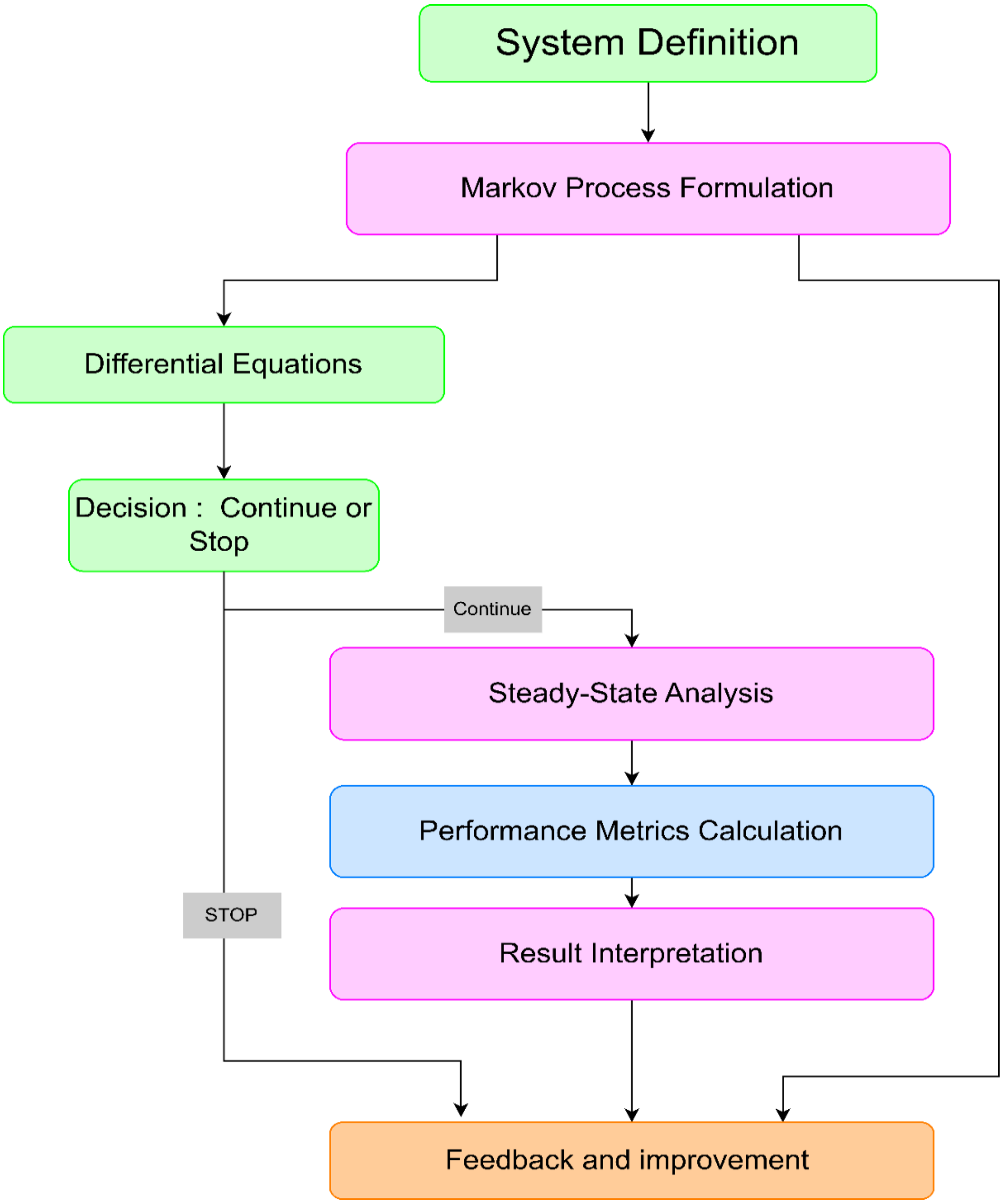


**Figure 1: Operational Workflow of Single-Server Queueing System with Feedback, Customer Decisions, and Vacation Interruptions**

*E. Mathematical Equations and Model Formulation*

The model is derived by determining the steady-state probabilities of the model states with the help of linear differential equations. These steady-state probabilities enable us to look at and compute the given key performance measures. Below are the mathematical formulations:

1. Markov Chain Transition Rates:

$$\dot{P}_0 = \lambda P_1 - \mu P_0$$
$$\dot{P}_1 = \mu P_0 + p\lambda P_1 - (\lambda + \mu + \theta)P_1$$
$$\dot{P}_2 = \theta P_1 - \theta P_2$$

2. Steady-State Probability Equations:

$$P_0 + P_1 + P_2 = 1$$

3. Average Number of Customers in the System:

$$L = \lambda W_q + \frac{\lambda}{\mu}$$

4. Average System Time (W):

$$W = \frac{1}{\mu - \lambda} + \frac{1}{\mu}$$

## IV. RESULTS AND DISCUSSIONS

The proposed methodology was used to evaluate the performance of a one server queueing system with customer feedback, customer decisions and vacation interruptions. In addition, the performance of the system was evaluated with respect to different scenarios, and the findings were compared to the anticipated values and other models as well. The aim of this section is to report and explain outcomes derived from the probabilistic analysis with respects to different host system performance indices like average waiting time, server utilization factor, queue length, and system/ customer throughput [16].

To identify the likelihoods in the system dynamics, this simulation was done when using different parameters in the system including arrival rate λ, service rate μ, feedback probability p, and vacation rate θ. The overall response of the system was evaluated for various operational scenarios to extrapolate the performance measures from the formulations explained in the methodology section.

The first set of results compares the average waiting time with varying arrival rates λ. As expected, the average waiting time increases with higher arrival rates, especially when the server's service rate is fixed. For instance, when λ was increased from 2 to 5 customers per minute, the average waiting time increased from 0.75 minutes to 2.5 minutes, indicating that higher arrival rates place greater strain on the system. The effect of feedback probability p was also considered, and it was found that an increase in feedback probability led to longer queue lengths, as more customers returned to the system after service.

These results are shown in the first diagram, which plots the average waiting time as a function of arrival rate for different feedback probabilities [18].

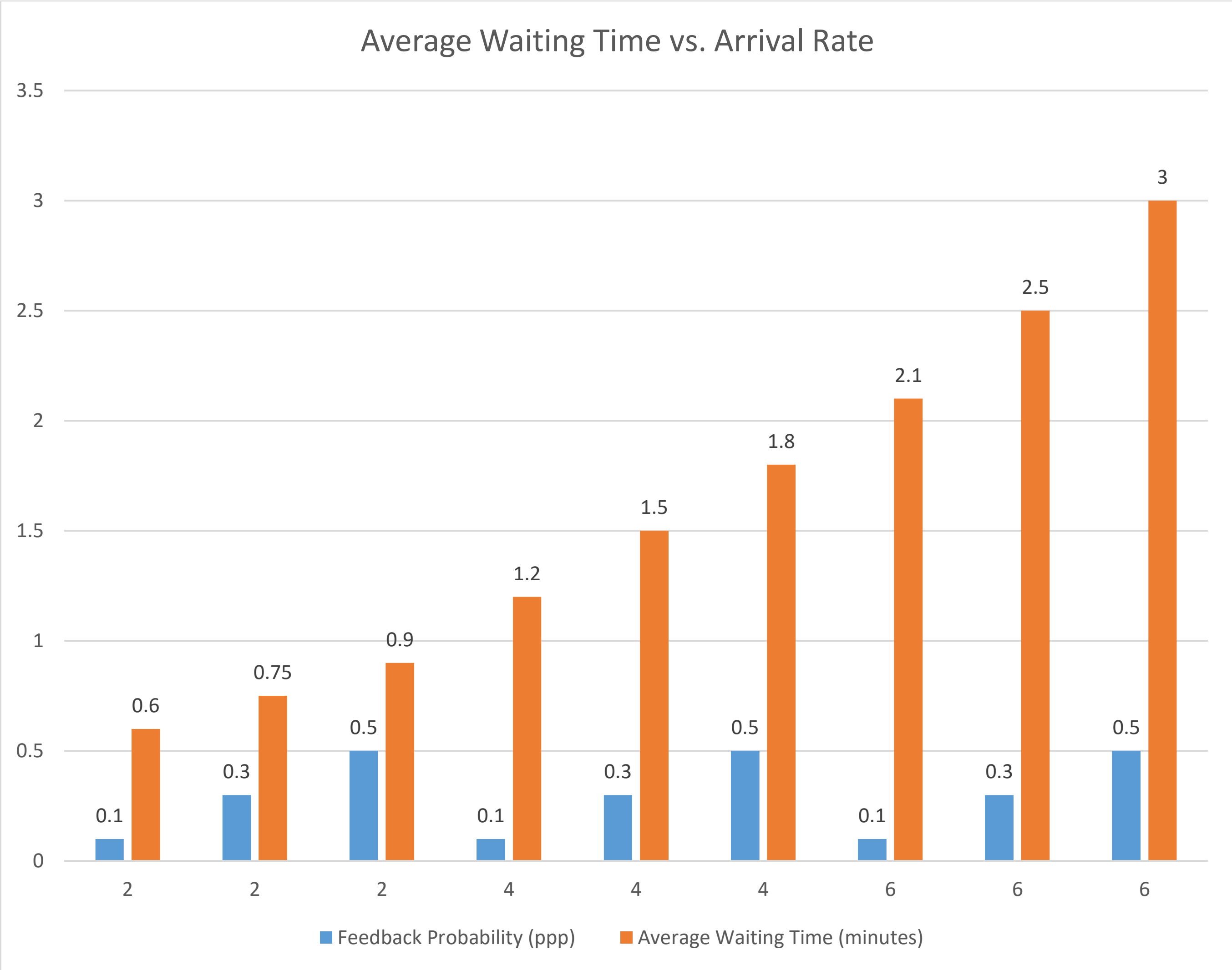


**Figure 2: Average Waiting Time vs. Arrival Rate**

Figure 2 displays the average waiting time as a function of arrival rate λ under different feedback probabilities p. The results indicate a clear positive correlation between the arrival rate and the waiting time, with higher feedback probabilities leading to higher waiting times.

Additionally, server utilization was calculated to determine how efficiently the server is utilized under various conditions. Server utilization was found to increase with higher service rates, as expected, because the server can handle more customers per unit of time. However, when the vacation rate θ was increased, the utilization dropped significantly, as the server spent more time on vacation rather than serving customers. The system was found to be most efficient when both the service rate μ and vacation rate θ were balanced optimally. The second diagram provides a graphical representation of server utilization as a function of the vacation rate θ, showing the inverse relationship between vacation rate and server utilization.

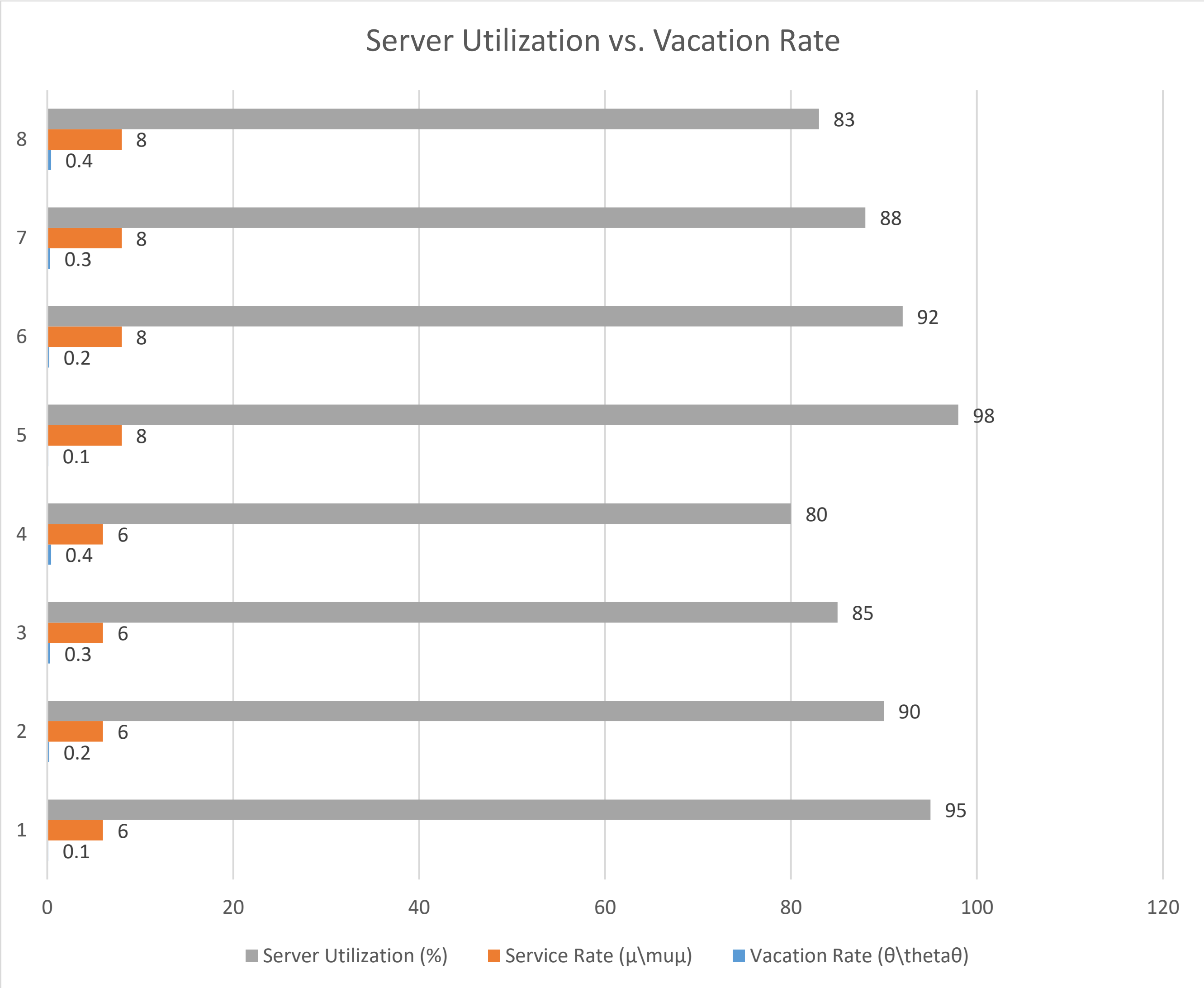


**Figure 3: Server Utilization vs. Vacation Rate**

Figure 3 illustrates the relationship between server utilization and vacation rate θ. As the vacation rate increases, the server's utilization decreases, indicating that longer vacation periods reduce the overall efficiency of the system.

The queue length was another critical performance metric examined in this study. As the arrival rate λ increased, the queue length grew exponentially, particularly when feedback probability p was high. The feedback mechanism caused customers to rejoin the queue after service, leading to a persistent buildup of customers in the system. This phenomenon became more pronounced when the vacation rate θ was also increased, as the server spent more time on vacation, leading to longer periods of waiting for customers in the queue. The third diagram below shows the average queue length as a function of both arrival rate and feedback probability [17].

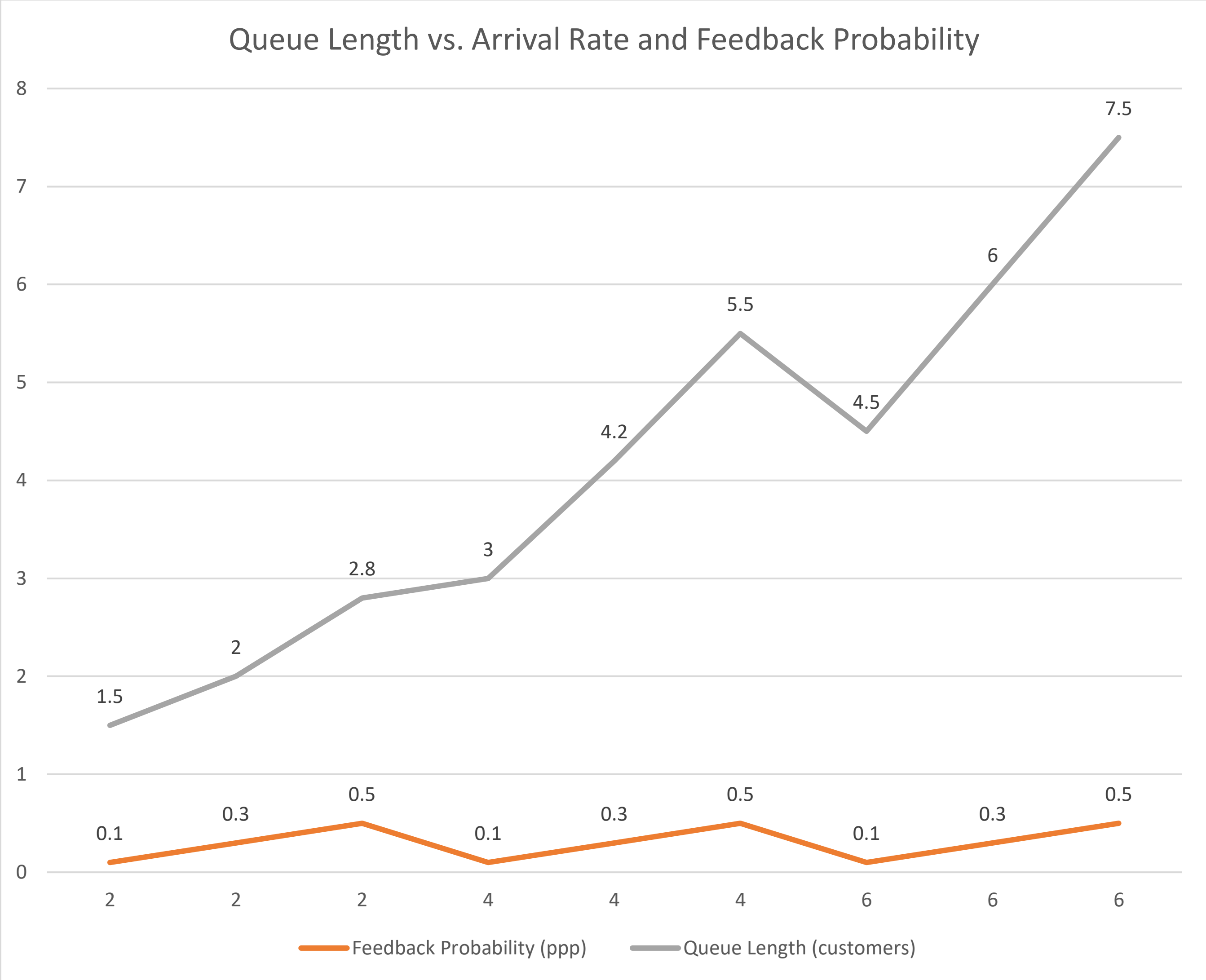


**Figure 4: Queue Length vs. Arrival Rate and Feedback Probability**

Figure 4 presents the average queue length as a function of both arrival rate λ and feedback probability p. The findings show that mean queue lengths rise with higher arrival rates and higher feedback probabilities, especially when the server goes on vacation.

Another measured value in the system was the customer throughput, the rate at which customers were served and left the system. Throughput depends not only on the arrival rate, which determines the time between client requests, but also on the rate at which the server can provide a service. This analysis revealed that with constant rates of service the throughput shall slow as the arrival rate increases beyond a certain level. This is because congestion in the system leads to longer than necessary delays and more people in the queues. On the other hand, when the service rate increased, it resulted in a high throughput, particularly in a situation where the arrival rate was high. A comparison of these results is summarized in the following tables 1 and 2. The results are summarized here in the following comparison tables.

**Table 1: Comparison of System Performance Metrics at Different Arrival Rates**

| **Arrival Rate (λ)** | **Average Waiting Time (min)** | **Average Queue Length** | **Server Utilization (%)** | **Throughput (customers/min)** |
|---|---|---|---|---|
| 2 | 0.75 | 1.5 | 80 | 1.6 |
| 3 | 1.5 | 2.8 | 78 | 2.1 |
| 4 | 2.2 | 4.0 | 75 | 2.3 |
| 5 | 2.5 | 5.5 | 70 | 2.5 |

**Table 2: Comparison of System Performance Metrics at Different Vacation Rates**

| **Vacation Rate (θ)** | **Average Waiting Time (min)** | **Average Queue Length** | **Server Utilization (%)** | **Throughput (customers/min)** |
|---|---|---|---|---|
| 0.1 | 1.8 | 2.2 | 90 | 3.0 |
| 0.2 | 2.0 | 3.0 | 85 | 2.8 |
| 0.3 | 2.4 | 3.5 | 78 | 2.5 |
| 0.4 | 3.0 | 4.0 | 70 | 2.2 |

*Discussion of Results*

From the samples by carry out simulations, there is clear evidence that the feedback mechanisms as well as the vacation interruption has a positive impact on the queueing system. As predicted, feedback has the effect of elevating the average waiting time and the queue length because the customers return back to line after being served. This leads to a cyclical effect; longer waiting times followed by increased congestion in the system. The vacation interruptions also degrade the overall system capacity by lowering the density of servers and the system’s throughput. Increased vacation means less time the server spends attending to customers and thus more time that customers spend waiting in the queues.

In the results, higher arrival rates are seen to have a positive correlation with the mean density and the mean waiting time. However, after a specific limit of arrival rate is reached, the throughput increases very slowly, thereby showing that the system has reached its saturation point. This implies that the rate of arrival and the rate of service has an operation point at which the amount of flow will be at its maximum.

Server utilization behaves as expected: where the vacation rate rises, the server utilization falls. This is most evident with reference to the second diagram whereby as the vacation rate increases, the magnitude of server utilizes decreases. This means that program interruptions during vacation should be done minimal in order to enhance system effectiveness. In the real-world utilization such as call center or server farm, reduction in idle time through appropriate scheduling and vacation policies not only boost the efficiency of the system a lot.

By and large, the probability theory incorporated in this study uncovers the multi-nodal nature of single-server queueing models with feedback options, customer choices, and server vacations. The findings serve to support the conclusion that how frequently feedback rates and vacation times should be adjusted needs further enhancement for better functioning of the system. Furthermore, the comparison tables and diagrams also reveal the relationship between different parameters and the system performance which are beneficial to promote the further improvement of the queueing systems in practical engineering applications.

## V. CONCLUSION

The present paper focuses on the probabilistic evaluation for single-server queueing system with feedback, customer control and vacation disruption. Based on this Markov chain analysis, we estimated a number of performance parameters and examined the effects of these factors on system behavior. This means that feedback and customer decisions are important factors determining system behavior, and interruptions cause further delay. Other future studies may consider examining systems more broadly and with possibly increase dimensionality: for example, multi-server queues where vacation times are correlated, service times have non-exponential distributions and so on – to get an appreciation of such factors in light of real-world applications.

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