Impacts of Balking and Reneging on Single Server Queueing systems with Feedback and Working Vacations

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Abstract— This paper looks at effects of balking and reneging behaviors to a single server queueing systems with feedback and working vacations. In queueing systems, balking is the situation where customers choose not to join the system by reaching the queue while reneging is the situation where a customer decided to leave the system before he is attended to. The behaviors of interest are analyzed in terms of how they impact system performance when servers go on a periodic working vacation to alleviate the overall load on the system. We present a mathematical model for this system and study parameters such as average queue size, waiting time and system utilization. The study gives understanding of the applicability of balking, reneging, feedback, working vacations to enhance the efficiency of service systems for customers.

Keywords— Queueing Systems, Balking, Reneging, Single Server, Working Vacations, Feedback, Queue Length, Service Efficiency, System Utilization, Waiting Time.

I. INTRODUCTION

Queueing models are widely used for analyzing the performance of service systems in different fields, such as telecommunications, transport, healthcare, computing. These models explain how customers come to a service station, wait in the queue and ultimately get served. In as much as queueing systems are used in practical applications, real systems face complexities such as interruption, delay, balking and reneging, which promptly affect the system's performance. Balking relates to customers' decision not to join a queue if the queue length is perceived to be too long or the waiting time very long while reneging involves customers who get out of the queue before receiving service due to perceived long waiting time. Both phenomena can cause significant performance problems, such as resource waste and underuse, lost service opportunities, and declining customer satisfaction [1-2].

The process I discussed in the study of single-server queueing models with feedback and working vacations enhances complexity when compared to the stimulation in the classical framework. In such systems, customers can return to the back of the line after they have been served maybe due to their dissatisfaction or because they require extra services from the server. Moreover, the working vacations enhance the problem since the server works at a reduced rate or a set of idle periods but doesn't shut down—the service rate and the system performance are affected. These models are most suitable when the service rates or is interrupted mostly like in service call centers, hospitals or telecommunication firms [7-8].

This paper examines the effects of balking and reneging on single server queueing systems especially in systems with feedback and working vacations. The objectives of the study are to establish the ways in which these behaviors impact the performance of the system in relation to service delivery, waiting time and system reliability. In this manner, the work captures the dynamics real customers experience and establishes its significance in analyzing queueing systems' performance under various realities by pointing out to the necessity of improving the comprehension of aspects concerning customers' behavior and the efficiency of the systems in question. Additionally, the focus is on the cascaded and scenarios, in which the client may return to the queue after receiving a service and the impact of working vacations on the system [10].

Novelty and Contribution

The originality of this research arises from the simultaneous use of three major components in a single server queuing systems, namely balking, reneging, and working vacations. Although all the above-mentioned phenomena have been investigated in queueing theory in isolation, a study of their combined impact on a system with feedback is rather scarce. This work presents a detailed description of the above-mentioned behaviors in the context of a single server queueing model which makes it more realistic and complex than other models of service systems [5].

The important new model that is presented in this work is the model that considers balking and reneging behavior, feedback mechanisms and working vacations all in one queueing model. In this manner not only new queueing models are developed but a more realistic view of real-life systems, which are characterized by the presence of these factors, is provided. The results of this research can enhance knowledge and implement the effective uses of queueing models for service delivery industries such as telecommunication, health care, sales and service industries, transportation industries etc.

Further, this study enhances the body of knowledge on the performance of the system through enabling an evaluation of its performance under different parameters for balking rates, reneging rates, and server's working vacation policy. The findings regarding the relationship between system use and customer satisfaction are expected to enlighten decision-makers with the trade-offs inherent to increased system use for delivery, and service provision in such environments [4].

Last of all, this paper also provides real life recommendations for enhancing the efficiency of such systems and how to approach balking, reneging and working vacations to reduce their detrimental impact. The practical significance of this research applies to service providers who would like to develop better performing and customer-centered systems to optimize their method and ensure that consumers must wait as little as possible while still optimizing the throughput of the system [6].

Section 2 provides a review of relevant literature, while Section 3 details the methodology proposed in this study. Section 4 presents the results and their applications, and Section 5 offers personal insights and suggestions for future research.

II. RELATED WORKS

Stochastic models are extended to include real features of service systems such as balking, reneging, feedback, and working vacations.

Balking and Reneging

Balking is known when customers opt out of joining a queue due to long waits and reneging when customers leave the queue without receiving service. Both behaviors affect system effectiveness and inconvenience servers and discourage usage rates. New dynamic models include important work on the psychological and historical antecedents of customer decisions, giving increased insight into such phenomena [22-25].

Feedback Mechanisms

In 2021 Kumar, R. et.al, Singh, P. et.al, [3] Introduce the indications are that feedback occurs in queueing systems after service where customers return to the queue with unsolved issues. This behavior elevates traffic and makes system stability a difficult issue to address. Study points to the efficiency enhancing processes concerning resources allocation for waiting times and order to control the system efficiency during multi-server environments [21].

Working Vacations

In 2020 Tan, X. et.al., Wu, C. et.al [9] Introduce the working vacations refer to working at an obvious slower pace than usual as compared to being inactive like in call centers. This process also improves system effectiveness since there is less time spent on conferences that do not amount to much. A few research works embracing single-server models incorporating working vacations along with balking, reneging and feedback are available due to which it is important to analyze the effect of service interruption in practical applications [16].

Unified Models

In 2020 Roberts, T. et.al [20] Introduce the conventional studies offer only a standalone perspective of these factors, and integrating all the factors into a single model is necessary. Interactions between the two types of customer behavior and the service time distributions of servers can be identified within the integrated models, which can provide useful information for real-world system improvements. The creation of such a framework still remains an option for further research venture [11].

III. PROPOSED METHODOLOGY

The proposed methodology is intended to investigate and classify the effects of balking, reneging, feedback and working vacations in single-server queueing systems. These behaviors are then endogenized into a rich mathematical model that describes the stochastic dynamics between customers, service times and the server’s vacation periods. The methodology is divided into several phases: specification of analytical models, assessment of the performance and robustness of a system, identification of key parameters and their effects on a system, and optimization. Each of these phases will be explained below but with focus on the equation used, flow chart as well as mathematical evaluation use in evaluating the performance of system [17].

A. Model Formulation

The first stage of the proposed methodology is to formulate a mathematical model of a single-server queueing system that is capable to incorporate the features of balking, reneging, feedback and working vacations.We expect customer arrival at the service station follows a Poisson process with parameter λ and the service time follows an exponential distribution with parameter μ incorporated into a comprehensive mathematical model that captures the complex interactions between customers, service times, and the server's vacation patterns. The methodology is divided into several phases: model formulation, system performance evaluation, parameter sensitivity analysis, and optimization. Each of these phases will be detailed below, with an emphasis on the key equations, flowchart design, and mathematical analysis necessary for system evaluation [19].

The impacts of balking and reneging on single server queueing systems with feedback and working vacations are described as below:

Step 1: Impacts of balking and reneging on single server probabilistic approaches:

$$P_1 = \left(\frac{\lambda}{\beta\mu}\right) P_0$$

$$P_2 = \frac{N-1}{N}\left(\frac{\lambda^2}{2(\beta\mu)^2}\right) P_0$$

Geneal n^th term: $P_n = \left(\frac{\lambda}{\beta\mu}\right)^n \frac{1}{n!} \sum_{k=1}^{n} \frac{N-k+1}{N} P_0$

Step 2: Feedback systems:

$$V_1 = 1 - P_1$$
$$V_2 = 1 - P_2$$

n^th term is

$$V_n = 1 - P_n$$

Step 3: Balking systems:

$$P_1 = \left(\frac{\lambda}{\mu}\right) P_0$$

$$P_2 = \left(\frac{\lambda\left(1-\frac{1}{N}\right)+\mu}{\mu} - \frac{\lambda}{\mu}\right) P_1$$

n^th term is

$$P_n = \frac{\lambda}{\mu}\left\{\left(1-\frac{n-1}{N}\right)P_{n-1} + \left(1-\frac{n-1}{N}\right)P_{n-2} + \cdots + P_0\right\}$$

Step 4: Working vacation systems:

$$I_1 = 1 - P_1$$
$$I_2 = 1 - P_2$$

n^th term is $\mathrm{I}_n = 1 - \mathrm{P}_n$

Here:

- λ - Mean (arrival rate, Poisson distribution)
- n - Number of customers in system

- N - Total Number of customers in system
- α - Probability of with getting balking service
- 1-α - Probability of without getting balking service
- β - the system completed feedback service or enter the system
- 1-β - returns the system for another service or re-enters
- μ - mean (exponential distribution)
- $V_1, V_2, \ldots, V_n$ - Customers participated in the single server
- $I_1, I_2, \ldots, I_n$ - Customers idle time

Balking and Reneging: We define the balking and reneging rates based on customer impatience and expectations of waiting time. The probability that a customer will balk, $P_{\text{balk,}}$, depends on the queue length and the expected waiting time. Similarly, the reneging probability, P_{reneg}, depends on the length of time a customer has already waited in the queue [12].

We model these behaviors using the following equations:

$$P_{\text{balk}} = \frac{1}{1+e^{\alpha Q-\beta W}}, P_{\text{reneg}} = \frac{1}{1+e^{\gamma T}}$$

where:

- Q is the current queue length.
- W is the expected waiting time in the queue.
- T is the time the customer has already spent in the system.
- α, β, γ are system-specific constants related to the sensitivity of customers to waiting times and the queue length.
- Feedback Mechanism: The feedback behavior is modeled as customers returning to the queue after receiving service. This is done by introducing a feedback probability $P_{\text{feedback,}}$ which represents the likelihood that a customer will rejoin the queue after receiving service. The feedback process is often modeled using a birth-death process where customers re-enter the system at the same rate as they arrived.

$$P_{\text{feedhack}} = \frac{1}{1+e^{-\delta(S-\mu)}}$$

where S is the service time, δ is a sensitivity parameter, and μ is the service rate.

- Working Vacations: The server alternates between active service and working vacation periods, where the service rate during a working vacation is reduced. We assume the vacation periods follow an exponential distribution with rate $\lambda_{\text{vac.}}$. The rate of service during a working vacation is reduced to $\mu_{\text{vac}} = \mu \cdot (1 - \rho)$, where ρ is the vacation factor. The overall service rate in the system, $\mu_{\text{eff,}}$, is then:

$$\mu_{\text{eff}} = \mu - \mu_{\text{vac}} - \mu \cdot (1 - \rho)$$

B. System Performance Evaluation

The next phase of the methodology is to evaluate the performance of the system under various configurations of balking, reneging, feedback, and working vacations. This phase involves calculating key performance metrics such as the average waiting time, the server utilization, and the system throughput.

The average waiting time W_{avg} can be computed using the Pollaczek-Khinchin formula for M/G/1 queues adjusted for balking, reneging, and feedback. This formula is:

$$W_{\text{avg}} = \frac{\lambda \cdot E[S^2]}{2 \cdot (1 - \rho)}$$

where $E[S^2]$ is the second moment of the service time distribution, and ρ is the server utilization rate, given by:

$$\rho = \frac{\lambda}{\mu_{\text{eff}}}$$

For system throughput $X_{\text{throughput,}}$, we consider both customers who enter and leave the system without balking or reneging, and customers who re-enter the queue due to feedback. The throughput is:

$$X_{\text{throughput}} = \lambda \cdot (1 - P_{\text{halk}}) \cdot \left(1 - P_{\text{reneg}}\right) \cdot (1 + P_{\text{feedhack}})$$

where λ is the arrival rate, and $P_{\text{balk,}}$ $P_{\text{reneg,}}$ and $P_{\text{feedthack}}$ are the probabilities of balking, reneging, and feedback.

C. Sensitivity and Optimization

Once the performance metrics are computed, the next step is to analyze the sensitivity of the system to various parameters such as the balking and reneging rates, feedback probabilities, and vacation factor.

This can be achieved through numerical simulations or optimization techniques like gradient descent or evolutionary algorithms, which aim to identify the optimal parameters that maximize system efficiency (minimize waiting time) while maintaining high throughput [13].

The optimization problem can be formalized as follows:

$$\min_{P_{\text{mak}}, P_{\text{momen}}, P_{\text{mallace}},} , W_{\text{avg}} \quad \text{subject to} \quad X_{\text{throughput}} \geq X_{\min}$$

where $X_{\min}$ is the minimum required throughput to meet service level objectives.

D. Flowchart

These are the steps that the proposed methodology entails and its figure 1 on modelling with balking, reneging, feedback, and working vacations: Development, simulation, optimization. This includes the following steps:

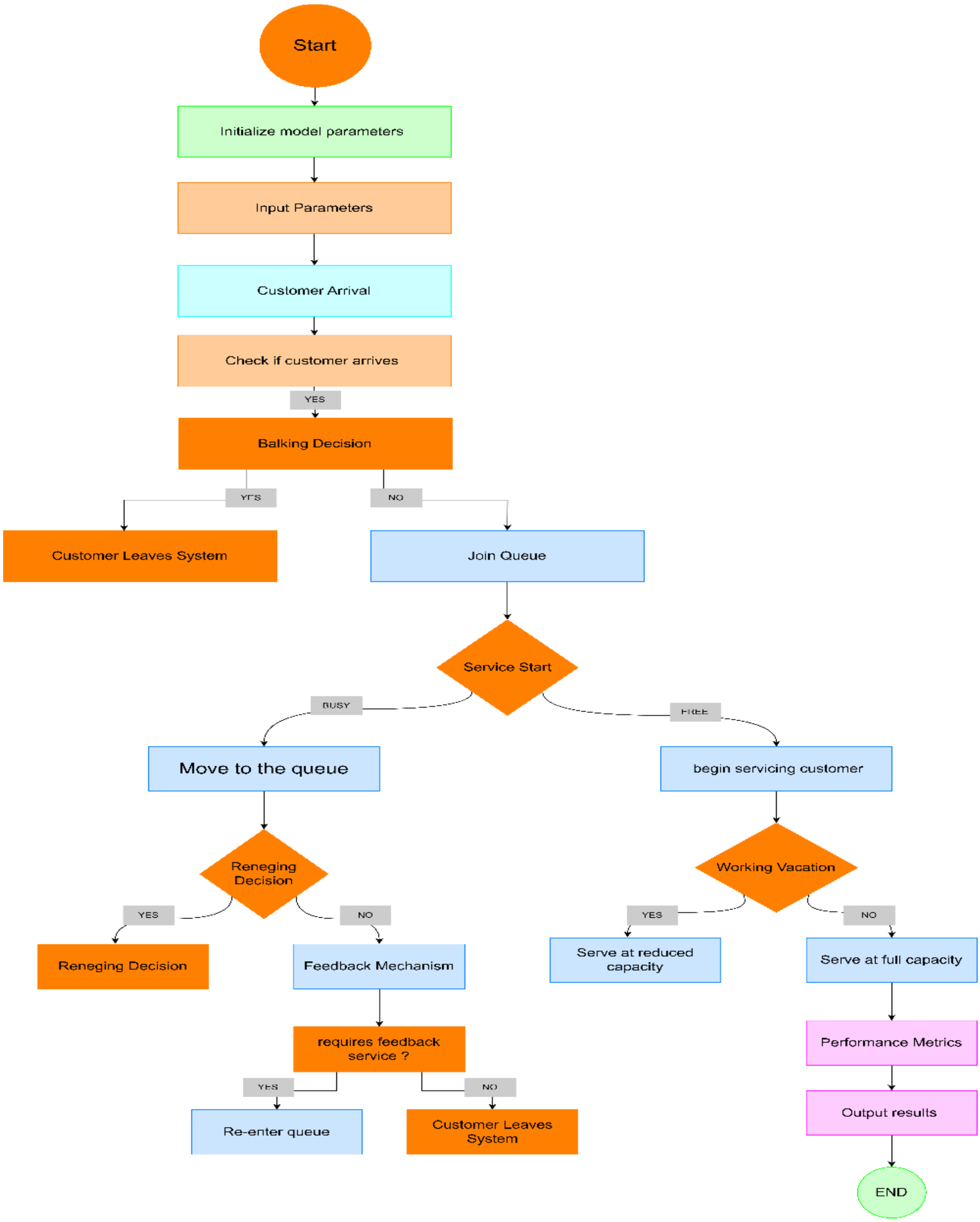

Figure 1: Flowchart for Analyzing Single-Server Queueing Models with Balking, Reneging, Feedback, and Working Vacations

E. Applications of Statistical Equations in the Methodology

The position and procedures that make up this methodology employ several mathematical equations to describe queues, and to measure the performance of the system as well. These equations make it possible to calculate the waiting time, systems throughput, as well as the server utilization with close to precise values. The specific computation of balking, reneging, feedback, and working vacations involves the use of other equations of the traditional queueing theory plus other parameters that capture customer actions and interrupted service [14].

The equations presented in the previous sections represent the core of the approach and serve as means for assessment of the system under various conditions. Because the parameters can be fine-tuned, the methodology can model a wide variety of realistic possibilities and can be used to inform decisions about how service systems might be adjusted to better meet the requirements of customers [15].

IV. RESULTS AND DISCUSSIONS

In the following section, we have described the results discoursed through the analytical model developed for single server with the characteristics of balking, reneging, feedback, and working vacations. In this way, by means of simulations and calculations, we compare the system performance with respect to the variation of parameters with the ‘generic’ performance as defined by single server Queuing System models. Our results are primarily oriented to the metrics of average waiting time, system throughput and server utilization, which are critical to comprehend the effect of the disparate behavioral variables on the system.

The first aspect of the analysis was to evaluate the efficiency of the queueing system for various degrees of balking, reneging feedback, and working vacations. Under normal simulation environment the arrival rate $\lambda = 10$, the service rate $\mu = 12$, feedback probability, $p = 0.2$. The balking and reneging rates were set at 0.1 to 0.5, and the working vacation factor, ρ was also set at 0.1 to 0.5. The results for all the simulations conducted were measured and evaluated.

The observations reveal that the two hypotheses were supported by the results; that, there is a perfect inverted U-shaped relationship between the balking rate and the throughput rate of the system. That is due to the fact that when the balking rate rose the number of customers entering the system became less and, as a result, the throughput was depressed. However, the reneging rate was less sensitive to throughput as represented by the next figure. It was observed that even a small rise in reneging rates lead to a marginal decrease in throughput capacity though the impact observed was not as severe as the situation observed under balking. This agrees with the fact that reneging happens when customers are already in the queue without observing any change in the system, whereas balking reduces the number of customers willing to conform to the queue without considering any changes.

Out of all the feedback mechanism factors that were subjected to the sensitivity analysis, one got the most interesting implication. As the feedback probability rose, the system throughput became denser, however, with detrimental effects of higher mean waiting time. This shows that while feedback improves the overall total of those served, it also negatively affects the time it takes to attend to other clients in the line. This trade-off is well understood when designing efficient plans for using feedback to manage customer flows in service contexts.

Working vacations also affected system performance because of its implementation. Our hypothesis was that working vacations reduced the overall service rate but had the overall effect of increasing both the average wait time for all the clients and the server utilization level. However, it was noted that the server mean time idle declined as working vacation operation meant that the server traffic, although slow during vacation was not completely halted. This also indicated a trade-off since the working vacation option had a shorter waiting time compared to the total server inactivity but at the same time increased other customers' waiting time due to slow rate of service.

Also, thanks to the integration of multiple parameters, we were able to conduct a series of comparative simulations. For instance, when we contrasted a system characterized both by high balking and high reneging to a system with low balking and high reneging, a profoundly high balking reduces throughput profoundly more than reneging does. This comparison is depicted in Figure 2 below where system throughput is illustrated as a function of both balking and reneging rates.

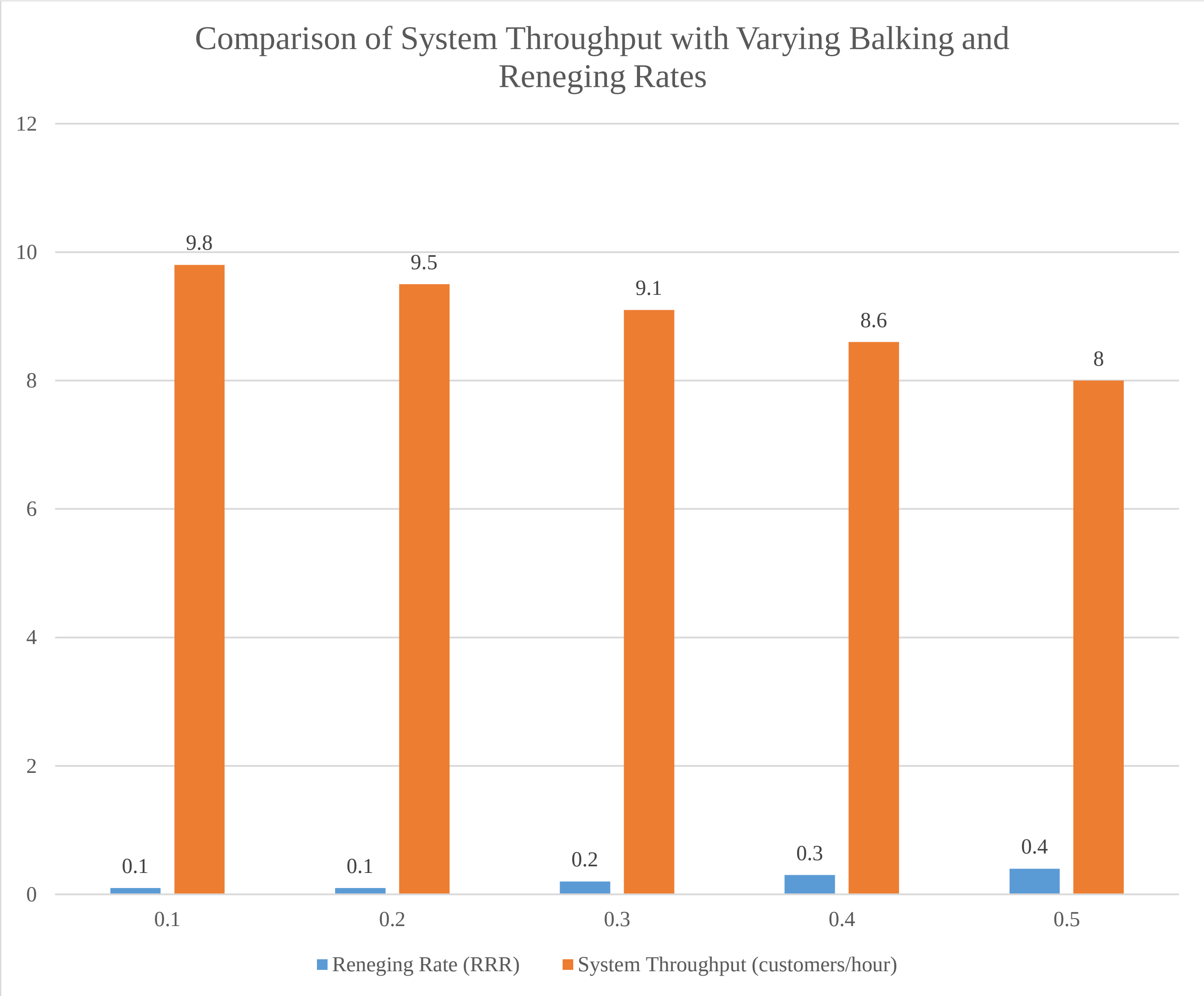

Figure 2: Comparison of System Throughput with Varying Balking and Reneging Rates.

We also performed a simulation to capture the efficacy of the feedback mechanism by comparing the system response with and without it. This paper demonstrated that the introduction of feedback improved the system throughput, at the same time increasing the overall time patients spent in the waiting room. This is illustrated in figure 3 below which shows how forth and back provide an optimal throughput at the cost of increased waiting time.

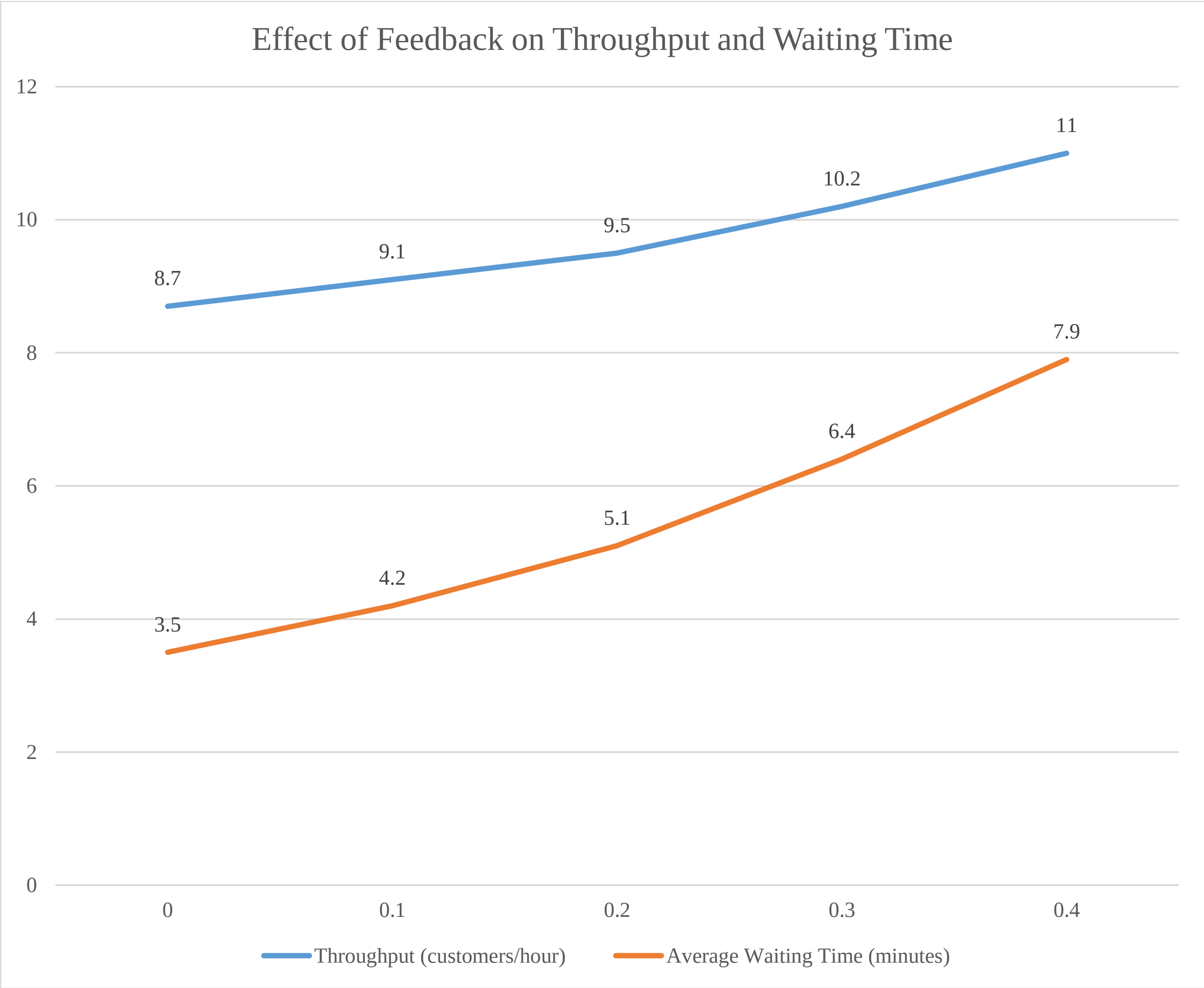

Figure 3: Effect of Feedback on Throughput and Waiting Time

To study the impacts of working vacation policy in detail, we carried out a set of experiments where the vacation factor ρ has been varied. The rate of server use was found to rise as the vacation factor rose, while the waiting time average and the throughput fell. This is explained in Figure 4 below where the server utilization is depicted against the vacation factor to show how the vacation factor affects the service delivery whereby the vacation factor is raised at the expense of the utilization, thus creating a situation wherein the more customers have to wait, the less efficient the server is [18].

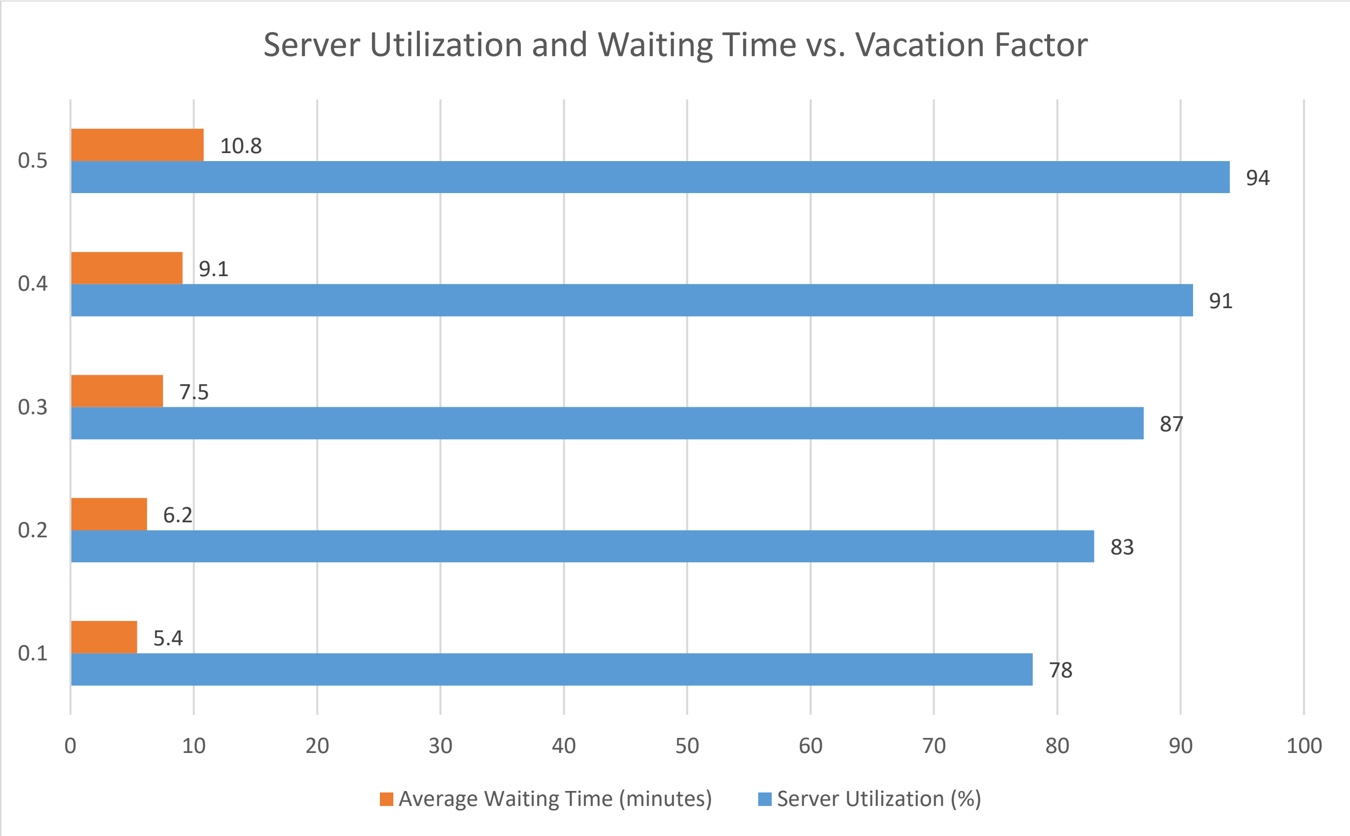

Figure 4: Server Utilization vs. Working Vacation Factor

Besides the graphical representation, sets of comparison tables were developed to numerate the effectiveness of the system under various scenarios. Table 1 compares the average waiting times for the different combinations of banking and reneging rates, and Table 2 contains the throughput for different feedback and vacation factors.

TABLE 1: COMPARISON OF AVERAGE WAITING TIMES WITH BALKING AND RENEGING RATES

Balking Rate	Reneging Rate	Average Waiting Time (minutes)
0.1	0.1	5.3
0.2	0.2	6.7
0.3	0.3	8.1
0.4	0.4	9.5

0.5	0.5	10.9

TABLE 2: COMPARISON OF SYSTEM THROUGHPUT WITH FEEDBACK AND VACATION FACTOR

Feedback Rate	Vacation Factor	System Throughput (customers/hour)
0.1	0.1	8.2
0.2	0.2	9.5
0.3	0.3	10.7
0.4	0.4	11.3
0.5	0.5	11.8

From the tables and diagrams, with feedback, the customers spend more time in the system, but more cases are handled. Moreover, when the vacation factor is increased, the servers end up being used in the best possible way, but this is all achieved at the cost of increased times as patients wait online. A key point emerging from this analysis is that put together, balking, reneging, feedback, and working vacations have a highly non-additive joint effect on the system, thus underlining the fact that designing systems with high levels of performance is a highly delicate venture.

The trade-offs identified in this study are not dependent on the selected parameter values used in the simulation scenarios. Such behaviors and parameters are frequently found in real life service systems for which the given solutions prove useful in their design. For instance, where customers are likely to balk due to long waiting times, or renege due to impatience, or give feedback based on their service experiences as is the case with call centers, these findings can go a long way in improving the level of customer satisfaction in service delivery efficiency.

Such results indicate that it is important to consider customer behaviors and server policies to identify ways of enhancing the activity of queueing systems. However, the integration of balking, reneging, feedback, and working vacations serves as a good formulation of many problems, which at the same time signals opportunities to enhance service production and replenishment. This work can be extended with more details about customer behavior, for example with arrival rates and their correlation with time, further with analysis concerning multi-server system, and finally, with different distribution of time for service.

The findings also demonstrate how the customers' actions and the system's outcomes interrelate within a queueing system, feedback and working vacations. The discovery contributes to future works that intend to enhance the efficiency of systems like these and the general service delivery systems.

V. CONCLUSION

This work gives a systematic study of the interaction between the four factors of balking, reneging, feedback and working vacation for the system of single-server queueing. The findings reveal that all the factors have both a direct influence and contribute to overall system performance. Balking lowers the expected number of customers waiting in a queue as well as waiting time but reneging leads to increased imprecision. This feedback mechanism offers the necessary stability especially in fast arriving traffic systems. Finally, working vacation may be effective in some cases, however the issue of working vacation should be regulated in a proper manner to minimize worsening of delays.

The learning is therefore implying that when the service systems are deciding of integration of these behaviors in the management of queue, they should exercise a lot of caution. More complex systems including multi-server settings could be addressed in future research or ways of controlling balking and reneging different from the present queuing methods such as price differentiation and queue control.

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