NEW GENERALIZED FORMS OF WEAKLY BINARY CLOSED SETS IN BINARY TOPOLOGICAL SPACES

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Abstract. In this paper, we introduce the notions of weakly binary πg-closed sets and weakly binary πg-open sets, which are weaker forms of binary πg-closed sets and binary πg-open sets, respectively. Also, the relationships among related binary generalized closed sets are investigated.

1 Introduction and Preliminaries

In 2011, Nithyanantha Jothi et al., [4] they have introduced topology between two sets and also studied some of their properties. Topology between two sets is the binary structure from X to Y which is defined to be the ordered pairs (A, B) where $A \subseteq X$ and $B \subseteq Y$. In this paper, we introduce the notions of weakly binary πg-closed sets and weakly binary πg-open sets, which are weaker forms of binary πg-closed sets and binary πg-open sets, respectively. Also, the relationships among related binary generalized closed sets are investigated.

Let X and Y be any two nonempty sets. A binary topology [4] from X to Y is a binary structure $\mathcal{M} \subseteq \mathbb{P}(X) \times \mathbb{P}(Y)$ that satisfies the axioms namely

1. (ϕ, ϕ) and $(X, Y) \in \mathcal{M}$,
2. $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$, and
3. If $\{(A_\alpha, B_\alpha): \alpha \in \delta\}$ is a family of members of $\mathcal{M}$, then $(\bigcup_{\alpha\in\delta} A_\alpha, \bigcup_{\alpha\in\delta} B_\alpha) \in \mathcal{M}$.

If $\mathcal{M}$ is a binary topology from X to Y then the triplet $(X, Y, \mathcal{M})$ is called a binary topological space and the members of $\mathcal{M}$ are called the binary open subsets of the binary topological space $(X, Y, \mathcal{M})$. The elements of $X \times Y$ are called the binary points of the binary topological space $(X, Y, \mathcal{M})$. If $Y = X$ then $\mathcal{M}$ is called a binary topology on X in which case we write $(X, \mathcal{M})$ as a binary topological space.

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Definition 1.1 [4] Let X and Y be any two nonempty sets and let (A,B) and $(C,D) \in \mathbb{P}(X) \times \mathbb{P}(Y)$. We say that $(A,B) \subseteq (C,D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 1.2 [4] Let $(X,Y,\mathcal{M})$ be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A,B) is called binary closed in $(X,Y,\mathcal{M})$ if $(X \backslash A, Y \backslash B) \in \mathcal{M}$.

Proposition 1.3 [4] Let $(X,Y,\mathcal{M})$ be a binary topological space and $(A,B) \subseteq (X,Y)$.
Proof. Let $(A,B)^{1*} = \cap \{A_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)\}$ and $(A,B)^{2*} = \cap \{B_\alpha : (A_\alpha, B_\alpha)$ is binary closed and $(A,B) \subseteq (A_\alpha, B_\alpha)\}$. Then $((A,B)^{1*}, (A,B)^{2*})$ is binary closed and $(A,B) \subseteq ((A,B)^{1*}, (A,B)^{2*})$.

Proposition 1.4 [4] Let $(X,Y,\mathcal{M})$ be a binary topological space and $(A,B) \subseteq (X,Y)$. Let $(A,B)^{1*} = \cup \{A_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}$ and $(A,B)^{2*} = \cup \{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.

Definition 1.5 [4] The ordered pair $((A,B)^{1*}, (A,B)^{2*})$ is called the binary closure of (A,B), denoted by $b\text{-}cl(A,B)$ in the binary space $(X,Y,\mathcal{M})$ where $(A,B) \subseteq (X,Y)$.

Definition 1.6 [4] The ordered pair $((A,B)^{1*}, (A,B)^{2*})$ defined in proposition 1.4 is called the binary interior of of (A,B), denoted by $b\text{-}int(A,B)$. Here $((A,B)^{1*}, (A,B)^{2*})$ is binary open and $((A,B)^{1*}, (A,B)^{2*}) \subseteq (A,B)$.

Definition 1.7 [4] Let $(X,Y,\mathcal{M})$ be a binary topological space and let $(x,y) \subseteq (X,Y)$. The binary open set (A,B) is said to be a binary neighbourhood of (x,y) if $x \in A$ and $y \in B$.

Proposition 1.8 [4] Let $(A,B) \subseteq (C,D) \subseteq (X,Y)$ and $(X,Y,\mathcal{M})$ be a binary topological space. Then, the following statements hold:

1. $b\text{-}int(A,B) \subseteq (A,B)$.
2. If (A,B) is binary open, then $b\text{-}int(A,B) = (A,B)$.
3. $b\text{-}int(A,B) \subseteq b\text{-}int(C,D)$.
4. $b\text{-}int(b\text{-}int(A,B)) = b\text{-}int(A,B)$.
5. $(A,B) \subseteq b\text{-}cl(A,B)$.
6. If (A,B) is binary closed, then $b\text{-}cl(A,B) = (A,B)$.
7. $b\text{-}cl(A,B) \subseteq b\text{-}cl(C,D)$.
8. $b\text{-}cl(b\text{-}cl(A,B)) = b\text{-}cl(A,B)$.

Definition 1.9 A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is called

1. a binary semi open set [8] if $(A,B) \subseteq b\text{-}cl(b\text{-}int(A,B))$.
2. a binary regular open set [7] if $(A,B) = b\text{-}int(b\text{-}cl(A,B))$.
3. a binary α-open [2] if $(A,B) \subseteq b\text{-}int(b\text{-}cl(b\text{-}int(A,B)))$.
4. a binary β-open [3] if $(A,B) \subseteq b\text{-}cl(b\text{-}int(b\text{-}cl(A,B)))$.
5. a binary π-open [10] if the finite union of binary regular-open sets.

Definition 1.10 A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called

1. a binary g-closed set [5] if $b\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open.
2. a binary sg-closed set [9] if $bs\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary semi open.
3. a binary $g\alpha$-closed set [1] if $b\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary α-open.
4. a binary αg-closed set [1] if $b\alpha\text{-}cl(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open.
5. a binary πg-closed [10] if $b\text{-}cl(A, B) \subseteq (U, V)$, whenever $(A, B) \subseteq (U, V)$ and (U,V) is binary π-open.
6. $brwg$-closed set [3] if $b\text{-}cl(b\text{-}int(A, B)) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary regular open.

2 Some weakly binary generalized closed sets

Definition 2.1 A subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is called

1. a weakly binary πg-closed (briefly $wb\pi g$-closed) set if $b\text{-}cl(b\text{-}int(A, B)) \subseteq (E, F)$ whenever $(A, B) \subseteq (E, F)$ and (E, F) is $b\pi$-open.
2. a wbg-closed set if $b\text{-}cl(b\text{-}int(A, B) \subseteq (U, V)$ whenever $(A, B) \subseteq (U, V)$ and (U, V) is binary open.

The complement of closed set is called an open set.

Proposition 2.2 In a binary topological space $(X, Y, \mathcal{M})$, every $b\pi g$-closed set is $wb\pi g$-closed. Let (A, B) be a $b\pi g$-closed set. Then $b\text{-}cl(A, B) \subseteq (G, H)$, whenever $(A, B) \subseteq (G, H)$ and (G, H) is $b\pi$-open in (X, Y). Since (G, H) is $b\pi$-open, we have the finite union of binary regular open sets. Thus $(G, H) = b\text{-}int(b\text{-}cl(G, H))$ and $b\text{-}cl(b\text{-}int(G, H)) \subseteq b\text{-}int(b\text{-}cl(G, H))$. Therefore, (G, H) is a $b\pi$-open in (X, Y). This shows that (A, B) is a $wb\pi g$-closed in (X, Y).

Remark 2.3 The converse of Proposition 2.2 is need not be true in general as shown in the following example.

Example 2.4 Let $X = \{a, b, c\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi, \phi), (\{a\}, \{1\}), (\{b\}, \phi), (\{b\}, \{2\}), (\{a, b\}, \{1\}), (\{a, b\}, Y), (X, Y)\}$. Then $b\pi g$-closed sets are (ϕ, ϕ), $(\{c\}, \phi)$, $(\{c\}, \{1\})$, $(\{c\}, Y)$, $(\{a, c\}, \phi)$, $(\{a, c\}, \{1\})$, $(\{a, c\}, \{2\})$, $(\{a, c\}, Y)$, $(\{b, c\}, \phi)$, $(\{b, c\}, \{1\})$, $\{b, c\}, \{2\})$, $(\{b, c\}, Y)$, (X, ϕ), $(X, \{1\})$, $(X, \{2\})$, (X, Y) and $wb\pi g$-closed sets are (ϕ, ϕ), $(\phi, \{1\})$, $(\phi, \{2\})$, (ϕ, Y), $(\{a\}, \phi)$, $(\{a\}, \{2\})$, $(\{c\}, \phi)$, $(\{c\}, \{1\})$, $(\{c\}, Y)$, $(\{a, c\}, \phi)$, $(\{a, c\}, \{1\})$, $(\{a, c\}, \{2\})$, $(\{a, c\}, Y)$, $(\{b, c\}, \phi)$, $(\{b, c\}, \{1\})$, $\{b, c\}, \{2\})$, $(\{b, c\}, Y)$, (X, ϕ), $(X, \{1\})$, $(X, \{2\})$, (X, Y). It is clear that the subset $(\phi, \{2\})$ is $wb\pi g$-closed but not $b\pi g$-closed in $(X, Y, \mathcal{M})$.

Proposition 2.5 In a binary topological space $(X, Y, \mathcal{M})$, every $wb\pi g$-closed set is $brwg$-closed.

Proof. Let (A, B) be any $wb\pi g$-closed set and let (E, F) be binary regular open set containing (E, F). Then (E, F) is a $b\pi$-open set containing (A, B). We have $b\text{-}cl(b\text{-}int(A, B)) \subseteq (E, F)$. Thus (A, B) is $brwg$-closed.

Remark 2.6 The converse of Proposition 2.5 is need not be true in general as shown in the following example.

Example 2.7 Let $X = \{1,2\}$, $Y = \{a, b\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{b\}), (\{1\}, \{a\}), (\{1\}, Y), (X, Y)\}$. Then the $wb\pi g$-closed sets are (ϕ, ϕ), $(\phi, \{a\})$, (ϕ, Y), $(\{1\}, \phi)$, $(\{2\}, \phi)$, $(\{2\}, \{a\})$, $(\{2\}, \{b\})$, $(\{2\}, Y)$, (X, ϕ), $(X, \{a\})$, $(X, \{a\})$, $(X, \{b\})$, (X, Y) and $brwg$-closed sets are (ϕ, ϕ), $(\phi, \{a\})$, (ϕ, Y), $(\{1\}, \phi)$, $(\{1\}, \{b\})$, $(\{1\}, Y)$, $(\{2\}, \phi)$, $(\{2\}, \{a\})$, $(\{2\}, \{b\})$, $(\{2\}, Y)$, (X, ϕ), $(X, \{a\})$, $(X, \{a\})$, $(X, \{b\})$, (X, Y). It is clear that the subset $(\{1\}, \{b\})$ is $brwg$-closed but not a $wb\pi g$-closed.

Proposition 2.8 In a binary topological space $(X, Y, \mathcal{M})$, every wbg-closed set is $wb\pi g$-closed.

Proof. Let (A, B) be any wbg-closed set and let (E, F) be $b\pi$-open set containing (A, B). Then (E, F) is an binary open set containing (A, B). We have $b\text{-}cl(b\text{-}int(A, B)) \subseteq (E, F)$. Thus (A, B) is $wb\pi g$-closed.

Remark 2.9 The converse of Proposition 2.8 is need not be true in general as shown in the following example.

Example 2.10 Let $X = \{a, b\}$, $Y = \{1,2\}$ and $\mathcal{M} = \{(\phi, \phi), (\phi, \{1\}), (\phi, \{2\}), (\phi, Y), (\{a\}, \{1\}), (\{a\}, Y), (\{b\}, \{1\}), (\{b\}, Y), (X, \{1\}), (X, Y)\}$. Then $wb\pi g$-closed sets are $\mathbb{P}(X) \times \mathbb{P}(Y)$ and wbg-closed sets are (ϕ, ϕ), $(\phi, \{2\})$, $(\{a\}, \phi)$, $(\{a\}, \{2\})$, $(\{b\}, \phi)$, $(\{b\}, \{2\})$, (X, ϕ), $(X, \{1\})$, $(X, \{2\})$, (X, Y). It is clear that the subset $(\{a\}, \{1\})$ is $wb\pi g$-closed but not a wbg-closed.

Theorem 2.11 If a subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is both binary closed and $b\alpha g$-closed, then it is $wb\pi g$-closed in $(X, Y, \mathcal{M})$.

Proof. Let (A, B) be an $b\alpha g$-closed set in $(X, Y, \mathcal{M})$ and (E, F) be an $b\pi$-open set containing (A, B). Then (E, F) is binary open containing (A, B) and so $(E, F) \supseteq b\alpha\text{-}cl(A, B) = (A, B) \cup b\text{-}cl(b\text{-}int(b\text{-}cl(A, B)))$. Since (A, B) is binary closed, $(E, F) \supseteq b\text{-}cl(b\text{-}int(A, B))$ and hence (A, B) is $wb\pi g$-closed in $(X, Y, \mathcal{M})$.

Theorem 2.12 If a subset (A, B) of a binary topological space $(X, Y, \mathcal{M})$ is both $b\pi$-open and $wb\pi g$-closed, then it is binary closed.

Proof. Since (A, B) is both $b\pi$-open and $wb\pi g$-closed, $(A, B) \supseteq b\text{-}cl(b\text{-}int(A, B)) = b\text{-}cl(A, B)$ and hence (A, B) is binary closed in $(X, Y, \mathcal{M})$.

Corollary 2.13 If a subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is both $b\pi$-open and $wb\pi g$-closed, then it is both binary regular open and binary regular closed in $(X,Y,\mathcal{M})$.

Proof. Since (A,B) is both $b\pi$-open and $wb\pi g$-closed, $(A,B) \supseteq b\text{-}cl(b\text{-}int(A,B)) \supseteq b\text{-}int(b\text{-}cl(A,B)) = b\text{-}cl(A,B) = (A,B)$ and hence (A,B) is both binary regular open and binary regular closed in $(X,Y,\mathcal{M})$.

Theorem 2.14 Let $(X,Y,\mathcal{M})$ be a $b\pi g\text{-}T_{1/2}$ space and $(A,B) \subseteq (X,Y)$ be $b\pi$-open. Then, (A,B) is $wb\pi g$-closed if and only if (A,B) is $b\pi g$-closed.

Proof. Let (A,B) be $b\pi g$-closed. By Proposition 2.2, it is $wb\pi g$-closed. Conversely, let (A,B) be $wb\pi g$-closed. Since (A,B) is $b\pi$-open, by Theorem 2.12, (A,B) is binary closed. Since (X,Y) is $b\pi g\text{-}T_{1/2}$, (A,B) is $b\pi g$-closed.

Theorem 2.15 A subset (A,B) is $wb\pi g$-closed if and only if $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non-empty $b\pi$-closed set.

Proof. Necessity. Let (U,V) be a $b\pi$-closed set such that $(U,V) \subseteq b\text{-}cl(b\text{-}int(A,B)) - (A,B)$. Since $(U,V)^c$ is $b\pi$-open and $(A,B) \subseteq (U,V)^c$, from the definition of $wb\pi g$-closed set it follows that $b\text{-}cl(b\text{-}int(A,B)) \subseteq (U,V)^c$. ie. $(U,V) \subseteq (b\text{-}cl(b\text{-}int(A,B)))^c$. This implies that $(U,V) \subseteq (b\text{-}cl(b\text{-}int(A,B))) \cap (b\text{-}cl(b\text{-}int(A,B)))^c = (\phi,\phi)$.

Sufficiency. Let $(A,B) \subseteq (G,H)$, where (G,H) is $b\pi$-open set in (X,Y). If $b\text{-}cl(b\text{-}int(A,B))$ is not contained in (G,H), then $b\text{-}cl(b\text{-}int(A,B)) \cap (G,H)^c$ is a non-empty $b\pi$-closed subset of $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$, we obtain a contradiction. This proves the sufficiency and hence the theorem.

Corollary 2.16 A $wb\pi g$-closed set (A,B) is binary regular closed if and only if $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ is $b\pi$-closed and $b\text{-}cl(b\text{-}int(A,B)) \supseteq (A,B)$.

Proof. Necessity. Since the subset (A,B) is binary regular closed, $b\text{-}cl(b\text{-}int(A,B)) - (A,B) = (\phi,\phi)$ is binary regular closed and hence $b\pi$-closed.

Sufficiency. By Theorem 2.15, $b\text{-}cl(b\text{-}int(A,B)) - (A,B)$ contains no non-empty $b\pi$-closed set. That is $b\text{-}cl(b\text{-}int(A,B)) - (A,B) = (\phi,\phi)$. Therefore (A,B) is binary regular closed.

Theorem 2.17 Let $(X,Y,\mathcal{M})$ be a binary topological space and $(G,H) \subseteq (A,B) \subseteq (X,Y)$. If (G,H) is $wb\pi g$-closed set relative to (A,B) and (A,B) is both binary open and $wb\pi g$-closed subset of (X,Y) then (G,H) is $wb\pi g$-closed set relative to (X,Y).

Proof. Let $(G,H) \subseteq (J,K)$ and (J,K) be a $b\pi$-open in $(X,Y,\mathcal{M})$. Then $(G,H) \subseteq (A,B) \cap (J,K)$. Since (G,H) is $wb\pi g$-closed relative to (A,B), $b\text{-}cl(b\text{-}int(G,H)) \subseteq (A,B) \cap (J,K)$. That is $(A,B) \cap b\text{-}cl(b\text{-}int(G,H)) \subseteq (A,B) \cap (J,K)$. We have $(A,B) \cap b\text{-}cl(b\text{-}int(G,H)) \subseteq (J,K)$ and then $((A,B) \cap b\text{-}cl(b\text{-}int(G,H))) \cup (b\text{-}cl(b\text{-}int(G,H)))^c \subseteq (J,K) \cup (b\text{-}cl(b\text{-}int(G,H)))^c$. Since (A,B) is $wb\pi g$-closed in $(X,Y,\mathcal{M})$, we have $b\text{-}cl(b\text{-}int(A,B)) \subseteq (J,K) \cup (b\text{-}cl(b\text{-}int(G,H)))^c$. Therefore $b\text{-}cl(b\text{-}int(G,H)) \subseteq (J,K)$ since $b\text{-}cl(b\text{-}int(G,H))$ is not contained in

$(b\text{-}cl(b\text{-}int(G,H)))^c$. Thus (G,H) is $wb\pi g$-closed set relative to $(X,Y,\mathcal{M})$.

Corollary 2.18 If (A,B) is both binary open and $wb\pi g$-closed and (J,K) is binary closed in a binary topological space $(X,Y,\mathcal{M})$, then $(A,B)\cap(J,K)$ is $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

Proof. Since (J,K) is binary closed, we have $(A,B)\cap(J,K)$ is binary closed in (A,B). Therefore $b\text{-}cl((A,B)\cap(J,K)) = (A,B)\cap(J,K)$ in (A,B). Let $(A,B)\cap(J,K)\subseteq(O,P)$, where (O,P) is $b\pi$-open in (A,B). Then $b\text{-}cl(b\text{-}int((A,B)\cap(J,K)))\subseteq(O,P)$ and hence $(A,B)\cap(J,K)$ is $wb\pi g$-closed in (A,B). By Theorem 2.17, $(A,B)\cap(J,K)$ is $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

Theorem 2.19 If (A,B) is $wb\pi g$-closed and $(A,B)\subseteq(J,K)\subseteq b\text{-}cl(b\text{-}int(A,B))$, then (J,K) is $wb\pi g$-closed.

Proof. Since $(A,B)\subseteq(J,K)$, $b\text{-}cl(b\text{-}int(J,K))-(J,K)\subseteq b\text{-}cl(b\text{-}int(A,B))-(A,B)$. By Theorem 2.15 $b\text{-}cl(b\text{-}int(A,B))-(A,B)$ contains no non-empty $b\pi$-closed set and so $b\text{-}cl(b\text{-}int(J,K))-(J,K)$. Again by Theorem 2.15, (J,K) is $wb\pi g$-closed.

Theorem 2.20 Let $(X,Y,\mathcal{M})$ be a binary topological space and $(A,B)\subseteq(J,K)\subseteq(X,Y)$ and (J,K) be binary open. If (A,B) is $wb\pi g$-closed in (X,Y), then (A,B) is $wb\pi g$-closed relative to (J,K).

Proof. Let $(A,B)\subseteq(J,K)\cap(G,H)$ where (G,H) is $b\pi$-open in $(X,Y,\mathcal{M})$. Since (A,B) is $wb\pi g$-closed in $(X,Y,\mathcal{M})$, $(A,B)\subseteq(G,H)$ implies $b\text{-}cl(b\text{-}int(A,B))\subseteq(G,H)$. That is $(J,K)\cap(b\text{-}cl(b\text{-}int(A,B)))\subseteq(J,K)\cap(G,H)$ where $(J,K)\cap b\text{-}cl(b\text{-}int(J,K))$ is binary closure of binary interior of (A,B) in $(U,V,\mathcal{M}')$. Thus (A,B) is $wb\pi g$-closed relative to $(U,V,\mathcal{M}')$.

Theorem 2.21 If a subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is binary nowhere dense, then it is $wb\pi g$-closed.

Proof. Since $b\text{-}int(A,B)\subseteq b\text{-}int(b\text{-}cl(A,B))$ and (A,B) is binary nowhere dense, $b\text{-}int(A,B)=(\phi,\phi)$. Therefore $b\text{-}cl(b\text{-}int(A,B))=(\phi,\phi)$ and hence (A,B) is $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

The converse of Theorem 2.21 need not be true as seen in the following Example.

Example 2.22 Let $X=\{a,b\}$, $Y=\{1,2\}$ and $\mathcal{M}=\{(\phi,\phi),(\phi,\{1\}),(\phi,\{2\}),(\phi,Y),(\{a\},\{1\}),(\{a\},Y),(\{b\},\{2\}),(\{b\},Y),(X,Y)\}$. Then the subset $(\{a\},\{2\})$ is $wb\pi g$-closed in $(X,Y,\mathcal{M})$ but not binary nowhere dense in $(X,Y,\mathcal{M})$.

Remark 2.23 The following example shows that, if any subsets (A,B) and (C,D) of binary topological space (X,Y) are $wb\pi g$-closed, then their intersection need not be $wb\pi g$-closed.

Example 2.24 In Example 2.7, the subsets (ϕ,Y) and $(\{2\},\{b\})$ are $wb\pi g$-closed but their intersection $(\phi,\{b\})$ is not $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

Proposition 2.25 In a binary topological space $(X,Y,\mathcal{M})$, every $bg\alpha$-closed set is $wb\pi g$-closed.

Proof. Let (A,B) be any $bg\alpha$-closed subset of $(X,Y,\mathcal{M})$ and let (G,H) be an $b\pi$-open set containing (A,B). Then (G,H) is $b\alpha$-open set containing (A,B). Now $(C,D) \supseteq b\alpha$-$cl(A,B) \supseteq b$-$cl(b$-$int(b$-$cl(A,B))) \supseteq b$-$cl(b$-$int(A,B))$. Thus (A,B) is $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

Remark 2.26 The following example shows that converse of Proposition 2.25 is need not be true in general.

Example 2.27 Let $X=\{a,b\}$, $Y=\{1,2\}$ and $\mathcal{M}=\{(\phi,\phi),\ (\phi,\{1\}),\ (\{a\},\{1\}),\ (\{b\},\{1\}),\ (X,Y)\}$. Then $wb\pi g$-closed sets are $\mathbb{P}(X)\times\mathbb{P}(Y)$ and $bg\alpha$-closed sets are (ϕ,ϕ), $(\phi,\{2\})$, $(\{a\},\{2\})$, $(\{b\},\{2\})$, $(X,\{2\})$, (X,Y). It is clear that the subset $(\{a\},\{1\})$ is $wb\pi g$-closed but not $bg\alpha$-closed in $(X,Y,\mathcal{M})$.

Remark 2.28 The following example shows that the family of $wb\pi g$-closedness is independent of the family of binary semi-closedness in $(X,Y,\mathcal{M})$.

Example 2.29 In Example 2.7, the semi-closed sets are (ϕ,ϕ), $(\phi,\{b\})$, $(\{1\},\{a\})$, $(\{2\},\phi)$, $(\{2\},\{b\})$, $(X,\{a\})$, (X,Y). Then the subset $(\phi,\{a\})$ is $wb\pi g$-closed in $(X,Y,\mathcal{M})$ but not binary semi-closed and also the subset $(\phi,\{b\})$ is binary semi-closed but not $wb\pi g$-closed in $(X,Y,\mathcal{M})$.

Theorem 2.30 1. Every $b\pi g$-open set is $wb\pi g$-open in a binary topological space $(X,Y,\mathcal{M})$;
2. Every bwg-open set is $wb\pi g$-open in a binary topological space $(X,Y,\mathcal{M})$.

Proof. The proof follows from Proposition 2.2 and 2.8.

Theorem 2.31 A subset (A,B) of a binary topological space $(X,Y,\mathcal{M})$ is $wb\pi g$-open if $(G,H)\subseteq b$-$int(b$-$cl(A,B))$ whenever $(G,H)\subseteq(A,B)$ and (G,H) is $b\pi$-closed.

Proof. Let (A,B) be any $wb\pi g$-open. Then $(A,B)^c$ is $wb\pi g$-closed. Let (G,H) be a $b\pi$-closed set contained in (A,B). Then $(G,H)^c$ is a $b\pi$-open set in (X,Y) containing $(A,B)^c$. Since $(A,B)^c$ is $wb\pi g$-closed, we have b-$cl(b$-$int((A,B)^c))\subseteq(G,H)^c$. Therefore $(G,H)\subseteq b$-$int(b$-$cl(A,B))$.

Conversely, we suppose that $(G,H)\subseteq b$-$int(b$-$cl(A,B))$ whenever $(G,H)\subseteq(A,B)$ and (G,H) is $b\pi$-closed. Then $(G,H)^c$ is a $b\pi$-open set containing $(A,B)^c$ and $(G,H)^c\supseteq(b$-$int(b$-$cl(A,B)))^c$. It follows that $(G,H)^c\supseteq b$-$cl(b$-$int((A,B)^c))$. Hence $(A,B)^c$ is $wb\pi g$-closed and so (A,B) is $wb\pi g$-open.

Conclusion

Main aim of this paper, we introduced the notions of weakly binary πg-closed sets and weakly binary πg-open sets, which are weaker forms of binary πg-closed sets and binary πg-open sets, respectively. Also, the relationships among related binary generalized closed sets are investigated with suitable examples are given.

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