Analysis of Neutrosophic Dominating Path-Coloring and Multivalued Star Chromatic Numbers in Neutrosophic Graphs

J. Muthuerulappan[1*] and Dr.S. Chelliah[2]

[1]Research Scholar, Reg.no:18221072091013, PG&Research Department of Mathematics, The M.D.T.Hindu College, Tirunelveli-627010, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

Email: muthuerulappanjeyaram@gmail.com

[2]Associate Professor (Rtd), PG &Research Department of Mathematics,The M.D.T.Hindu College, Tirunelveli-627010. Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.

Email: kscmdt@gmail.com

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Abstract

Graph products are a mathematical concept used to derive information about large graphs based on the properties and characteristics of smaller, simpler graphs. By understanding the smaller graphs and how they combine, insights about the larger, more complex graphs can be obtained efficiently. In this research, a new method for analyzing the neutrosophic dominating path-coloring number and the dominant path-coloring number in neutrosophic graphs is presented. Additionally, it computes their multivalued star chromatic number and suggests a novel technique for multivalued star coloring on the corona product of two neutrosophic graphs. Within neutrosophic graphs, the neutrosophic dominant path-coloring number is a measure of connectedness. A neutrosophic graph's multivalued star chromatic number also shows how few colors are needed to color the vertices so that no two neighboring vertices have the same color. This trait is retained in the edge corona product of a star graph with several kinds of graphs, such as path graphs, cycle graphs, complete graphs, or any other simple graphs. The edge corona product of a star graph with various types of graphs, including path graphs, cycle graphs, complete graphs, or any other simple graphs, retains this property.

Keywords: *Domination in neutrosophic graph, Neutrosophic graph,* Vertices, Edges, *Multi–Valued Neutrosophic Graph and Graph coloring*

1. Introduction

Introduced by Azriel Rosenfeld in 1975, fuzzy graph theory is still a relatively new science, but it has grown quickly and has important applications in many other fields. The fuzzy set was first proposed by Garrett [1] and is essentially described by a membership function. Hamidi [2] eventually developed this idea into the intuitionistic fuzzy set, which has a membership function as well as a non-membership function. The total of the degrees of membership and non-membership, in Atanassov's opinion, does not equal one. Broumi discussed several features and went on to examine ideas like independent and connected dominance inside fuzzy and intuitionistic fuzzy graphs[3,4]. Further investigations have explored the concepts of chromatic numbers, super-closing numbers, closing numbers, and neutrosophic chromatic numbers in connection with (dual)coloring, (dual)resolving, and (dual)dominating in (neutrosophic) n-SuperHyperGraphs[5-7]. Research has also looked into independent sets in neutrosophic graphs, the neutrosophic chromatic number based on connectedness, and the many kinds of neutrosophic chromatic numbers, dimensions, and coloring alongside dominance in neutrosophic hypergraphs [8-12]. In this study, three forms of neutrosophic alliances based on connectedness and edges are examined, polynomials linked to numbers in classes of neutrosophic graphs are explored, and the characteristics of SuperHyperGraphs and neutrosophic SuperHyperGraphs are investigated [13-15].

With an emphasis on shared edges and the neutrosophic cardinality of these edges, Garrett *et al.* [16] established the idea of path coloring numbers in neutrosophic graphs, with a variety of useful applications. Think of the following neutrosophic graph: *(V, E, σ, μ)*. Any two vertices, x and y, can be connected by more than one path. Two pathways from x to y are given different colors if they share an edge. The path-coloring set from x to y is the name given to the set of colors utilized in this procedure. The path-coloring number, *L(NTG)*, is the least cardinality $\sum_{e\in s}\sum_{i=1}\mu_i(e)$ among all path-coloring sets between two vertices. Furthermore, the neutrosophic path-coloring number, *Ln(NTG)*, is the minimal neutrosophic cardinality of all path-coloring sets, indicated by *Ss*.

The idea of neutrosophic path-coloring numbers in respect to endpoints inside neutrosophic graphs has been investigated by Garrett *et al*. [17]. Think of the following neutrosophic graph: (*V, E, σ, μ)*. There are several pathways that can link any two specified vertices, x and y. Two pathways from x to y that share an endpoint need to have distinct colors assigned to them. The path-coloring set from x to y is the collection of unique colors employed in this procedure. The path-coloring number, *V(NTG)*, is the least cardinality between all path-coloring sets for two vertices. Furthermore, the neutrosophic path-coloring number, denoted by *Vn(NTG*), is the lowest neutrosophic cardinality $\sum_{x\in Z}\sum_{i=1}^{3}\sigma_i(x)$among all sets *Zs*, which comprise the endpoints corresponding to the path-coloring set *Ss*.

The notion of a worldwide offensive coalition in strong neutrosophic graphs was first presented by Garrett *et al.* [18]. In addition, they looked at sets, minimum sets, numbers, and neutrosophic numbers for a variety of classes of neutrosophic graphs, including full, empty, route, cycle, star, and wheel. This allowed them to define the neutrosophic number in a new way. The minimum-global-offensive-alliance number and the minimal-global-offensive-alliance-neutrosophic number are formed by the minimal set and the optimum set, respectively, and this new description helps to identify them. The global-offensive alliance and the minimal-global-offensive alliance are two sorts of sets that are specified. The minimal-global-offensive alliance concentrates on sets where eliminating any vertex is not feasible, whereas the global-offensive alliance identifies sets more generally. The smallest cardinality among all minimal global offensive alliances in a particular neutrosophic graph is represented by the minimal-global-offensive-alliance number. The m-family of odd complete graphs with a common neutrosophic vertex set, the m-family of neutrosophic stars with a common neutrosophic vertex set, and an additional m-family of odd complete graphs with a common neutrosophic vertex set were also studied.

A technique for determining the longest weakest pathways in specific kinds of neutrosophic graphs has been presented by Garrett *et al*. [19]. Consider of the following neutrosophic graph:

(V, E, σ, μ). If a path from vertex x to vertex y has the highest length, it is said to be the weakest path; this length is known as the weakest number between x and y. The weakest number of NTG is *(V, E, σ, μ)*, which is the maximum value among all vertices and is represented by *W(NTG)*. Furthermore, if a road from x to y has a strength, *μ(uv)*, that is smaller than the strengths of all other pathways from x to y, it is called the neutrosophic weakest path. Between x and y, this strength is known as the neutrosophic weakest number. Between x and y, this strength is known as the neutrosophic weakest number. The neutrosophic weakest number of NTG in this case is the largest value among all vertices *(V, E, σ, μ)*, denoted by *Wn(NTG)*.

Neutronosophic graphs provide significant paths for defining connectivities investigated by Garrett and others [20]. If a path has the least length, or the strongest number between x and y, it is considered the strongest path from vertex x to vertex y in the neutrosophic graph *NTG: (V, E, σ, μ)*. *S(NTG)* is a representation of the strongest number of *NTG (V, E, σ, μ)*, which is the highest value among all vertices. Furthermore, if the strength of a road from x to y, μ(uv), is greater than the strengths of all other paths from x to y, then that path is called the neutron sophic strongest path.

A discovery of Hamiltonian neutrosophic cycles in classes of neutrosophic graphs has been reported by Garrett et al. [21]. Suppose we have a neutrosophic graph *NT G:(V, E, σ, μ)*. Then, for a neutrosophic graph *NTG:(V, E, σ, μ)*, the n-hamiltonian neutrosophic cycle N(HNC) is the number of sequences of consecutive vertices *x1, x2, · · ·, xO(NTG), x1* which are neutrosophic cycles. Additionally, the hamiltonian neutrosophic cycle *M(NTG)* for a neutrosophic graph *NTG:(V, E, σ, μ)* is a sequence of consecutive vertices *x1, x2, · · · ·, xO(NTG), x1* which is a neutrosophic cycle. The final findings include some assertions, comments, illustrations, and explanations regarding a few classes of strong neutrosophic graphs: (strong-)path-, (strong-)cycle-, complete-, and (strong-)star-, (strong-)complete-bipartite-, (strong-)complete-t-partite-, and (strong-)wheel-neutrosophic graphs among other classes.

The notions of minimum dense sets and dense numbers in neutrosophic graphs were first presented by Garrett et al. [22]. In a neutrosophic graph *NTG:(V, E, σ, μ),* a collection of vertices is referred to as a dense set if, for any vertex y outside the set, there is at least one vertex x inside the set such that x and y are endpoints of an edge *xy∈E* and x has more neighbors than y does. The dense number of *NTG,* represented by *D(NTG),* is the lowest cardinality of all dense sets.

The aim of this work is to discover new ideas that are applicable to any class of hypergraphs that are neutrosophic. This study also serves as motivation for practical timetabling and scheduling applications. Colors, dominant sets, and domination are determined in large part by the links between two elements. As a result, they are employed in the definition of novel concepts that result in the coloring, dominating sets, and dominance structures. Everyone was motivated to investigate the behavior of general neutrosophic hyperedges in three different sorts of coloring numbers: dominating numbers, resolving sets, and individuals and families. This was due to the notion of having a general neutrosophic hyperedge. Thus, it is possible to compute the multivalued star chromatic number of a neutrosophic graphs (MVSCNGs) by taking into consideration valid edges and applying the idea of dominance, as inspired by [12–15]. The following is a summary of the study's main contributions:

- This paper introduces a new conceptualization of domination on multivalued star chromatic neutrosophic graphs (MVSCNGs) based on valid edges and vertices.
- Along with examples, this paper also covers the ideas of valid edges, cardinality, and nearby vertices in MVSCNGs.

- This study demonstrates how to compute the corona product of two neutrosophic graphs using MVSCNGs.
- The practical applications of the idea of dominance in multivalued neutrosophic incidence graphs (MVNIGs) are also covered in this study.

The remainder of this essay is structured as follows:

The concepts of dominance in fuzzy graphs, fuzzy incidence graphs, and neutrosophic graphs are presented in this introduction along with the history of incidence and domination graphs and a brief overview of pertinent literature on incidence graphs in fuzzy and neutrosophic settings. Graphs and neutrosophic sets as they are utilized in this study are introduced in Section 2. Section 3 formulates the concept of neutrosophic dominating path-coloring number in multivalued star chromatic neutrosophic graphs (MVSCNGs) and derives key properties. Section 4 summarizes the findings and discusses the limitations and suggested avenues for further research.

2. Preliminaries

The foundational information from the previous section is presented in this subsection along with some updated concepts and explanations.

Definition 2.1 A Neutrosophic set X is contained in another neutrosophic set Y , (i.e) $X \subseteq Z$ if $\forall_A \in A, TM_X(A) \le TM_Y(B), IM_X(A) \le IM_B(A)$and $FM_X(A) \ge FM_B(A)$.

Definition 2.2 A pair $G^* = (V, E)$ is neutrosophic graph defind as,

(i) $V = \{v_1, v_2, ..., v_X\}$denotes the degree of the truth-membership function, indeterminacy function, and falsity-membership function, respectively $TM_1 = V \to [0,1]$, $IM_1 = V \to [0,1]$and $FM_1 = V \to [0,1]$, is called a neutrosophic graph.

$$0 \le TM_X(A) + IM_X(A) + FM_X(A) \le 3$$

(ii) $E \subset V * V$ when, $TM_2 = E \to [0,1]$, $IM_2 = E \to [0,1]$ and $FM_2 = E \to [0,1]$ and are situated so as to

$$TM_2(pq) \le Min\{TM_1(p), TM_1(q)\} ,$$

$$IM_2(pq) \le Min\{IM_1(p), IM_1(q)\} ,$$

$$FM_2(pq) \le Min\{FM_1(p), FM_1(q)\} ,$$

$$\text{and } 0 \le TM_2(pq) + IM_2(pq) + FM_2(pq) \le 3, \forall pq \in E \ .$$

Definition 2.3 A vertex $p \in V$ of a neutrosophic graphs (NG) $G = (V, E)$ is said to be an isolated vertex if $\delta_2(p,q) = 0$ and $\lambda_2(p,q) = 0$ for everyone $q \in V$. That is $N(p) = \phi$;. As a result, no vertex in G is dominated by an isolated vertex [23].

Definition 2.4 If no appropriate subset ψ of Ψ is a dominating set, then a dominating set Ψ of NG is referred to as a minimum dominating set. The inituitionistic fuzzy dominating number, represented by $\gamma_{nt}(G)$, is the minimum cardinality among all minimal dominating sets.

Definition 2.5 If the induced neutrosophic subgraph $\psi = ([V - \Psi], V'', E'')$ is unconnected, then a dominant set Ψ of a neutrosophic graph $G = (V, E)$ is divided. A split domination number is the minimal fuzzy cardinality of a split dominating set, and it is represented as $\gamma_{sd}(G)$.

Definition 2.6 Lower split domination number of neutrosophic graphs of G is the minimum cardinality among all minimal split dominating set and is represented by $\Psi_{sd}^{L}(G)$.

Definition 2.7 The term "highest split domination number of neutrosophic graphs of G" refers to the maximum cardinality among all maximum split dominating sets and is represented by $\Psi_{sd}^{H}(G)$.

Definition 2.8 Let $X = (TM_X, IM_X, FM_X)$ and $Y = (TM_Y, IM_Y, FM_Y)$ be single valued neutrosophic sets on a set A. If $X = (TM_X, IM_X, FM_X)$ is a single valued neutrosophic relation on a set X, then $X = (TM_X, IM_X, FM_X)$ is called a single valued neutrosophic relation on $Y = (TM_Y, IM_Y, FM_Y)$ if

$TM_Y(a,b) \leq Min(TM_X(a), TM_X(b)) \geq Max(IM_X(a), IM_X(b))$
$FM_Y(a,b) \geq Max(FM_X(a), FM_X(b))$ for all a, b in A.

The term "symmetric" refers to a single-valued neutrosophic relation X on A if,

$TM_X(a,b) = TM_X(b,y), IM_X(a,b) = IM_X(b,y), FM_X(a,b) = FM_X(b,y)$ and

$TM_Y(a,b) = TM_Y(b,y), IM_Y(a,b) = IM_Y(b,y), FM_Y(a,b) = FM_Y(b,y)$, for all a,b in A.

Definition 2.9 Let G be a graph that is neutrosophic. If there is no suitable subset of that dominating set Ψ_S of G, then that dominating set of G is called a minimum dominating set Ψ_S.

Definition 2.10 In a neutrosophic graph G, a vertex A is considered isolated if and only if

$TM_Y(a,b) < Min\{TM_Y(a), TM_Y(b)\}$
$IM_Y(a,b) < Max\{IM_Y(a), IM_Y(b)\}$ and

$FM_Y(a,b) < Max\{FM_Y(a), FM_Y(b)\}$,for all $b \in V - \{a\}$.

i.e $N(a) = \varphi$.

Definition 2.11 An isolated edge of a neutrosophic graph G is defined as follows:

$TM_Y(a,b) < Max\{TM_Y(a), TM_Y(b)\}$
$IM_Y(a,b) < Min\{IM_Y(a), IM_Y(b)\}$ and

$FM_Y(a,b) < Min\{FM_Y(a), FM_Y(b)\}$,for all $b \in V - \{a\}$.

i.e $N(a) = NULL$.

Definition 2.12 A set of vertices Ψ_S of a neutrosophic graph G is said to be independent [24] if,

$TM_X(p,q) < Min\{TM_X(p), TM_X(q)\}$
$IM_X(p,q) < Max\{IM_X(p), IM_X(q)\}$ and

$FM_X(p,q) < Max\{FM_X(p), FM_X(q)\}$, for all $p,q \in \Psi_s$.

Definition 2.13 Then Let's say that neutrosophic graphs $G = (V, E)$.

(i) A route is a set of successive vertices $I : a_0, a_1, \ldots, a_{O(NG)}$ where $a_x a_{x+1} \in E, x = 0,1,\ldots O(NG) - 1$

(ii) Path's strength $I : a_0, a_1, \ldots, a_{O(NG)}$ is $\Lambda_{x=0,\ldots,O(NG)-1}\, \omega(a_x a_{x+1})$

(iii) Interconnectivity between vertices a_0 and a_k is

$$\omega^{\infty}(a_x, a_k) = \bigvee_{I:a_0,a_1,\ldots,a_k} \bigwedge_{x=0,\ldots,k=1} \omega(a_x a_{x+1})$$

(iv) A cycle is a series of successive vertices $I : a_0, a_1, \ldots, a_{O(NG)}$ where, $a_x a_{x+1} \in E, x = 0,1,\ldots O(NG) - 1$, $a_{O(NG)} a_0 \in E$ there exist two edges ab and pq such that $\omega(ab) = \omega(pq) = \Lambda_{x=0,1,\ldots i-1}\, \omega(q_x q_{x+1})$

(v) The edge ab indicates $x \neq y$ where, $a \in V_x^{d_x}$ and $b \in V_b^{d_b}$ it is t-partite where $V_1^{d1}, V_2^{d2}, \ldots\ldots V_t^{dt}$, is divided into t pieces. In the event that it is complete $x \neq V_x$, it is indicated by $\mathrm{T}_{\alpha_1,\alpha_2,\ldots\alpha_t}$ where α_x is on α on $V_x^{d_x}$ rather than induces $\alpha_x(a) = 0$. Furthermore, $\left|V_x^{d_x}\right| = d_x$

(vi) If t = 3, then t-partite is [25] full bipartite, and it is represented by $\mathrm{T}_{\alpha_1,\alpha_2,\alpha_3}$

(vii) Full bipartite is represented by $SR_{1,\alpha 3}$ the symbol if $|V| = 2$.

(viii) If a vertex in V links to every other vertex in a cycle, then that vertex is a center. Next, it is a wheel, indicated by $WH_{1,\alpha 3}$

(ix) When it's finished, where $\forall_{pq} \in V, \omega(pq) = \alpha(p) \wedge \alpha(q)$

(x) Its strong where $\forall_{pq} \in E, \omega(pq) = \alpha(p) \wedge \alpha(q)$.

3. Proposed Neutrosophic Dominating Path-Coloring Number

The neutrosophic dominating path-coloring number of a neutrosophic graph is the minimum number of colors needed to color the paths in the graph in such a way that every vertex is dominated by at least one path of each color.

Definition 3.1. (Dominating path-coloring numbers)

Let $NG : G(V, E, \alpha, \beta)$ be a graph that is neutrosophic. Next

(i) There exist certain pathways from a to b for any two vertices, a and b. Two pathways from a to b are given different colors if they share an edge. In this method, the collection of colors η is referred to as the dominant path-coloring set from a to b if, for each edge outside, there are at least two edges inside that share a vertex. The term "dominating path-coloring number" refers to the minimal relationship across all domineering path-coloring sets between two given vertices, and it is represented by $\Omega_{DPC}(NG)$;

(ii) There exist certain pathways that connect x and y given a pair of vertices, a and b. Two pathways from a and b are given different colors η if they share an edge. In this method, the collection of colors is referred to as the dominating path-coloring set from a and b if, for each edge outside, there are at least two edges inside that share a vertex. The term "neurosophic dominating path-coloring number" refers to the lowest neutrosophic cardinality $\sum_{e \in \eta} \sum_{x=1}^{3} \omega_x(e)$, between any dominating path-coloring set $\sum_{e \in \eta} \sum_{x=1}^{3} \omega_x(e)$ and is represented by $\Omega_{NDPC}^{N}(NG)$.

The term "neutronosophic," which was used in the prior definition, would typically not be utilized for practical purposes.

Example 3.1. Figure 1 illustrates a complete neutrosophic graph.

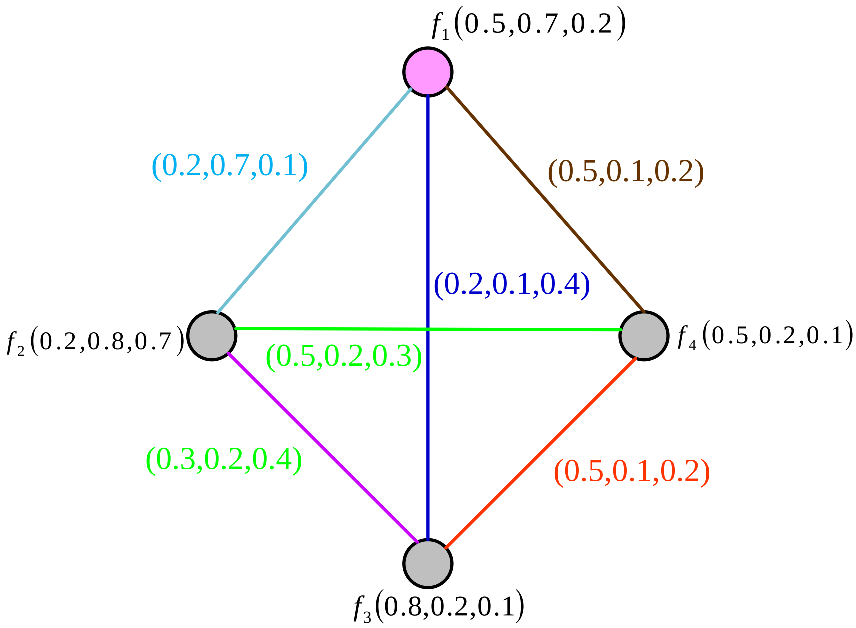


Figure 1. A Neutrosophic Graph in the Viewpoint of its dominating path-coloring number and its neutrosophic dominating path-coloring number

(i) Consider two vertices f_1 and f_4. All paths are as follow:

$$W_1 : f_1, f_4 \rightarrow blue.$$

$$W_2 : f_1, f_2, f_4 \rightarrow blue.$$

$$W_3 : f_1, f_3, f_4 \rightarrow blue.$$

$$W_4 : f_1, f_2, f_3, f_4 \rightarrow red.$$

$$W_5 : f_1, f_3, f_2, f_4 \rightarrow green.$$

The paths W_1, W_2 and W_3 has no shared edge so they have been colored the same as red. The path W_4 has shared edge $f_1 f_2$ with W_2 and shared edge $f_3 f_4$ with W_3 thus it is been colored the different color as blue in comparison to them. The path W_5 has shared edge $f_1 f_3$ with W_3 and shared edge $f_3 f_4$ with W_4 thus it is been colored the different color as yellow in comparison to different paths in the terms of different colors. Thus $\eta = \{blue, red, green\}$ is dominating path-coloring set and its cardinality 3 is dominating path-coloring number. In summary, given two vertices, a and b, there are certain pathways that connect a and b. Two pathways from a to b are given different colors if they share an edge. In this method, the color set $\eta = \{blue, red, green\}$ is referred to as the prevailing path-coloring set from a to b. A dominant path-coloring number is the least cardinality between all sets of dominating path-colorings from two vertices. It is represented by $\Omega_{DPC}(NG) = 3$;

(ii) All vertices have the same positions in the matter of creating paths. So, for every two given vertices, the number and the behaviors of paths are the same

(iii) There are three different paths which have no shared edges. So, they have been assigned to same color

(iv) Shared edges form a set of representatives of colors. Each color is corresponded to an edge which has minimum neutrosophic cardinality

(v) The minimum neutrosophic cardinality of a graph can be obtained by assigning each color to an edge with the smallest possible neutrosophic cardinality is η , or by using all shared edges to form a set S and then finding the minimum neutrosophic cardinality of S

(vi) Two edges $f_1 f_2$ and $f_3 f_4$ are shared with W_4 by W_3 and W_4. The minimum neutrosophic cardinality is 0.5 corresponded to $f_4 f_3$. Other corresponded color has only one shared edge $f_2 f_3$ and minimum neutrosophic cardinality is 0.7. Thus minimum neutrosophic cardinality is 1.2. And corresponded set is $\eta = \{f_3 f_2, f_4 f_3\}$. In summary, given two vertices, a and b, there are certain pathways that connect a and b. Two pathways from a to b are given different colors if they share an edge. In this approach, the collection of common edges is referred to as the dominant path-coloring set $\eta = \{f_3 f_2, f_4 f_3\}$from x to y. Neutronosophic dominating path-coloring number Ω_{NDPC}^{N} is the minimal neutrosophic cardinality $\sum_{e\in\eta}\sum_{x=1}^{3}\omega_x(e)$, between all dominating path-coloring sets and is represented by $\Omega_{NDPC}^{N}(NG) = 1.2$.

Theorem 3.1.

Let Ex be an extra arc in G_{Ex}^{best} and let G be an NG. In the event that nodes Ex are present, an NDPC is G_{Ex} created when $u-v$ the strongest neutrosophic route G_{Ex} between Ex two nodes u and v is discovered.

Evidence. Let $Ex = uv$ there be an NDPC there G_{Ex}. Next,

$$TM_{Po}(uv) > TM_{Po}^{\infty}(uv), IM_{Po}(uv) < IM_{Po}^{\infty}(uv), FM_{Po}(uv) \le FM_{Po}^{\infty}(uv).$$

If we let $u = a$ and $v = b$, then the proof is clear.

Conversely, if there exist nodes u, v where u − v neutrosophic path W_{Ex} of G_{Ex} that includes $Ex = uv$ is the unique strongest neutrosophic path between two nodes u and v, then for each $a-b$ neutrosophic path W without arc $Ex = uv$ in G, we have,

$$TM_{Po}(uv) > TM_{W}(uv), IM_{Po}(uv) < IM_{W}(uv), FM_{Po}(uv) \le FM_{W}(uv).$$

Hence,

$$TM_{Po}(uv) > TM_{Po}^{\infty}(uv), IM_{Po}(uv) < IM_{Po}^{\infty}(uv), FM_{Po}(uv) \le FM_{Po}^{\infty}(uv).$$

Therefore, $Ex = uv$ be a NDPC in G_{Ex}.

Theorem 3.2. Let $NG : G(V, E, \alpha, \beta))$ be a path-neutrosophic graph. Then,

$$\Omega^{N}_{NDPC}(NG)_{Path} = \underset{\eta,|\eta|=\left[\frac{Y_{Path}}{3}\right]}{Min} \sum_{e\in\eta} \sum_{x=1}^{3} \omega_x(e).$$

Evidence.

Let $NG_{Path} : G(V, E, \alpha, \beta)$ be a graph that is path-neutrosophic. There is just one path connecting any two vertices, a and b. Two pathways from a and b that share an edge are given different colors, but there is only one path with a defined beginning and ending. In this procedure, the set η of common edges is referred to as the dominant path-coloring set from x to y. Neutronosophic dominating path-coloring number η'' is the minimal neutrosophic cardinality $\sum_{e\in\eta} \sum_{x=1}^{3} \omega_x(e)$ between all dominating path-coloring sets and is represented by $\Omega^{N}_{NDPC}(NG)_{Path}$. Consequently,

$$\Omega^{N}_{NDPC}(NG)_{Path} = \underset{\eta,|\eta|=\left[\frac{Y_{Path}}{3}\right]}{Min} \sum_{e\in\eta} \sum_{x=1}^{3} \omega_x(e).$$

Example 3.2.

1. Figure 2(a) depicts an odd-path neutrosophic graph. The following path presents some new viewpoints that suggest a variety of more investigative and reasonable definitions:

(i) All paths are as follows

$$f_1, f_4 \rightarrow Blue$$

$$f_1, f_4, f_3 \rightarrow \text{Re}\, d$$

$$f_1, f_4, f_2, f_3 \rightarrow Green$$

$$f_1, f_4, f_2, f_3, f_5 \rightarrow yellow$$

$$f_4, f_3 \rightarrow Blue$$

$$f_4, f_3, f_2 \rightarrow Orange$$

$$f_4, f_3, f_2, f_5 \rightarrow Black$$

$$f_3, f_2 \rightarrow Blue$$

$$f_3, f_2, f_5 \rightarrow red$$

$$f_3, f_5 \rightarrow blue$$

The number is 4;

(ii) 1-paths have same color

(iii) $\Omega_{NDPC}^{N}(NG)_{Path} = 4$

(iv) In order to create a route and control its behaviors, the positions of the provided vertices may differ.

(v) every color is corresponded to some shared edges. Minimum neutrosophic cardinality of edges corresponded to specific color is a representative for that color. Thus every color is corresponded one neutrosophic cardinality of some edges since edges could have same neutrosophic cardinality with exception of initial color. So the summation of 4 numbers is neutrosophic dominating path-coloring number. Every color is compared with its previous color. The way is a consecutive procedure.

(vi) All paths are as follows.

$$f_1, f_4 \rightarrow Blue$$

$$f_1, f_4, f_3 \rightarrow \mathrm{Re}d \rightarrow f_1 f_4 \rightarrow 0.6$$

$$f_1, f_4, f_2, f_3 \rightarrow Green \rightarrow f_1 f_4, f_1 f_3 \rightarrow 1.1$$

$$f_1, f_4, f_2, f_3, f_5 \rightarrow yellow \rightarrow f_1 f_4, f_4 f_3, f_3 f_2 \rightarrow 2$$

$$f_4, f_3 \rightarrow Blue$$

$$f_4, f_3, f_2 \rightarrow Orange \rightarrow f_4 f_3 \rightarrow f_3 f_2 \rightarrow 1.1$$

$$f_4, f_3, f_2, f_5 \rightarrow Black \rightarrow f_4 f_3, f_2 f_5 \rightarrow 1.1$$

$$f_3, f_2 \rightarrow Blue$$

$$f_3, f_2, f_5 \rightarrow red \rightarrow f_3 f_2 \rightarrow 0.5$$

$$f_3, f_5 \rightarrow blue$$

$\Omega_{NDPC}^{N}(NG)_{Path}$ is 5.4.

2. A graph with even paths and neutrosphic graphs is shown in Figure 2(b). Below are few points that are explained. This section applies a new meaning.

(i) All paths are as follows.

$$f_1, f_4 \rightarrow Blue$$

$$f_1, f_4, f_3 \rightarrow Blue$$

$$f_1, f_4, f_2, f_3 \rightarrow Blue$$

$$f_1, f_4, f_2, f_3, f_5 \rightarrow Blue$$

$$f_1, f_4, f_2, f_3, f_5, f_6 \rightarrow Blue$$

$$f_4, f_3 \rightarrow Blue$$

$$f_4, f_3, f_2 \rightarrow Blue$$

$$f_4, f_3, f_2, f_5 \rightarrow Blue$$

$$f_3, f_2 \rightarrow Blue$$

$$f_3, f_2, f_5 \rightarrow Blue$$

$$f_3, f_5 \rightarrow blue$$

The number is 1.

(ii) 1- Paths are all the same color

(iii) $\Omega_{NDPC}^{N}(NG)_{Path} = 1$

(iv) In terms of building a path and the behaviors inside it, the positions of the provided vertices may differ.

(v) Only one way is available. It suggests there isn't a common edge.

(vi) The following are all possible routes.

$$f_1, f_4 \rightarrow Blue$$

$$f_1, f_2, f_3 \rightarrow Blue \rightarrow no \quad shared \quad edge \rightarrow 0$$

$$f_1, f_4, f_2, f_3 \rightarrow Blue \rightarrow no \quad shared \quad edge \rightarrow 0$$

$$f_1, f_4, f_2, f_3, f_5 \rightarrow Blue \rightarrow no \;\; shared \;\;\; edge \rightarrow 0$$

$$f_2, f_3 \rightarrow Blue$$

$$f_4, f_3, f_2 \rightarrow Blue \rightarrow no \;\; shared \;\;\; edge \rightarrow 0$$

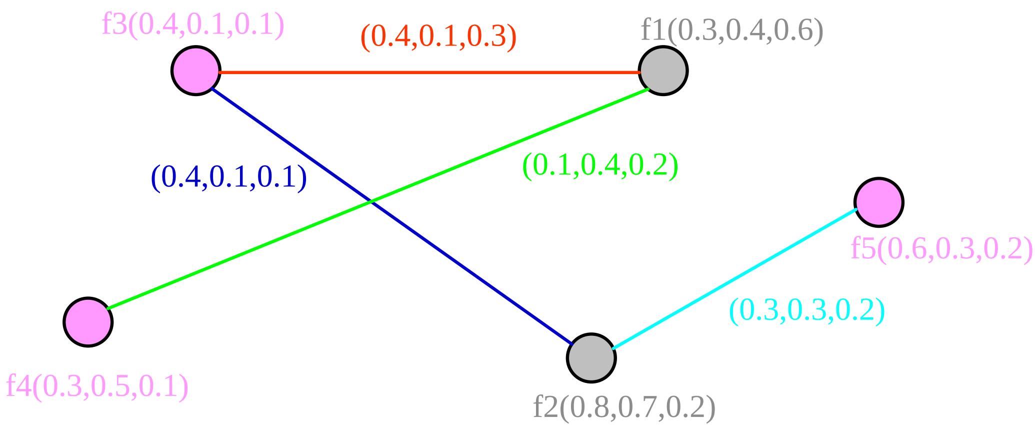
$$f_4, f_3, f_2, f_5 \rightarrow Blue \rightarrow no \;\; shared \;\;\; edge \rightarrow 0$$

$$f_3, f_2 \rightarrow Blue$$

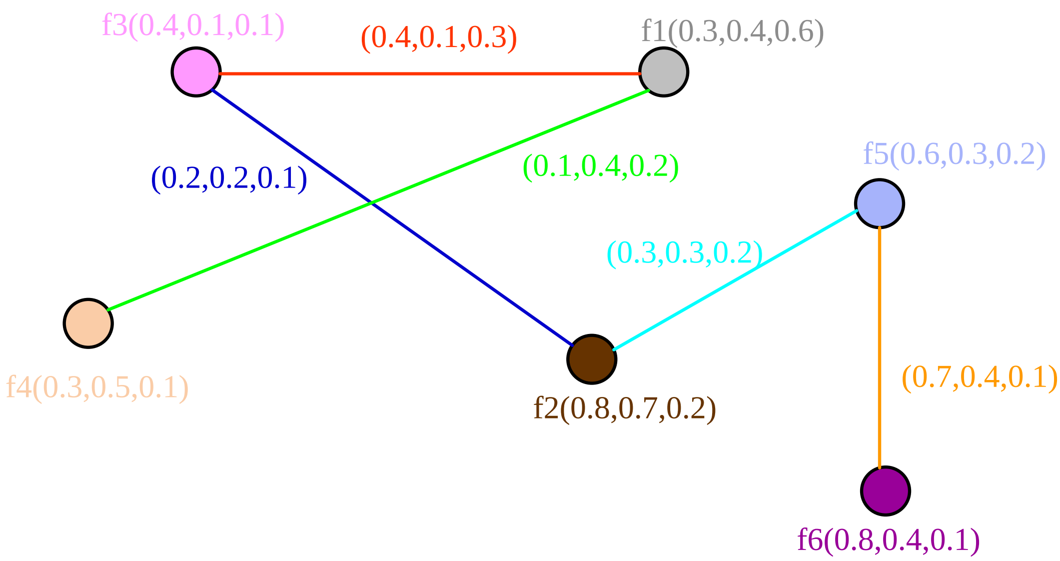
$$f_3, f_2, f_5 \rightarrow red \rightarrow no \;\; shared \;\;\; edge \rightarrow 0$$

$$f_3, f_5 \rightarrow blue$$

$\Omega^{N}_{NDPC}(NG)_{Path}$ is 0.



(a)



(b)

Figure 2: From the perspective of (a) dominant path-coloring number and its neutrosophic dominating path, a neutrosophic graph-coloring number and (b) the neutrosophic dominant path-coloring number that results from it

3.1. Multivalued Star Chromatic number of neutrosophic graph using corono product

When a vertex's color is a multivalued collection of colors, the Multivalued Star Chromatic number of a neutrosophic network is the bare minimum of colors needed to color its vertices so that no two neighboring vertices have the same color. The Corono product is a way of constructing new graphs from existing graphs. It is a useful tool for studying the properties of graphs. The corono product of two graphs G and H is a new graph $G \circ H$ that is constructed by taking one copy of G for each vertex of H and connecting the corresponding vertices of G and H by edges. In this research, we offer a new approach to use corono product to compute the Multivalued Star Chromatic number of neutrosophic graphs. The following steps form the foundation of the suggested method:

- Construct the corono product of the neutrosophic graph G with the star graph $H\{1, n\}$, where n is the number of colors in the multivalued set.
- Using the fewest possible colors, ensure that no two neighboring vertices have the same color while coloring the vertices of the corono product graph $G \circ H \in \{1, n\}$.
- The number of colors utilized to color the vertices of the corono product graph $G \circ H \in \{1, n\}$ is known as the Multivalued Star Chromatic number of the neutrosophic graph G.

Theorem 3.3 For any $n \geq 1$,

$$\gamma_{Sr}\left(G_n \circ H_n\right) = n + 3 \quad (1)$$

Proof: Let $V(G_n) = \{p_1, p_2, ..., p_n\}$ and $V(H_n) = \{q_1, q_2, ..., q_n\}$. Let $V(G_n \circ H_n) = \{p_x | 1 \leq i \leq n\} \cup \{q_{jk} | 1 \leq j \leq n-1; 1 \leq k \leq n\}$. Similar to how the edge corona product graph is created, end vertices $p_i, p_{i+1} \in V(G_n)$ are next to each vertex in $\{q_{jk} | 1 \leq j \leq n-1; 1 \leq k \leq n\}$.

The following is the multivalued star chromatic number of the vertices $G_n \circ H_n$ of using $n+3$ colors:

1. For every $i \in \{1, 2, ..., n\}$, assign the color $Cr_i + p_i$ to.

2. For $1 \leq j \leq n-1$ and $1 \leq k \leq n$.

- If $j + k + 1 \leq n + 3$, then assign the color Cr_{j+k+1} to q_{jk}.
- If $j + k + 1 \geq n + 3$ so, designate the coloring using the multivalued star chromatic number as follows:

Cr_1 *to* p_{jk} *when* $j+k \equiv 0 \pmod{n+3}$
Cr_{21} *to* p_{jk} *when* $j+k \equiv 1 \pmod{n+3}$
M
Cr_{n+1} *to* p_{jk} *when* $j+k \equiv n \pmod{n+3}$

Therefore $\gamma_{Sr}(G_n \text{o} H_n) \le n+3$.

To prove $\gamma_{Sr}(G_n \text{o} H_n) \ge n+3$, let us, on the contrary, assume that $\gamma_{Sr}(G_n \text{o} H_n)$ is less than $n+3$, say $\gamma_{Sr}(G_n \text{o} H_n) = n+2$. For accurate star coloring $\{p_1, p_2, q_{jk} : 1 \le k \le n\}$, $n+2$ now provide the vertices colors. Due to the fact that causes $\{p_1, p_2, q_{jk} : 1 \le k \le n\}$ a clique of order $n+2$ (say $n+2$), star color the order n clique that is caused by the second copy H_n, $\{q_{jk} : 1 \le k \le n\}$ using preexisting colors in such a way $Cr(p_2) \ne Cr(q_{jk})$. One of the order four pathways connecting these cliques is bicolored, which is contradicted by giving the same colors $n+2$ to the verties of another clique created by the third copy of H_n, $\{p_3, p_4, q_{3k} : 1 \le k \le n\}$. As a result, coloring $n+2$ a star with colors is not feasible. Consequently $\gamma_{Sr}(G_n \text{o} H_n) \le n+3$. thus $\gamma_{Sr}(G_n \text{o} H_n) \ge n+3$.

Example 3.3 By Theorem 3.3, n=6 we have $\gamma_{Sr}(G_n \text{o} H_n) = 6+3 = 9$ shown in figure 3.

Now assign the multivalued star coloring as follows:

$$Cr(q_1) = Cr(p_{36}) = Cr_1; Cr(q_2) = Cr_2$$

$$Cr(q_3) = Cr(p_{11}) = Cr_3; Cr(q_4) = Cr_{12} = Cr(p_{21}) = Cr_4$$

$$Cr(q_{13}) = Cr(q_{15}) = Cr(p_{22}) = Cr(q_{31}) = Cr_5$$

$$Cr(q_{14}) = Cr(q_{15}) = Cr(q_{16}) = Cr(p_{23}) = Cr(q_{32}) = Cr_6$$

$$Cr(q_{24}) = Cr(q_{26}) = Cr(p_{33}) = Cr_7$$

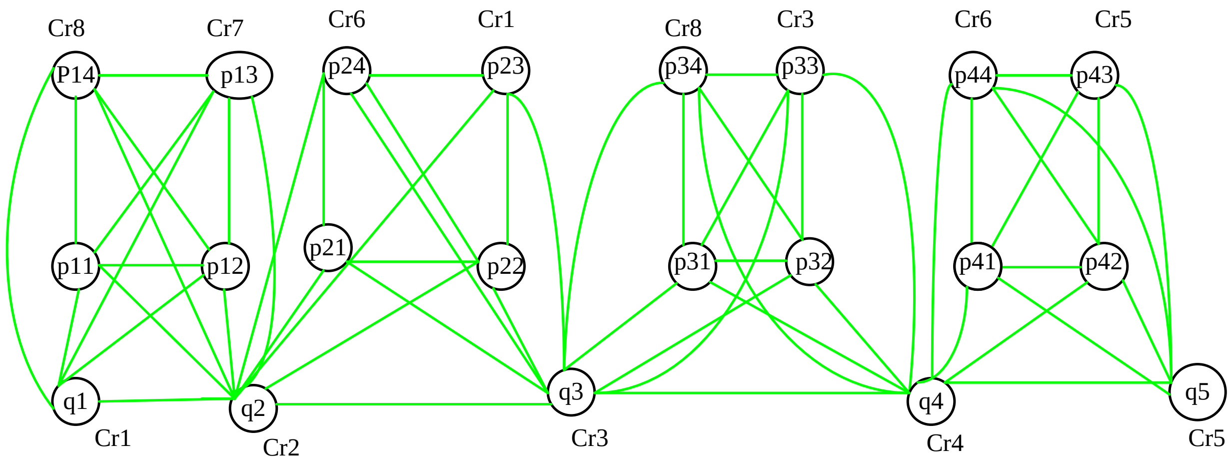


Fig.3: $\gamma_{Sr}(G_n \circ H_n)$

Theorem 3.4 For any $n \geq 6$,

$$\gamma_{Sr}(G_n \circ H_n) = \gamma_{Sr}(H) + 3$$

Proof: Let $V(G_n) = \{p_1, p_2, ..., p_n\}$ and $\gamma_{Sr}(H) = \mu$.

The vertex colors of $G_n \circ H$ should be $\mu + 3$ colored in the following manner, as indicated by λ.

- Apply color Cr_k to the vertex p_i such that $i \equiv m(\text{mod}\, \mu + 3)$ it matches each $i \in \{1, 2, ..., n\}$ and $0 \leq m \leq \mu + 3$.
- Assign the star coloring using the color set $\{Cr_k / j + k + 1 \equiv m(\text{mod}\, \mu + 3);\ 1 \leq j \leq n; 0 \leq k \leq \mu + 3\}$ to every copy $G_n \circ H$ of G.

That is clearly a proper λ coloration. We now need to demonstrate that λ the coloring is starry.

Let p_4 be any order four route in $G_n \circ H$. It is clear that p_4 is not bicolored if $|V(p_4) \cap V(G_n)| = 6$ or 0. Any three consecutive vertices of in have three different colors if $|V(p_4) \cap V(G_n)| = 5$ and only if p_4 is not bicolored. Then $|V(p_4) \cap V(G_n)| = 4$ either has p_4 two vertices on every copy of G of or it has two vertices on two G_n in $G_n \circ H$ distinct copies of G, let's say, g_x and g_y of $G_n \circ H$.

In the first scenario, let $\mu(p_i) = Cr_m$; $i \equiv m(\text{mod}\, \mu + 3)$, $\mu(p_{i+1}) = Cr_a$; $i + 1 \equiv a(\text{mod}\, \mu + 3)$ and $Cr_m \neq Cr_a \neq Cr_b$, where $i + j \equiv b(\text{mod}\, \mu + 3)$ then p_4 has a minimum of three colored vertices. Consequently p_4, it is not bicolored. In the latter instance $\mu(p_{i+1}) = Cr_a$, $i+1 \equiv a(\mathrm{mod}\,\mu+3)$, $\mu(p_{i+2}) = Cr_b$; $i+2 \equiv b(\mathrm{mod}\,\mu+3)$, since every vertex of g_y has color g_{i+j+1}, it follows that three consecutive vertices have different colors, i.e., p_4 are not bicolored. It is not bicolored if $|V(p_4) \cap V(G_n)| = 2$ there are at least two different colors on each of the three consecutive vertices p_4, and $\mu(p_i)$ if those colors are distinct from one another. Consequently $\gamma_{Sr}(G_n \circ H) \le \gamma_{Sr}(H) + 3$.

In order to $\gamma_{Sr}(G_n \circ H) \ge \gamma_{Sr}(H) + 3$ demonstrate the opposite, let us suppose that $\gamma_{Sr}(G_n \circ H)$ is fewer than, $\mu+3$, approximately $\gamma_{Sr}(G_n \circ H) = \mu+2$. For $\mu+2$ accurate star coloring, assign colors to the vertices $\{p_1, p_2, q_{jk} : 1 \le j \le n-1$, starting at k and going up to s} (p_{jk} see the label of vertices of copies of H) has a star chromatic number $\mu+2$, hence the subsequent duplicate with colors is star colored. One of the order four paths connecting these vertex sets is bicolored, which is contradicted by giving H'' the same $\mu+2$ colors to another copy of. As a result, coloring a star with colors $\mu+2$ is not feasible. Consequently $\gamma_{Sr}(G_n \circ H) \ge \mu+3$. thus $\gamma_{Sr}(G_n \circ H) = \gamma_{Sr}(H) + 3$.

Theorem 3.5 For any $n \ge 4$,

$\gamma_{Sr}(G_n \circ H_{1,n}) = n+4$.

Proof.

Let $V(G_n) = \{p_1, p_2, \ldots p_{n+1}\}$ and $V(H_{1,n}) = \{q_1, q_2, \ldots q_n\}$. Let $V(G_n \circ H_{1,n}) = \{p_i : 1 \le i \le n+1\} \cup \{q_{jk} : 1 \le j \le n+1 \le k \le n\}$. Every end vertex $p_i, p_{i+1} \in V(H_{1,n})$ in is next to every other vertex in $\{q_{jk} : 1 \le j \le n+1 \le k \le n\}$, just like in the definition of an edge corona graph. Assign the following $n+2$ chromatic coloring $H_{1,n} \circ H_n$ to be used as a star:

- Assign color Cr_{n+2} to the vertex p_1.
- For every $i \in \{2,3,\ldots,n+1\}$, assign the color Cr_{n+1} to p_1.
- For every $j \in \{2,3,\ldots,n\}$ and $k \in \{1,2,3,\ldots,n\}$, color the vertices q_{jk} with color Cr_k.

Assuming such $\gamma_{Sr}(H_{1,n} \circ H_n) \le n+2$ is the case, we can demonstrate that the multivalued star chromatic number of the edge corona product of a route graph with a complete graph is $\gamma_{Sr}(H_{1,n} \circ H_n) \ge n+2$. Then, there exists a star coloring of this graph with $\gamma_{Sr}(H_{1,n} \circ H_n) = n+1$ colors. This is not feasible, though, as each vertex in the whole subgraph $\{p_1, p_2, q_{1k} : 1 \le k \le n\}$ creates an order group $n+2$ (say H_{n+2}). Consequently, the multivalued star chromatic number is $\gamma_{Sr}\left(H_{1,n} \circ H_n\right) \geq n+2$ proven to be since a colored $n+1$ star coloring is not conceivable $\gamma_{Sr}\left(H_{1,n} \circ H_n\right) = n+2$.

Example 3.4 According to Theorem 3.5, we can notice that n = 4 is the multivalued star chromatic number of the edge corona product of a route graph with a complete graph $\gamma_{Sr}\left(H_{1,4} \circ H_4\right) = 4+4=8$. This is shown in Figure 4, where the colors are assigned in the following way,

$$Cr(p_1) = Cr_8, Cr(p_2) = Cr(p_3) = C_6, Cr(q_{11}) = Cr_1$$
$$Cr(q_{14}) = Cr(q_{15}) = Cr(q_{17}) = Cr_3, Cr(p_4) = Cr_2$$
$$Cr(p_{16}) = Cr_5, Cr(q_{14}) = Cr(q_{13}) = Cr_4$$
$$Cr(p_{11}) = Cr(p_{12}) = Cr_7$$

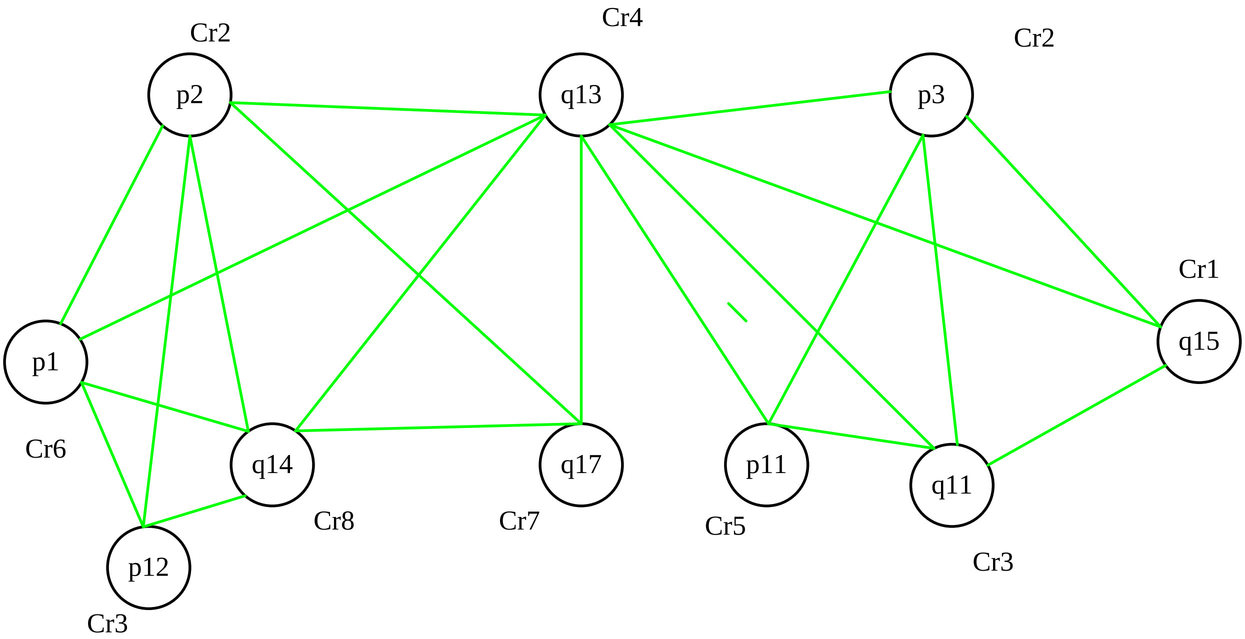


Fig 5. $\gamma_{Sr}\left(H_{1,4} \circ H_4\right)$

4. Conclusion and Future Research

The idea of dominance is fundamental to graph theory from both a theoretical and practical standpoint. Neutrosophic models are more flexible and compatible with real-world applications than fuzzy and intuitionistic fuzzy models. In this work, certain theorems about the dominant path-coloring number in neutrosophic graphs are developed and a description of multivalued star coloring on the corona product in a neutrosophic scenario is proposed. Additionally, the notions of cardinality in multivalued star chromatic neutrosophic graphs (MVSCNGs) and single-valued neutrosophic incidence valid edges are introduced by the authors. The multivalued star chromatic number of the edge corona product of a route graph with a full graph, a complete bipartite graph, and any simple graph is finally found by the authors. They also do the same actions for any simple graph, a full graph, and the edge corona product of a star graph with a route graph.

Other graph types, such intuitionistic fuzzy graphs and fuzzy graphs, will be covered by our method's extension. We also plan to investigate the relationship between the Multivalued Star Chromatic number of a neutrosophic graph and other graph properties.

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