Application of Generalized exponential distribution for the glucose levels under various experimental conditions

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Abstract

We employed an exponentiated Weibull family to analyze predictions made with time-to-event data. This paper, suggested a three-parameter exponentiated Weibull distribution. The exponentiated Weibull distribution encompasses the two-parameter exponentiated exponential, often known as the generalized exponential distribution. The Generalized Exponential Distribution has a right-skewed unimodal density function and a monotone hazard function, similar to the gamma and Weibull distributions. This approach is useful for assessing lifetime data, substituting the gamma, Weibull, and log-normal distributions. This article explains the model's origins, properties, estimating methodology, stress-strength parameter estimates, and how it compares to well-known distribution functions. The current investigation confirms that a glucose level reduces for Type -2 diabetics' patients, consistent with life-time evidence.

Key words: Type -2 diabetics, Hazard function, Maximum likelihood estimator.

1. Introduction

Weibull models explain various failures of components and phenomena. Weibull analyses focus on a single failure class and are widely used for real-world issues [1]. There are other Weibull-related distributions available in the statistics literature, in addition to the standard three-parameter Weibull distribution. Standard theoretical distributions are used in various industries, including medicine, engineering, insurance, economics, and finance. The author [2] found that generalizing standard distributions leads to more flexible compound distributions than base line distributions. Bourguignon et al. (2014) provide more information on how researchers have attempted to generate new distributions. Exponential distribution, a well-known continuous probability model, is widely used for data analysis and other applications. After 3 weeks of short-term high-intensity exercise training, the probability, hazard rate, survival, and reserved hazard-rate functions were analyzed [4]. Several life-time data sets are examined using statistical analysis based on a specific statistical distribution [5], a novel stochastic model for the generalization of the Sujatha distribution for the effects of two forms of exercise on plasma growth hormone [6]. The hazard function of the generalized exponential distribution differs from that of the Weibull distribution, but is more comparable to that of the gamma distribution, A new stochastic model on the generalization of Sujatha distribution for the effects of two forms of exercise on plasma growth hormone [7].

For further information, see Gupta and Kundu [8]. The Weibull-G exponential distribution provides researchers with new options for analyzing lifespan data. This page provides graphical solutions for this study.

2. Background

The study included nine type 2 diabetics and nine people with normal glucose tolerance. Each participant underwent an oral glucose challenge (75g/30ml) [9]. The tests were conducted in the morning following an overnight fast. Oral glucose was provided immediately after obtaining basal blood specimens. The values shown in table 1.

Treatment Group	Baseline Glucose(pmol/L)	Physiological Gulucose (pmol/L)	Hyperglycemic Glucose (pmol/L)
Control	60.1	90.5	120.2
CIP alone	65.8	95.6	125.6
GLP-1 alone	62.5	92.3	123.4
GIP+GLP-1	67.2	98.1	128.9

Table 1

Type 2 diabetic individuals had higher basal levels of immunoreactive GLP-1, but their GLP-1-integrated incremental responses after oral glucose were lower than in normal participants. However, the peak concentrations obtained in type-2 diabetic patients and normal people were given (Fig. 1).

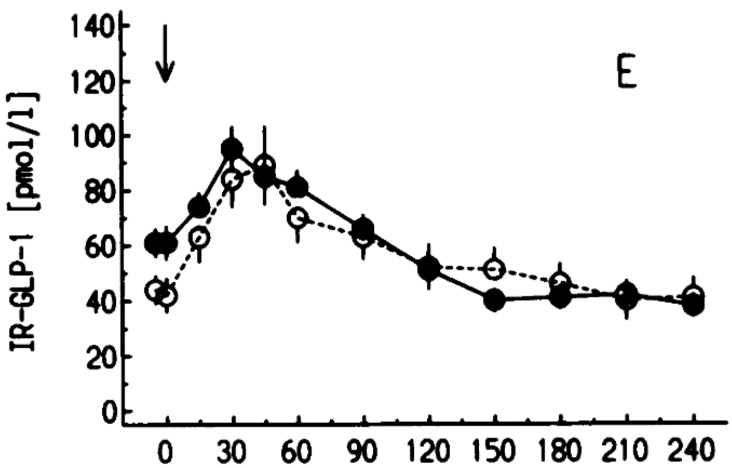

Fig.1

3. Mathematical model

3.1. Generalized exponential distribution

A continuous random variable X whose probability density function is given by

$$f(x) = \begin{cases} \beta e^{-\beta x} & if\ x \geq 0 \\ 0 & if\ \ x < 0 \end{cases} \qquad \beta > 0,$$

It is said to have an Exponential distribution.

$$f(x, \alpha, \beta) = \alpha\beta\left(1 - e^{-\beta x}\right)^{\alpha-1} e^{-\beta x}, x > 0\ for\ \ \alpha, \beta > 0$$

$$f(x,\alpha,\beta) = 0, \text{ Otherwise}$$

The MGF is given by

$$M_X(t) = E(e^{tX}) = \int_0^\infty e^{tx}\ \alpha\beta\left(1 - e^{-\beta x}\right)^{\alpha-1} e^{-\beta x}\, dx$$

$$= \alpha B\left(\propto, 1 - \frac{t}{\beta}\right),$$

Where $B(m,n) = \int_0^1 y^{n-1}(1-y)^{m-1}dy$

Replacing $\beta\ by\ 1/\lambda$ and x by $(x-\mu)$, then get the following form of generalized exponential distribution

$$f(x,\alpha,\lambda,\mu) = \frac{\alpha}{\lambda}\left(1 - e^{-(\alpha-1)/\lambda}\right)^{\alpha-1}\ e^{-(\alpha-1)/\lambda}$$

$$x > \lambda.\alpha > 0, \lambda > 0.$$

The major goal of this work is to provide intriguing extensions of the Generalized Exponential Distribution in various ways and investigate their moment generating functions. The authors will first define Generalized Exponential Distribution in terms of a new parameter k > 0, naming it k-Generalized Exponential Distribution. In fact, they demonstrate the following result by combining Generalized Exponential Distribution as an exceptional case.

Theorem 1.

Let X be a random variable of continuous type and let $\alpha > 0, \beta > 0$, and $k > 0$ be the parameters; then the function

$$f(x,\propto,\beta,k) = \propto\beta\left(1 - e^{-\beta x/k}\right)^{\alpha-1} x^{k-1} e^{-\beta x/k}, x > 0$$

$$f(x,\propto,\beta,k) = 0, \text{ Elsewhere}$$

Is the probability density function of random variable X of continuous type

Remark 1. If we take $k=1$,it reduces to Generalized Exponential Distribution. By above theorem. Clearly

$$f(x,\propto,\beta,k) \geq 0\ \ \forall\ x > 0, \propto > 0, \beta > 0, k > 0.$$

Now

$$\int_0^\propto f(x,\propto,\beta,k)dx$$

$$=\int_0^{\propto} \propto \beta \left(1-e^{-\beta x/k}\right)^{\alpha-1} x^{k-1} e^{-\beta x/k}\, dx$$

$$=\propto \int_0^{\propto} \left(1-e^{-\beta x/k}\right)^{\alpha-1} \left(x^{k-1} e^{-\beta x/k}\right) d \propto \left[\frac{\left(1-e^{-\beta x/k}\right)^{\alpha}}{\propto}\right]_0^{\propto}$$

$$=\left[\left[\left(1-e^{e^{-\beta x/k}}\right)^{\alpha}\right]\right]_0^{\infty}$$

As a result, $f(x, \alpha, \beta, k)$ is a continuous probability density function for X. The generalized exponential distribution in terms of a new variable, which is derived from the Weibull distribution, is presented here with several theorems and notes.

Theorem 2. Let X be a random variable of continuous type and let $\alpha > 0$, $\beta > 0$, and $k > 0$ be the parameters; then the function

$$f\ (x, \alpha, \beta, k)\ =\ k\alpha\beta \left(1-e^{-\beta x^k}\right)^{\alpha-1} x^{k-1\, e^{-\beta x^k}}\, 0 < x < \infty$$

$$f\ (x, \alpha, \beta, k) = 0, elsewhere,$$

Is the p.d.f.of random variable X of continuous type.

Remark 2. For $k = 1$, K- Generalized Exponential theorem 4 reduces to classical Exponential Distribution.

Proof of previous theorem. clearly

$$f(x, \alpha, \beta, k) \geq 0 \forall x > 0, \propto > 0, \beta > 0, k > 0.$$

$$\int_0^{\propto} f(x, \propto, \beta, k) dx$$

$$= k\alpha\beta \int_0^{\infty} \left(1-e^{-\beta x^k}\right)^{\alpha-1} x^{k-1\, e^{-\beta x^k}}\, dx$$

$$= \alpha \int_0^{\propto} \left(1-e^{-\beta x/k}\right)^{\alpha-1} (k\beta x^{k-1}) e^{-\beta x^k})\, dx$$

$$\propto \left[\frac{\left(1-e^{-\beta x/k}\right)^{\alpha}}{\propto}\right]_0^{\propto} = \left[\left[\left(1-e^{e^{-\beta x/k}}\right)^{\alpha}\right]\right]_0^{\infty}$$

$$=1 - 0 = 1.$$

Hence $f(x, \propto, \beta, k)$ is a p.d.f.of random variable X of continuous type.

Theorem 3.

Let X be a continuous random variable, then the function

$$f(x,\alpha,\beta,\delta) = \frac{\alpha\beta\delta}{1-(1-\delta)^{\alpha}}\left(1-\delta e^{-\beta x}\right)^{\alpha-1} e^{-\beta x}$$
$$x > 0, \alpha > 0, \beta > 0, 0 < \delta \leq 1,$$

$f(x,\alpha,\beta,\delta) = 0$, Otherwise is the p.d.f. of random variable X of continuous type.[10]

The moment generating function of the above theorem referrer [10] is,

$$B_{\delta} = \frac{\alpha}{(1-(1-\delta)^{\alpha})} B_{\delta}\left(\alpha, 1-\frac{t}{\beta}\right)$$

Where

$$B_{\delta} = B_{\delta}(m,n) = \int_0^1 (1-\delta y)^{m-1} y^{n-1} dy. m > 0, n > 0,\ 0 < \delta \leq 1.$$

Remark 3. for $\delta = 1, we$ have

$$B_1(m,n) = \int_0^1 (1-y)^{m-1} y^{n-1} dy = B(m,n).$$

Survival function and hazard rate function as follows,

$$f(x,\alpha,\beta,k) = p(X \leq x) = \int_0^x f(x,\propto,\beta,x)dx$$
$$=(1-e^{-\beta x^{k/K}})^{\propto},$$
$$x > 0, \propto > 0 >, \beta > 0.$$

The survival function $S(x,\propto,\beta,k) = 1 - F(x,\propto,\beta,k)$

$$=1-(1-e^{-\beta x^{k/K}})^{\propto}, x > 0$$

The hazard rate $h(x,\alpha,\beta,k)$is given by

$$S(x,\propto,\beta,k) = \frac{F(x,\propto,\beta,k)}{S(x,\propto,\beta,k)}$$

$$= \frac{(1-e^{-\beta x^{k/K}})^{\propto} x^{k-1} e^{-\beta x^{k/K}}}{1-(1-e^{-\beta x^{k/K}})^{\propto}}, x > 0.$$

4. Mathematical Results

The result of Generalized Exponential distribution for the reference data [8] shown as follows, MATLAB software supported to this work.

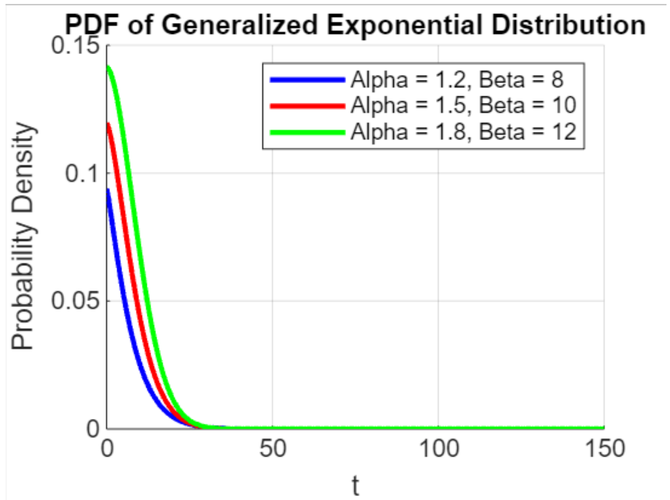

Fig 2.

The probability densities function of Generalized Exponential distribution plot shown in fig 2.

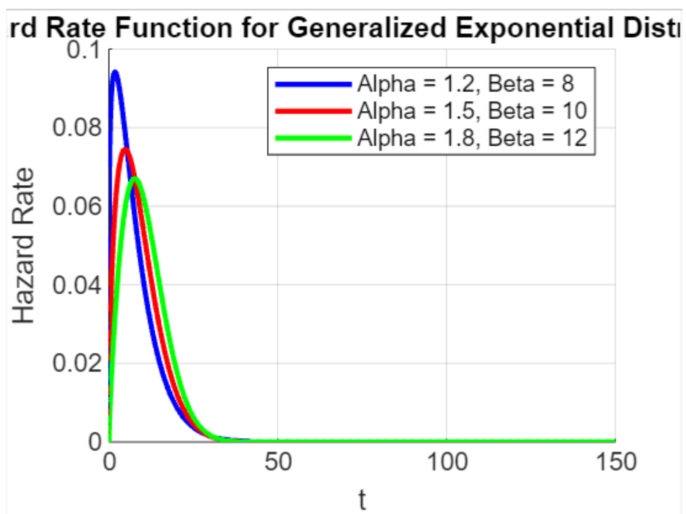

Fig 3.

The hazard rate function of Generalized Exponential distribution plot shown in fig 3.

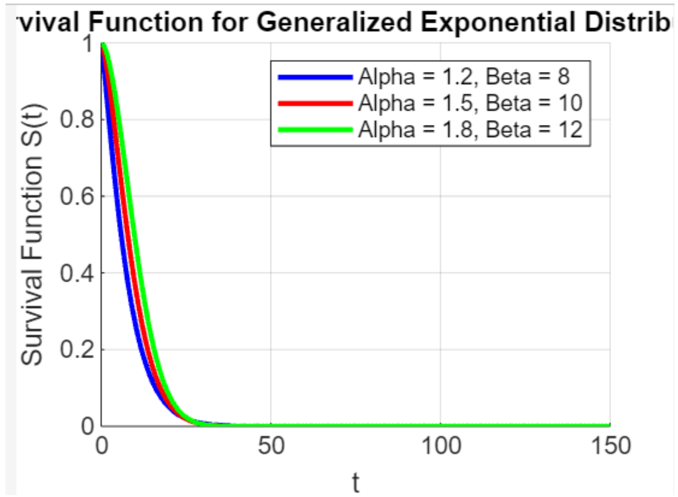

Fig 4.

The survival function of Generalized Exponential distribution plot depicts in fig 4.

5. Results and Discussion

In Results, According to the study's findings, the combination of GIP and GLP-1 leads to greater glucose levels than solo therapies. the Generalized Exponential Distribution's PDF, Hazard Rate, and Survival functions provide complimentary perspectives on the probability, risk, and survival aspects of an event as it unfolds over time. Every function is critical in connecting and modeling survival data, reliability engineering, and risk management applications; all of these functions contribute to making informed decisions and predictions about real-time data.

6. Conclusion

The Generalized Exponential Distribution is a mathematical tool for modeling to the real time data, such that the glucose levels, under different types of experimental settings. In this work, compared the medical data to GED parameters, we may understand and predict how various therapies alter glucose regulation and insulin secretion dynamics. These outcomes the adequacy related mathematical models for the medical research to measure and clear experimental results rigorously and quantitatively.

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