Mathematical Analysis of the Performance of A Longitudinally Rough Finite Porous Plane Slider Bearing

Girish C. Panchal[1*], Jatin V. Adeshara[2], Vijay C. Makwana[3], Nirav D. Patel[4], Vijay P. Soni[5], Bhavin S. Patel[6], Aswin K. Rathod[7]

[1*,2]Assistant Professor of Mathematics,Vishwakarma Government Engineering College, Chandkheda, Gujarat(India),
[3,5,6]Assistant Professor of Mathematics, Government Engineering College, Patan, Gujarat(India),
[4,7]Assistant Professor of Mathematics, Government Engineering College, Gandhinagar, Gujarat(India)

Email: [1*]gcpanchal@vgecg.ac.in
* Corresponding Author: Girish C. Panchal

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ABSTRACT:
The present article examines important factors like roughness parameters, porosity, and parameter of roughness-pattern (PRP) on a finite porous plane slider bearing (PSB). These factors influence significantly the behaviour of the bearing's long-lasting execution. The present article describes the efforts for improving the bearing's life period, and load-tolerating capacity (LTC) while adopting a conventional fluid as a lubricant and longitudinal-roughness in the PSB. Here the piston and ring assemblage in internal combustion engine (ICE) is assumed to have the shape of the ring to be a plane shape. It reveals from the graphical and numerical results of the mathematical model that the LTC of the bearing can be improved to a certain extent with proper combinations of various factors like roughness parameters, porosity, and PRP.

Keywords: Plane slider bearing(PSB), Porosity, Load-tolerating capacity, longitudinal roughness.

1. INTRODUCTION AND PRELIMINARY INFORMATION

The study of roughness parameters, like variance, standard deviation and skewness of slider's random roughness, porosity of the surfaces of a bearing, and parameter of roughness-pattern (PRP) play very crucial role while designing bearing system which provides better load-tolerating capacity and lower friction and wear. Many researchers have carried study on different types of bearings in the last few years.

Christensen and Tonder [1-4] set up the stochastic concept and presented the mathematical model representing the average pressure in a bearing system having longitudinal-roughness and transverse-roughness with the hydrodynamic lubrication. A number of investigations by Panchal et al. [5-7], Andharia et al. [8] have been carried out with stochastic approach introduced by Christensen and Tonder.

Panchal [9] examined the effect of different lubricant film shapes in a transversely rough plane slider bearing equipped with ferro-fluid as a lubricant on the load-tolerating capacity by mathematical modelling. Prakash & Vij [10-11] have been discussed the lubricationg charecteristics of the plane bearing when the lubricating surface is porous and backed by solid surface. They observed that the porousity also acts out a pivotal role in the bearing's LTC.

Ishaq, Mohammad, et al [12] also established that the porosity parameter decreases the motion of the liquid films. Awan, A. U. et al [13] observed the performance of an infinite vertical plate enclose by an in-compressible and viscous fluid and Wehgal et al.[14] carried out a numerical study for steady incompressible viscous magneto hydrodynamic asymmetric-flow of an electrically conducting lubricant between two infinite parallel stationary coaxial porous disks of dissimilar permeability in the presence of magnetic field. Patir and Cheng [15-16] adapted the Averaged-Reynolds' Type Equation (ARTD) for rough PSB and defined shearing and pressure flow-factors.

In this paper, we have inculcated porosity to one of the slider of the PSB supported by a concrete plate (housing). The lubricant is not allowed to pass out from the bearing's surface due to the housing. We have modified the ARTD and solved the modified Reynolds equation numerically by using Simpson's rule to calculate incorporated integrals. We have analysed a combined effect of porosity and roughness parameters on the performance of bearing's LTC. The results have been presented in tabular forms and graphically.

2. MATHEMATICAL ANALYSIS

In the study, we adopt usual hypothesis for conventional lubrication theory that the lubricant is incompressible, the flow is steady and the flow through the homogeneous and isotropic porous matrix satisfies Darcy's law. Also the pressure due to the fluid-velocity in this porous medium obeys Laplace's equation. owing to these postulations, the equation governing the pressure distribution in the lubricant film satisfies a derived form of Reynolds' type equation that has been established by Prakash et al.[10].

Owing to these postulations, Prakash et al.[10] have formulated the governing equation for the generated pressure in the fluid-film from the Reynolds' equation, which is of the form,

$$\frac{\partial}{\partial x}\left[h^3\ \frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial y}\left[h^3\ \frac{\partial p}{\partial y}\right]=-6\mu U\ \frac{dh}{dx}+12\mu V_h+12\phi\left(\frac{\partial p}{\partial y}\right)_{y=0} \tag{1}$$

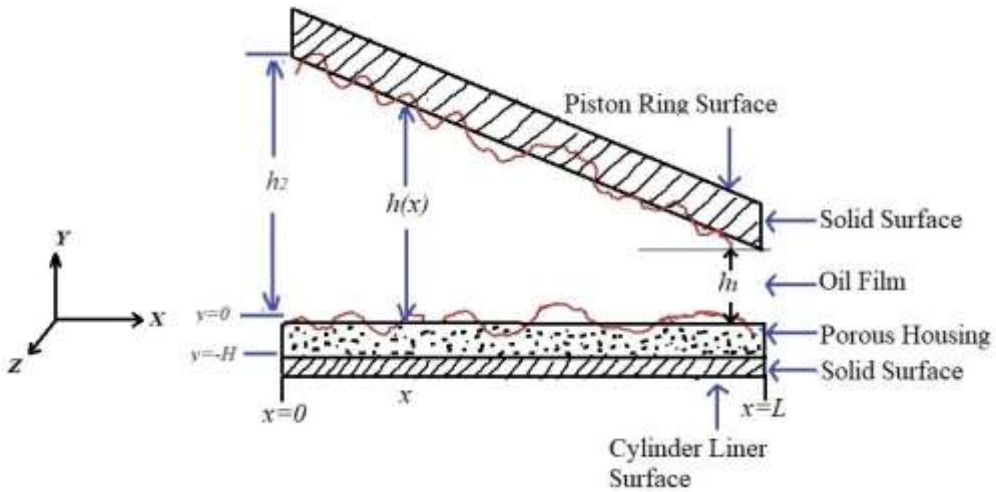

Fig.1. PSB Configuration

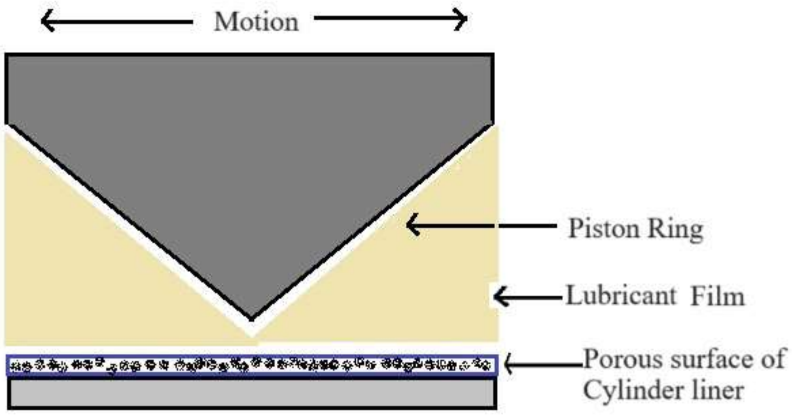

Fig.2. Configuration of PSB in Piston and Ring Assemblage in ICE

Here, $\left(\frac{\partial p}{\partial y}\right)_{y=0}$ reveals the pressure gradient at the interface between the fluid-film and the bearing's porous surface (cylinder liner of the piston and ring assemblage in ICE) as shown in the figure-1. U shows the velocity of bearing surface. Let us assume that the bearing's (cylinder liner's) length is l, the thickness of the porous surface lying on the solid housing is H, minimum and maximum film thickness of the lubricant is h_1 and h_2 respectively.

If P and p show the fluid's pressure in porous plate and in the fluid- film respectively and since the velocity (v) of the lubricant in the porous layer satisfy the Darcy's law,

$$v = -\frac{\Phi}{\eta}\,\nabla P$$

Where Φ is represents the porous material's permeability and η shows viscosity of the lubricant. Since v satisfies the continuity equation, P satisfies the Laplace equation. Also, consider that the pressure function is continuous at the lubricant and the porous surface boundary (at $y=0$). Hence the term $\left(\frac{\partial p}{\partial y}\right)_{y=0}$ can be approximated as,

$$\left(\frac{\partial p}{\partial y}\right)_{y=0} = -H\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \tag{2}$$

Also, $V_h = U\frac{dh}{dx}$ represents the upper surface's velocity element due to movement in Y-direction as the surface of the piston- ring is not porous. Hence Equation-1 is reformed into,

$$\frac{\partial}{\partial x}\left[h^3\,\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[h^3\,\frac{\partial p}{\partial y}\right] = -6\mu U\,\frac{dh}{dx} + 12\mu U\frac{dh}{dx} - 12\Phi H\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) \tag{3}$$

$$\therefore \quad \frac{\partial}{\partial x}\left[\left(\frac{h^3}{12\mu} + \frac{\Phi H}{\mu}\right)\frac{\partial p}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\frac{h^3}{12\mu} + \frac{\Phi H}{\mu}\right)\frac{\partial p}{\partial y}\right] = \frac{U}{2}\,\frac{dh}{dx} \tag{4}$$

Going along with the averaging process mentioned by Patir [16], Equation-4 turns out to be an ARTE,

$$\frac{\partial}{\partial x}\left[\varphi_x\left(\frac{{h_T}^3}{12\mu} + \frac{\Phi H}{\mu}\right)\frac{\partial \bar{p}}{\partial x}\right] + \frac{\partial}{\partial y}\left[\varphi_y\left(\frac{{h_T}^3}{12\mu} + \frac{\Phi H}{\mu}\right)\frac{\partial \bar{p}}{\partial y}\right] = \frac{U}{2}\,\frac{\partial \overline{h_T}}{\partial x} + \frac{U\sigma}{2}\,\frac{\partial \varphi_s}{\partial x} \tag{5}$$

Where, h shows the thickness of local fluid- film, $\delta = \delta_1 + \delta_2$ is the arbitrary amplitude of the roughness of the bearing's surfaces, $h_T = h + \delta$, $\overline{h_T}$ is the averaged thickness of the fluid- film, σ is a standard-deviation (S.D.) of the arbitrary roughness, $\bar{p}$ is the mean pressure, φ_x, φ_y are pressure flow-factors in *x* and *y* directions respectively and φ_s represents the shearing flow-factor.

In the piston and ring assemblage of an ICE, we neglect the side leakage in Z-direction (Figure-1). Since the movement of the bearing is considered in X-direction with the velocity-U, the flow is presumed to be steady and considering longitudinal ($\gamma > 1$) rough surface, the variations in roughness heights in X-direction is undersized (Figure-1), so the effect of φ_s may be not worth mentioning.

Therefore, Equation-5 is reduced to,

$$\frac{d}{dx}\left[\frac{\varphi_x}{12\mu}\left({h_T}^3 + 12\Phi H\right)\frac{d\bar{p}}{dx}\right] = \frac{U}{2}\frac{d\overline{h_T}}{dx} \tag{6}$$

$h(x)$ for a rough plane slider bearing geometry(Fig.2) is given by,

$$h(x) = h_1 + (h_2 - h_1)(1 - \frac{x}{l})$$

The estimation of $\overline{h_T}$ as mentioned by Patir [15-16] is expressed as the expected value of h_T,

$$\overline{h_T} = \int_{-c}^{c}(h + \delta)f(\delta)d\delta = E(h_T)$$

Where,
$h_T = h + \delta$

Here, $f(\delta)$, $-c < \delta < c$ the probability density functions (p.d.f.), where *c* is highest possible deviation in δ.

Deheri et al.[18] have already described the mean $E(\delta) = \alpha$, the S.D. $\sqrt{E[(\delta - \alpha)^2]} = \sigma$ and the skewness-parameter $E[(\delta - \alpha)^3] = \varepsilon$ in terms of the expectancy operator $E(r) = \int_{-c}^{c} r\, f(\delta)d\delta$,

Hence Equation-6 can be modified to,

$$\frac{d}{dx}\left[\frac{\varphi_x}{12\mu}(m_h^{-1} + 12\Phi H)\frac{d\overline{p'}}{dx}\right] = \frac{U}{2}\frac{dn_h^{-1}}{dx} \tag{7}$$

Where $\overline{p'}$ represents mean pressure's expected value and

$$m_h = h^{-3}[1 - 3\alpha h^{-1} + 6h^{-2}(\sigma^2 + \alpha^2) - 10h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)]$$
$$n_h = h^{-1}[1 - \alpha h^{-1} + h^{-2}(\sigma^2 + \alpha^2) - h^{-3}(\varepsilon + 3\sigma^2\alpha + \alpha^3)]$$

We can modify the Equation-7 into dimensionless form by introducing the dimensionless quantities as below,

$h^* = \frac{h}{h_1}$, $X = \frac{x}{l}$, $p^* = \frac{h_1^2\overline{p'}}{\mu U l}$, $\psi = \frac{\Phi H}{h_1^3}$ (permeability parameter of the porous surface) ,

$$M_h = h^{*-3}\left[1 - 3\alpha^* h^{*-1} + 6h^{*-2}\left(\sigma^{*2} + \alpha^{*2}\right) - 10h^{*-3}(\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})\right]$$

and

$$N_h = h^{*-1}\left[1-\alpha^* h^{*-1} + h^{*-2}\left(\sigma^{*2}+\alpha^{*2}\right) - h^{*-3}(\varepsilon^* + 3\sigma^{*2}\alpha^* + \alpha^{*3})\right]$$

Hence, Equation-7 becomes,

$$\frac{d}{dX}\left[\varphi_X\{M_h^{-1}+12\psi\}\,\frac{dp^*}{dX}\right] = 6\,\frac{d}{dX}\left[N_h^{-1}\right] \qquad (8)$$

Patir [15] recognized the investigational relation for φ_X as,

$\varphi_x = 1 + C\,R^{-r}$ $\quad (for\ \gamma > 1)$

$\varphi_X = 1 + C\,(h^* R_m)^{-r}$ $\quad (for\ \gamma > 1)$ (Dimensionless form)

Where

$$R = \frac{h}{\sigma}\,,\quad R_m = \frac{h_1}{\sigma}$$

And the values of the roughness pattern (longitudinal pattern) parameter (γ) depends on the constants r and C which are given in Table-1 (Patir [15]),

γ	*C*	*r*	*R*
3	0.225	1.5	> 0.5
6	0.520	1.5	> 0.5
9	0.870	1.5	> 0.5

Table-1. Association between $C, r, R\ and\ \gamma$

Also the ratio (h_2/h_1) is presumed as per the Table-2 (Prakash et al.[11]).

ψ	0	0.0001	0.001	0.01	0.1	1.0
h_2/h_1	2.1887	2.1891	2.1940	2.2373	2.5727	3.9605

Table-2. Association between $\psi\ and\ h_2/h_1$

Depending on the following boundary conditions:

$$p^* = 0\,, \text{at } X = 0 \text{ and } 1$$

$$\frac{dp^*}{dX} = 0\,, \text{at which the mean gap is maximum, say } q^* \text{(Constant)}$$

Equation-8 leads us to,

$$p^*(X) = \int_0^X \frac{6\,N_h^{-1} - q^*}{\varphi_X\{M_h^{-1}+12\psi\}}\,dX \qquad (9)$$

Where,

$$q^* = \frac{\int_0^1 \frac{6\,N_h^{-1}}{\varphi_X\left\{M_h^{-1}+12\psi\right\}}\,dX}{\int_0^1 \frac{1}{\varphi_X\left\{M_h^{-1}+12\psi\right\}}\,dX}$$

The dimension-less LTC (L^*) is seen as,

$$L^* = \frac{w.h_1^2}{\mu U l^2} = \int_0^1 p^*\;dX$$

$$\therefore L^* = \int_0^1 \left[\int_0^X \frac{6\, N_h^{-1} - q^*}{\varphi_X \{M_h^{-1} + 12\psi\}} \, dX \right] dX \tag{10}$$

3. RESULTS AND DISCUSSION

Equation-10 reveals the LTC (Load Tolerating Capacity) in dimensionless form. The incorporated integrals all the way through the mathematical analysis are estimated by Simpson-1/3 rule and the conclusions are obtained in tabular as well as graphical manner. We have analysed a combined effect of the permeability parameter of the porosity, surface roughness perameter, standard deviation and skewness on the performance of bearing's LTC.

L* versus (σ* & γ) (α*= -0.05, ε*= -0.025, ψ=0.1)						
σ*	**0**	**0.025**	**0.05**	**0.075**	**0.1**	
L*	0.59441	0.59662	0.60322	0.61417	0.62939	**γ=3**
L*	0.57449	0.57663	0.58302	0.59362	0.60835	**γ=6**
L*	0.55261	0.55467	0.56083	0.57105	0.58524	**γ=9**

Table-3. LTC versus S.D.

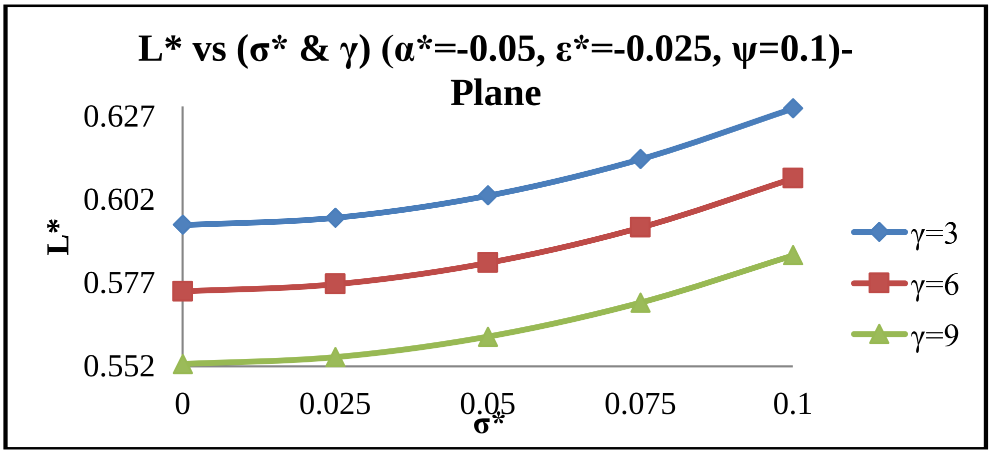

Fig.3. LTC versus S.D.

Fig.3 and Table-3 show how the LTC can be augmented by escalating the S.D. (σ*) of the surface roughness by treating the dimensionless mean α*= -0.05, skewness ε*=-0.025, and permeability parameter of the porous surface ψ=0.1 fixed in the inclined plane longitudinally rough slider bearing. The figure and table also reveal that the LTC can be enhanced by decreasing the PRP, that means that more longitudinality decays the LTC of the bearing. Hence it shows that the roughness parameters play a vital role in designing the bearing so that it improves the performance of the bearing system.

L* versus (ε* & γ) (σ*=0.1, α*= -0.05, ψ =0.1)						
ε*	**-0.025**	**-0.01**	**0**	**0.01**	**0.025**	
L*	0.3754834	0.3442512	0.3201395	0.2925978	0.2425943	**γ=3**
L*	0.3676816	0.337106	0.31353	0.2866349	0.2379112	**γ=6**
L*	0.3589849	0.3291406	0.3061594	0.27998	0.2326655	**γ=9**

Table-4. LTC versus Skewness

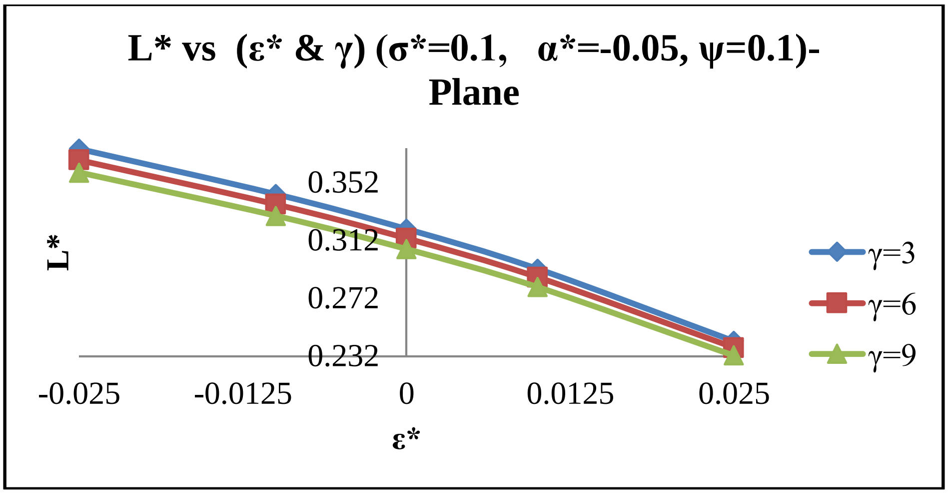

Fig.4. LTC versus Skewness

Fig.4 and Table-4 demonstrate that more negatively skewed ($\varepsilon^* < 0$) surface roughness enhances the LTC of the bearing system while keeping the S.D. ($\sigma^*=0.1$), mean ($\alpha^*= -0.05$), and permeability parameter of the porous surface ($\psi=0.1$) fixed. The Fig.4 and Table-4 have similar trend of the LTC with respect to the PRP which also shows that increased longitudinality decreases the LTC of the bearing.

L* versus (ψ & γ) (σ*=0.1, α*=-0.05, ε*=-0.025)						
ψ	**0**	**0.0001**	**0.001**	**0.01**	**0.1**	
L*	0.1968317	0.1968119	0.1966878	0.1951075	0.1713057	**γ=3**
L*	0.1912464	0.1912288	0.1911269	0.1897491	0.1674106	**γ=6**
L*	0.1850473	0.185032	0.1849537	0.1837925	0.1630438	**γ=9**

Table-5. LTC versus Permeability Parameter

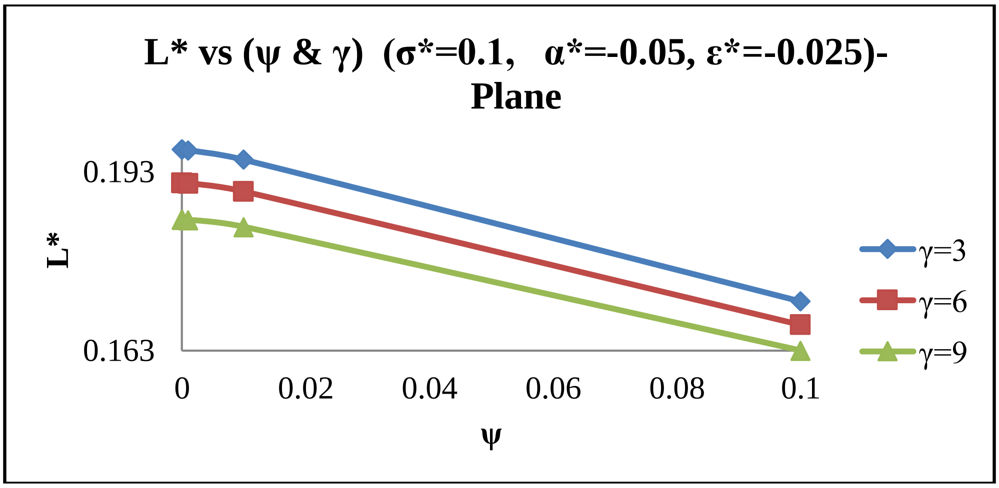

Fig.5. LTC versus Permeability Parameter

Fig.5 and Table-5 convey the behaviour of the LTC with respect to the permeability parameter of the porosity of the surface and PRP assuming the S.D. ($\sigma^*=0.1$), mean ($\alpha^*= -0.05$), and skewness of the surface ($\varepsilon^*= -0.025$) fixed. The Fig.5 and Table-5 reveal very important information about the performance of the bearing that increased porosity may decrease the LTC of the bearing. Hence, it is very clear that the LTC can be increased while the porosity parameter (ψ) is tending to zero.

L* versus γ (σ*=0.1, α*=-0.05, ε*=-0.025)				
γ	**3**	**6**	**9**	
L*	0.1968317	0.1927589	0.1881359	**ψ=0**
L*	0.1954178	0.1913902	0.1868174	**ψ=0.001**
L*	0.1840594	0.1803867	0.1762092	**ψ=0.01**
L*	0.1260471	0.1240572	0.1217659	**ψ=0.1**

Table-6. LTC versus PRP

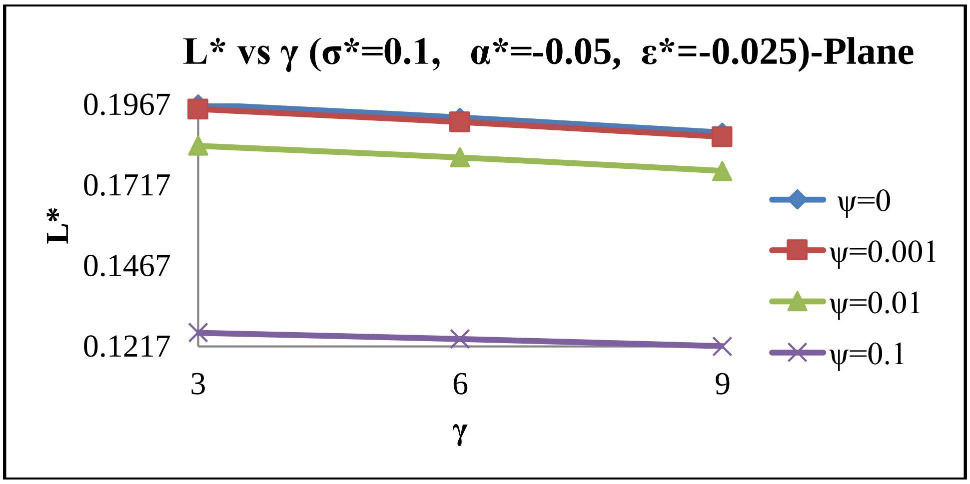

Fig.6. LTC versus PRP

Fig.6 and Table-6 are showing the behaviour of the LTC against the PRP and the permeability parameter of the longitudinal rough surface. They disclose that the trend of the LTC matches with the Fig.3-5 with respect to γ and ψ. The Fig.6 and Table-6 shows that both increased longitudinality and increased permeability decrease the LTC of the bearing.

4. CONCLUSION

The present article justifies that proper combinations of the negatively-skewed roughness (ε*), the S.D. (σ*), the PRP (γ), and the permeability parameter of the porous and longitudinal rough surface may result in a better performance of the inclined plane slider bearing. All the above numerical and graphical results justify the endeavour to improve the bearing's life period, and load-tolerating capacity (LTC) while adopting a conventional fluid in a longitudinally rough finite plane porous slider bearing. It also reveals from the graphical and numerical results of the mathematical model that the LTC of the bearing can be improved to a certain extent with proper combinations of various factors like roughness parameters, porosity, and parameter of roughness-pattern (PRP) while designing the bearing.

5. REFERENCES

1. Christensen H.,(1969): "Stochastic Models for Hydrodynamic Lubrication of Rough Surfaces", Proceedings of the Institution of Mechanical Engineers, vol. 184 no. 1 1013-1026.

2. Christensen H., Tonder K.,(1971): “The Hydrodynamic Lubrication of Rough Bearing Surfaces of Finite Width”, J. of Lubrication Tech, 93(3), 324-329.
3. Christensen H., Tonder K.,(1972): “Waviness and Roughness in Hydrodynamic Lubrication”, Proceedings of the Institution of Mechanical Engineers, vol. 186 no. 1 807-812.
4. Christensen H.,(1972): “A Theory of Mixed Lubrication”, Proceedings of the Institution of Mechanical Engineers, vol. 186 no. 1 421-430.
5. Panchal Girishkumar C., Himanshu C. Patel, G. M. Deheri,(2016): “Influence of surface roughness through a series of flow factors on the performance of a longitudinally rough finite slider bearing”. Annals of the Faculty of Engineering Hunedoara-International Journal of Engineering, vol.14.2, 227-232.
6. Panchal G. C., H. C. Patel, G. M. Deheri,(2016): “Influence of magnetic fluid through a series of flow factors on the performance of a longitudinally rough finite slider bearing”. Global Journal of Pure and Applied Mathematics vol.12.1: 783-796.
7. Panchal, Girish C., et al.(2018) "The Effect of Magnetic Fluid Together with Transverse Parameter of roughness-pattern on the Performance of a Plane Slider Bearing." Proceedings of the 1st International Conference on Numerical Modelling in Engineering: Numerical Modelling in Mechanical and Materials Engineering, Ghent University, Belgium. Springer Singapore, vol.2, 151-167.
8. P. I. Andharia, J. L. Gupta, G. M. Deheri,(2001): “Effect of Surface Roughness on Hydrodynamic Lubrication of Slider Bearings” Tribology Transactions, Volume 44, Issue 2, pages 291-297.
9. Panchal, G. C., H. C. Patel, and H. A. Patel. (2021) "Effect of Various Shapes of Rough Transverse Slider Bearing Equipped with Ferro-Lubricant on the Load-Tolerating Capability by Mathematical Modeling." Journal of Scientific Research, vol.13.3, 522-527.
10. Prakash, J., and S. K. Vij. "Load capacity and time-height relations for squeeze films between porous plates." Wear 24.3 (1973): 309-322.
11. Prakash, J., and S. K. Vij. "Hydrodynamic lubrication of a porous slider." Journal of Mechanical Engineering Science 15.3 (1973): 232-234.
12. Ishaq, Mohammad, et al. (2020)"Nanofluid film flow of eyring powell fluid with magneto hydrodynamic effect on unsteady porous stretching sheet." Punjab University Journal of Mathematics 51.3, 131-153.
13. Awan, A. U., et al. (2016) "Effects of chemical reaction on the unsteady flow of an incompressible fluid over a vertical oscillating plate." Punjab University Journal of Mathematics 48.2, 167-182.
14. Wehgal, A. R., and Muhammad Ashraf. (2020) "MHD asymmetric flow between two porous disks." Punjab University Journal of Mathematics vol.44.1, 9-21
15. Patir N.,(1978): “Ëffects of surface roughness on Partial film lubrication using an average flow model based on numerical simulation”, Northwestern University, Ph.D. thesis, Chapter-2.
16. Patir, Nadir, H. S. Cheng,(1978): "An average flow model for determining effects of three-dimensional roughness on partial hydrodynamic lubrication." Journal of Tribology 100.1: 12-17.
17. Rouleau, W. T. "Hydrodynamic lubrication of narrow press-fitted porous metal bearings." (1963): 123-128.
18. G.M. Deheri, P.I. Andharia, R.M. Patel, (2004): "Longitudinally rough slider bearings with squeeze film formed by a magnetic fluid", Industrial Lubrication and Tribology, Vol. 56 Issue: 3, pp.177 – 187.