A USE OF LAPLACIAN OPERATOR WITH MULTIVARIABLE I – FUNCTION

Yogesh Kumar Tiwari[1], Neelam Panday[2], Bhupendra Tripathi[3*]
[1]Department of Mathematics, A.P.S. University, Rewa, Madhya Pradesh, India – 486002
https://orcid.org/0009-0006-5419-2919
[2]Department of Mathematics, Model Science Collage, Rewa, Madhya Pradesh, India - 486001
[3*]Department of Mathematics, Lakshmi Narain College of Technology & Science Bhopal, Madhya Pradesh, India – 462002
Email: yogesh.rewa@gmail.com, dr.pandeyneelam@gmail.com, bhupendrat@lnct.ac.in,
Corresponding Author; https://orcid.org/0009-0001-7538-346X

Abstract

The present paper deals with the interesting results for the double Laplacian operator involving a product of generalized polynomial and the multivariable I – functions. The established results are motivated by well-known authors in the field of special functions.

2020 Mathematical Sciences Classification: 33C20, 33C45, 33C60.

Keywords- Hypergeometric function, Laplace transformation, Multivariable I-functions.

1 Introduction

Author of this paper is establishing Integral formula involving a product of generalized polynomials and multivariable I – Function with the help of double Laplacian operator the new generalized polynomials itself indicate the importance of the results. He is deriving the number of formulae which are known as in term of orthogonal polynomial.

There are some results which will be use to latter on. Debnath and Bhatta [1] was defined by double Laplacian operator in (2015) as failures.

$$L_{(\alpha,\beta)}^{(\lambda,\mu)}\{\ \} = \left[B(\alpha,\beta)\,\Gamma(\alpha+\beta+\mu)\,\lambda^{-\mu^{-\beta}-\mu^{-1}}\right]$$

$$\int_0^\infty\int_0^\infty e^{-\lambda(\mu+v)}u^{(\alpha-1)}v^{(\beta-1)}(u+v)^{\mu}\ dudv;$$

$$Re(\lambda)>0,\quad Re(\alpha+\beta+\mu)>0, \tag{1.1}$$

The generalized polynomials:

$$S_{n_1\dots n_r}^{m_1\dots m_r}\begin{bmatrix}c_1\\ \vdots\\ c_r\end{bmatrix}$$

Defined Gupta and Agrawal [3] will be as following form.

$$S_{n_1\cdots n_r}^{m_1\cdots m_r}\begin{bmatrix} c_1 \\ \vdots \\ c_r \end{bmatrix} = \sum_{m=0}^{\infty}\sum_{k_1=0}^{(n_1/m_1)}\cdots\sum_{k_r=0\cdots k_i!}^{(n_r/m_r)\cdots(n_i/m_ik_i)} A\left[n_1k_1;\cdots n_rk_{r_{1,n_1}}\right] c_1^{k_1}\cdots c_r^{k_r} \qquad (1.2)$$

Where $n_i = 0,1,2\ldots\ldots\ldots$

$m_i \neq 0(1,2,r)$

That is m_i is an arbitrary positive integer and the coefficient.

$A(n_1k_1,\cdots n_rk_r)$ are arbitrary real or complex.

The multivariable I – functions defined by Sharma and Ahmad [4, 5] also see Sharma and Tiwari [6] given as follows. Subsequently, Chandel and Gupta in 2012, the fractional Laplace transform was formulated and is applied to solve various multivariable distribution [1].

We will take

$$I(x_1,\ldots,x_r) = I_{R_i:R_i';\ldots;R_i(r)}^{m,n:m_1,n_1;\ldots;m_r,n_r}\left(\begin{matrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ x_r \end{matrix}\middle| \begin{matrix} A(r): C^{(1)};\ldots;C^{(r)} \\ \\ B(r): D^{(1)};\ldots;D^{(r)} \end{matrix}\right)$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\psi(s_1,\cdots,s_r)\prod_{j=1}^{r}\theta(s_j)x_j^{s_j}\,\mathrm{d}s_1\cdots\,\mathrm{d}s_r \qquad (1.3)$$

where

$$\psi(s_1,\ldots,s_r) = \frac{\prod_{j=1}^{m}\Gamma\left(b_j-\sum_{k=1}^{r}\beta_j^{(k)}s_k\right)\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{k=1}^{r}\alpha_j^{(k)}s_k\right)}{\sum_{i=1}^{r}\left[\prod_{j=n+1}^{p_i}\Gamma\left(a_{ji}-\sum_{k=1}^{r}\alpha_{ji}^{(k)}s_k\right)\prod_{j=m+1}^{q_i}\Gamma\left(1-b_{ji}+\sum_{k=1}^{r}\beta_{ji}^{(k)}s_k\right)\right]} \qquad (1.4)$$

and

$$\phi_j(s_j) = \frac{\prod_{l=1}^{m_j}\Gamma\left(d_l^{(j)}-\delta_l^{(j)}s_j\right)\prod_{l=1}^{n_j}\Gamma\left(1-c_l^{(j)}+\gamma_l^{(j)}s_j\right)}{\sum_{i=1}^{r}\left[\prod_{l=m_j+1}^{q_i}\Gamma\left(1-d_{lj}^{(j)}+\delta_{lj}^{(j)}s_j\right)\prod_{l=n_j+1}^{p_i}\Gamma\left(c_{lj}^{(j)}-\gamma_{li}^{(j)}s_j\right)\right]} \qquad (1.5)$$

where $j = 1$ to r and k = 1 to r .

Suppose, as usual, that the parameters $a_i, j = 1,\ldots,n; a_ji, j = n+1,\ldots,p_i; b_{ji}, j =$

$1,\ldots,q_i; c_j^{(k)}, j=1,\ldots,n c_{ji^{(k)}}, j=n_k+1,\ldots,q_{i(k)}; d_j^{(k)}, j=1,\ldots,m_k; d_{ji^{(k)}}^{(k)}, j=m_k+1,\ldots,q_{i^{(k)}}$; with $\mathrm{k}=1$ to r, $\mathrm{i}=1$ to R, $i^{(k)}=1$ to R, are complex numbers.

To established the results, detail can be seen in [1], [4-6]. More specific results can be seen in [7-10]

2 Main Results

$$L_{(\alpha,\beta)}^{(\lambda,\mu)}\left\{S_{n_1\cdots n_r}^{m_1\cdots m_r}\begin{bmatrix} c_1 u^{\alpha_1} v^{\beta_1}(u+v)^{\mu_1} \\ \vdots \\ c_r u^{\alpha_r} v^{\beta_r}(u+v)^{\mu_r}\end{bmatrix}\ I\begin{bmatrix} Z_1 u^{\alpha'_1} v^{\beta'_1}(u+v)^{\mu'_1} \\ \vdots \\ Z_r u^{\alpha'_r} v^{\beta'_r}(u+v)^{v'_r}\end{bmatrix}\right\}$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)\ \ \Gamma(\alpha+\beta+\mu)}$$

$$\sum_{m=0}^{\infty}\sum_{k_1=0}^{(n_1/m_1)}\cdots\sum_{k_r=0}^{(n_r/m_r)}\frac{(-n)_{m_1k_1}}{k_1!}\cdots\frac{(-n)_{m_rk_r}}{k_r!}\quad A[n_1k_1;\cdots n_rk_r]\, c_1^{k_1}\cdots c_r^{k_r}$$

$$I_{\mathrm{p+3,q+4};\,R':\,(p_i,\mathrm{q}_i;\ R');\cdots;\,(p_{i(r)},\mathrm{q}_{i(r)}:\,R^r)}^{\mathrm{o,\,n+3\,:\,(m_1,n_1);\cdots;\,}m_r,\,n_r}\begin{bmatrix} Z_1\,\lambda^{(-\alpha'_1-\beta'_1-\mu'_1)} \\ \vdots \\ Z_r\,\lambda^{(-\alpha'_r-\beta'_r-\mu'_r)}\end{bmatrix}$$

$$\left(-\mu-\sum_{j=1}^{r}\mu_j k_j;\ \mu'_i\cdots\mu'_r\right),\left(1-\alpha-m-\sum_{j=1}^{r}\alpha_j k_j;\ \alpha'_1\cdots\alpha'_r\right),$$

$$(1-\beta-\mu+m)\sum_{j=1}^{r}(\beta_j+\mu_j)k_j;\ \beta'_1+\mu'_1\cdots\beta'_r+\mu'_r\ \cdots;\cdots;\cdots;,$$

$$\left.(m-\mu)\ \sum_{j=1}^{r}(\mu_j k_j;\ \mu'_1\cdots\mu'_r)\right]$$

$$\frac{\lambda^{\sum_{j=1}^{r}(\alpha_j+\beta_j+\mu_j)k_j}}{m!} \tag{2.1}$$

Where $Re(\lambda)>0,\quad Re\left\{\alpha+\beta+\mu+\sum_{i=1}^{r}(\alpha_i+\beta_i+\mu_i)k_i\right\}>0,$

Again

$$L_{(\alpha,\beta)}^{(\lambda,\mu)} \left\{ S_{n_1 \cdots n_r}^{m_1 \cdots m_r} \begin{bmatrix} c_1 u^{\alpha_1} v^{\beta_1} (u+v)^{\mu_1} \\ \vdots \\ c_r u^{\alpha_r} v^{\beta_r} (u+v)^{\mu_r} \end{bmatrix} I \begin{bmatrix} Z_1\, u^{\alpha'_1}\, v^{\beta'_1} (u+v)^{\mu'_1} \\ Z_2\, u^{\alpha'_2} v^{\beta'_2} \\ \vdots \\ Z_r\, u^{\alpha'_r} v^{\beta'_r} \end{bmatrix} \right\}$$

$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)\ \ \Gamma(\alpha+\beta+\mu)}$$

$$\sum_{m=0}^{\infty} \sum_{k_1=0}^{(n_1/m_1)} \cdots \sum_{k_r=0}^{(n_r/m_r)} \frac{(-n)_{m_1 k_1} \cdots (-n)_{m_r k_r}}{k_1! \cdots k_r!} \quad A[n_1 k_1; \cdots n_r k_r]\, c_1^{k_1} \cdots c_r^{k_r}$$

$$I_{p+2,q+1\,,\,'\,R':\,(p_{i'},q_{i'+1}:R');\cdots;\,(p_{i(r)},q_{i(r)}:\,R^r)}^{0',n+2\,:\,(m_{1+1},n_1);\cdots;\,m_r,\,n_r}$$

$$\begin{bmatrix} Z_1\, \lambda^{(-\alpha'_1-\beta'_1-\mu'_1)} \\ Z_2\, \lambda^{(-\alpha'_2-\beta'_2)} \\ Z_r\, \lambda^{(-\alpha'_r-\beta'_r)} \end{bmatrix} \left| \left(1-\alpha-m-\sum_{j=1}^{r} \alpha_j k_j;\ \alpha'_1 \cdots \alpha'_r\right) \right.$$

$$\left(1-\beta-\mu+m-\sum_{j=1}^{r}(\beta_j+\mu_j)k_j;\ \beta'_1+\mu'_1;\beta'_2\ \ \cdots \beta'_r\right)$$

$$\left(m-\mu-\sum_{j=1}^{r}\mu_j k_j;\ \mu'_1 \cdots \mu'_r\right) \cdots;\cdots;\cdots;\,-\mu-\sum_{j=1}^{r}(\mu_j k_j;\ \mu'_1 \cdots\cdots;\cdots;\cdots;)\Big]$$

$$\frac{\lambda^{\sum_{j=1}^{r}(\alpha_j+\beta_j+\mu_j)k_j}}{m!} \tag{2.2}$$

Where $Re(\lambda) > 0,\quad Re\left\{\alpha+\beta+\mu+(\alpha_1+\beta_1+\mu_1)k_1 \sum_{i=1}^{r}(\alpha_i+\beta_i+\mu_i)k_i\right\} > 0,$

Lemma: If $Re(\lambda) > 0,\quad Re(\alpha+\beta+\mu) > 0,$

then

$$\int_0^{\infty}\int_0^{\infty} e^{-\lambda(\mu+v)} u^{(\lambda-1)} v^{(\beta-1)} (u+v)^{\mu}\ \{\ \}\, d\mu dv;$$

$$= \sum_{m=0}^{\infty} \binom{\mu}{m} \Gamma(\alpha+m)\, \Gamma(\beta+\mu-m)\, \lambda^{\alpha-\beta-\mu} \qquad (2.3)$$

Proof: Using binomial expression for $(u+v)^{\mu}$ collecting the power of u and v and changing the order of the integration and solving the inner integral with the help of Erdely al. [2] we obtain supposed to be new results.

3 Conclusion: This paper aims to establish some simple examples and applications of the double Laplace transform are discussed. The nature of convergence of Laplace Transform and hypergeometric function is observed in (2.1) and (2.2). Laplace transforms are tested and evaluated according to the criteria and application of the problems to various types of functions for numerical accuracy, computational efficiency, and ease of programming implementation. In future partial differential equations will be discussed in a subsequent paper.

Acknowledgement: The author is thankful to Professor M.A. Pathan Ex. head department of Mathematics, Aligarh Muslim University Aligarh (U.P) India, for his help in the preparation of this paper.

References

[1] L. Debnath, D. Bhatta, *Integral Transforms and Their Applications*, **3rd edn.**, CRC Press, Chapman and Hall, Boca Raton (2015).

[2] Arthur Erdelyi, *Tables of Integral Transforms*, **V.1**, McGraw-Hill, New Yark US (1954).

[3] K.C. Gupta, R. Jain, and R. Agarwal, On existence conditions for a generalized Mellin- Barnes type integral, *Natl, acad., sci. let.*, **30(5-6)**, (2007),169-172.

[4] C. K. Sharma and Ahmad, ‘ Multivariable I-function, *Vijnana Parishad Anusandhan Patrika*, 1997.63-70

[5] C. K. Sharma and S. S. Ahmad, On the multivariable I-function, *Acta Ciencia Indica Math*, **Vol 20, No2**, (1994), 113- 116.

[6] C. K. Sharma and I. M. Tiwari, Certain integrals and series expansion involving the I function, *Acta Ciencia India*, **Vol. XXM No. 2**, (1994), 146-152.

[7] Pandey Awadhesh Kumar, Bhardwaj Ramakant, Wadhwa Kamal and Thakur Nitesh Singh, "Some Recurrences for Generalized Hypergeometric Functions", International Journal of Theoretical and Applied Sciences, Vol. 5(1), (2013), pp. 14-21.

[8] Pandey Awadhesh Kumar, Bhardwaj Ramakant, Wadhwa Kamal and Thakur Nitesh Singh, "Some New Generalized Summation Formulae for Basic Hypergeometric Functions Leibnitz Rule in q-Fractional Calculus", Research Journal of Pure Algebra (RJPA), Vol. 3 (2), (2013), pp. 73-77.
[9] Saxena, Vinod Prakash, Praveen Agarwal, and Altaf Ahmad Bhat. *I-Function and Its Applications*. CRC Press, 2024.
[10] Saxena, Ram Kishore, Jeta Ram, and Dinesh Kumar. "Generalized fractional integration of the product of two ℵ-functions associated with the Appell function F3." *ROMAI J* 9.1 (2013): 147-158.