Fixed Point Results Related to Soft Metric Space

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Abstract

The present papers deal with fixed point results related to soft metric spaces. Specially the results are proved for soft parametric space for random operator. The established results are generalized form of many known results in fixed point theory.

Keywords: - Random operator, Soft Parametric metric space, Soft Metric Space, soft Convergent Sequence, fixed point. ξ

Mathematics Subject Classification: - 47H10, 54H25.

2. **Introduction and Preliminaries:**

The fixed-point theory is very useful tool to solve the real-life problems. [1-2] but many real-life problems cannot be explained through crisp set. The uncertainty can be explained by theory of probability, fuzzy set [21]. Molodstov [12] initiated a novel concept of soft set theory as new mathematical tool for dealing with uncertainty. The detail about soft set theory, can be studied in [10,11] and brief for soft metric space [6, 13-14, 16-17, 9-20]. The detail about fixed point theorem for random operator and Convergence of an iteration leading to a solution of a random operator equation can be viewed in [3-5, 18]. The definition can be modified for random operator easily

Throughout this paper (Ω, Σ) denotes a measurable space consisting of a set Ω and sigma algebra Σ of subset of Ω . X stands for a Banach space, and C is non empty subset of X.

Definition 2.1: A mapping $\tilde{\rho}: SP(\tilde{X}) \times SP(\tilde{X}) \times (0,\infty) \to \mathbb{R}(E)^*$, is said to be a soft parametric metric for random operator on the soft set $\tilde{X}$ if $\tilde{\rho}$ satisfies the following conditions:

(M1) $\tilde{\rho}(\widetilde{\xi x}_{e_1}, \xi\tilde{y}_{e_2}, t) = \bar{0}$ if and only if $\widetilde{\xi x}_{e_1} = \widetilde{\xi y}_{e_2}$ for all $t > 0$,

(M2) $\tilde{\rho}(\tilde{x}_{e_1}, \widetilde{\xi y}_{e_2}, t) = \tilde{\rho}(\xi\tilde{y}_{e_2}, \widetilde{\xi x}_{e_1}, t)$ for all $\widetilde{\xi x}_{e_1}, \widetilde{\xi y}_{e_2} \tilde{\in} \tilde{X}$ & $t > 0$,

(M3) $\tilde{\rho}(\widetilde{\xi x}_{e_1}, \xi\tilde{z}_{e_3}, t) \tilde{\leq} \tilde{\rho}(\xi\tilde{x}_{e_1}, \widetilde{\xi y}_{e_2}, t) + \tilde{\rho}(\widetilde{\xi y}_{e_2}, \widetilde{\xi z}_{e_3}, t)$

for all $\xi\tilde{x}_{e_1}, \xi\tilde{y}_{e_2}, \widetilde{\xi z}_{e_3} \tilde{\in} \tilde{X}, t > 0$.

and the pair $(\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma)$ is called soft parametric metric space for random operator.

Definition 2.2: Let $\{\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ be a sequence in a Soft parametric metric space $(\tilde{X}, \tilde{\rho}, E)$,

(i) $\{\xi\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ is said to be convergent to $\xi\tilde{x}_\lambda \in \tilde{X}$, if

$$\lim_{n\to\infty} \tilde{\rho}(\widetilde{\xi x}^n_{\lambda_n}, \widetilde{\xi x}_\lambda, t) = 0.$$

written as $\lim_{n\to\infty} \xi\tilde{x}^n_{\lambda_n} = \widetilde{\xi x}_\lambda$, for all $t > 0$,

(ii) $\{\xi\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ is said to be a Cauchy sequence in $\tilde{X}$, if

$$\lim_{n,m\to\infty} \tilde{\rho}\left(\xi\tilde{x}^n_{\lambda_n}, \widetilde{\xi x}^m_{\lambda_m}, t\right) = 0\,, \quad \text{for all } t > 0,$$

(iii) $\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma$ is said to be complete if every Cauchy sequence is a convergent sequence for random operator.

Definition 2.3: Let ($\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma$)be a Soft parametric metric space for random operator and a function $(f,\varphi) : \left(\tilde{X}, \tilde{\rho}, E\right) \to \left(\tilde{X}, \tilde{\rho}, E\right)$ is continuous at $\xi\tilde{x}_\lambda \in \tilde{X}$, if for any sequence $\{\xi\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ in $\tilde{X}$ such that $\lim_{n\to\infty}\widetilde{\xi x}^n_{\lambda_n} = \widetilde{\xi x}_\lambda$, then $\lim_{n\to\infty}(f,\varphi)\xi\tilde{x}^n_{\lambda_n} = (f,\varphi)\widetilde{\xi x}_\lambda$.

Lemma 2.4: Let $\{\xi\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ be a sequence in a Soft parametric metric space ($\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma$) for random operator such that

$$\tilde{\rho}\left(\widetilde{\xi x}^n_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \leq h\, \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^n_{\lambda_n}, t\right)$$

Where $h \in [0,1)$ and $n = 1,2, \ldots.$

Then $\{\xi\tilde{x}^n_{\lambda_n}\}_{n=1}^{\infty}$ is a Cauchy sequence in ($\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma$) for random operator.

3 MAIN RESULTS

Theorem 3.1: Let ($\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma$) be a complete soft parametric metric space for random operator and let (f,φ) be a continuous mapping. Let mapping $(f,\varphi) : (\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma) \to (\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma)$ is defined such that

for all $\widetilde{\xi x}_\lambda, \widetilde{\xi y}_\mu \in \tilde{X}, \widetilde{\xi x}_\lambda \neq \xi\tilde{y}_\mu$, and for all $t > 0$, where $\tilde{k} \in [0,\frac{1}{2})$. Then (f,φ) has a unique random fixed point in $\tilde{X}$, if satisfies the following condition:

$$\tilde{\rho}\left((f,\varphi)\xi\tilde{x}_\lambda, (f,\varphi)\xi\tilde{y}_\mu, t\right) \leq \tilde{k}\max\begin{bmatrix} \tilde{\rho}\left(\widetilde{\xi x}_\lambda, \widetilde{\xi y}_\mu, t\right), \tilde{\rho}\left(\xi\tilde{x}_\lambda, (f,\varphi)\xi\tilde{x}_\lambda, t\right), \\ \tilde{\rho}\left(\xi\tilde{y}_\mu, (f,\varphi)\widetilde{\xi y}_\mu, t\right), \tilde{\rho}\left(\xi\tilde{x}_\lambda, (f,\varphi)\widetilde{\xi y}_\mu, t\right), \\ \tilde{\rho}\left((f,\varphi)\widetilde{\xi x}_\lambda, \widetilde{\xi y}_\mu, t\right), \frac{\tilde{\rho}\left(\xi\tilde{x}_\lambda,(f,\varphi)\xi\tilde{x}_\lambda,t\right).\tilde{\rho}\left(\widetilde{\xi y}_\mu,(f,\varphi)\xi\tilde{y}_\mu,t\right)}{1+\tilde{\rho}\left((f,\varphi)\xi\tilde{x}_\lambda,(f,\varphi)\widetilde{\xi y}_\mu,t\right)} \end{bmatrix}$$

[3.1.1]

Proof: Let $\xi\tilde{x}^0_\lambda$ be any soft point in $SP(X)\ for : (\tilde{X}, \tilde{\rho}, E, \Omega, \Sigma)$

Taking iteration $\widetilde{\xi x}^1_{\lambda_1} = (f,\varphi)\left(\widetilde{\xi x}^0_\lambda\right) = \left(f\left(\widetilde{\xi x}^0_\lambda\right)\right)_{\varphi(\lambda)}$

$$\xi\,\tilde{x}^2_{\lambda_2} = (f,\varphi)\left(\widetilde{\xi x}^1_{\lambda_1}\right) = \left(f^2\left(\xi\tilde{x}^0_\lambda\right)\right)_{\varphi^2(\lambda)}$$

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$$\widetilde{\xi x}^{n+1}_{\lambda_{n+1}} = (f,\varphi)\left(\xi\tilde{x}^n_{\lambda_n}\right) = \left(f^{n+1}\left(\xi\tilde{x}^0_\lambda\right)\right)_{\varphi^{n+1}(\lambda)}, - -$$

Now

$$\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) = \tilde{\rho}\left((f,\varphi)\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}\right), (f,\varphi)\left(\xi\tilde{x}^{n}_{\lambda_n}\right), t\right)$$

$$\leq \tilde{k}\max\begin{bmatrix} \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, t\right), \\ \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, (f,\varphi)\xi\tilde{x}^{n}_{\lambda_n}, t\right), \\ \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)\widetilde{\xi x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left((f,\varphi)\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \\ \frac{\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, t\right).\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, (f,\varphi)\xi\tilde{x}^{n}_{\lambda_n}, t\right)}{1+\tilde{\rho}\left((f,\varphi)\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, (f,\varphi)\xi\tilde{x}^{n}_{\lambda_n}, t\right)} \end{bmatrix}$$

$$\leq \tilde{k}\max\begin{bmatrix} \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \\ \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right), \\ \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right), \tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \\ \frac{\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right).\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)}{1+\tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)} \end{bmatrix}$$

$$\tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \xi\tilde{x}^{n+1}_{\lambda_{n+1}}, t\right) \leq \tilde{k}\max\begin{bmatrix} \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \xi\tilde{x}^{n+1}_{\lambda_{n+1}}, t\right), \\ \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \end{bmatrix} \quad \text{----- [3.1.2]}$$

Case (I): If $\max\begin{bmatrix} \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right), \\ \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \end{bmatrix}$

$$= \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right)$$

$$\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \leq \tilde{k}\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right)$$

$$\tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \leq \tilde{k}^{n}\tilde{\rho}\left(\xi\tilde{x}^{0}_{\lambda_0}, \xi\tilde{x}^{1}_{\lambda_1}, t\right)$$

$\Rightarrow$ sequence $\left\{\xi\tilde{x}^{n}_{\lambda_n}\right\}$ is a Cauchy sequence in $\widetilde{X}$.

$\tilde{X}$ is a complete soft parametric metric space for random operator, hence $\left\{\widetilde{\xi x}^{n}_{\lambda_n}\right\}$ is converges.

here is $\xi\ \tilde{x}^{*}_{\lambda} \in \tilde{X}$:

$\xi\ \tilde{x}^{n}_{\lambda_n} \to \widetilde{\xi x}^{*}_{\lambda},\ n \to \infty$.

Using continuity of (f,φ) ,

$$(f,\varphi)\widetilde{\xi x}^{*}_{\lambda} = (f,\varphi)(\lim_{n\to\infty}\widetilde{\xi x}^{n}_{\lambda_n}) = \lim_{n\to\infty}(f,\varphi)\widetilde{\xi x}^{n}_{\lambda_n} = \lim_{n\to\infty}\widetilde{\xi x}^{n+1}_{\lambda_{n+1}} = \widetilde{\xi x}^{*}_{\lambda}.$$

$$i.e.\,(f,\varphi)\xi\tilde{x}^{*}_{\lambda} = \xi\tilde{x}^{*}_{\lambda}.$$

Thus, (f,φ) is a fixed point in $\widetilde{X}$.

Case (II): If $\max\left[\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right), \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n+1}_{\lambda_{n+1}}, t\right)\right]$
$= \tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)$

$$\Rightarrow \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \xi\tilde{x}^{n+1}_{\lambda_{n+1}}, t\right) \leq \tilde{k}\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)$$

Since $0 \leq \tilde{k} < \frac{1}{2}$, which gives contradiction.

Case (III): If $\max\left[\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right), \tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right), \tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n+1}_{\lambda_{n+1}}, t\right)\right]$

$= \tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)$

$$\Rightarrow \quad \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \leq \tilde{k}\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)$$

$$\leq \tilde{k}\left[\tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \widetilde{\xi x}^{n}_{\lambda_n}, t\right) + \tilde{\rho}\left(\widetilde{\xi x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right)\right]$$

$$\leq \left(\frac{\tilde{k}}{1-\tilde{k}}\right)\tilde{\rho}\left(\xi\tilde{x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right)$$

$$\leq s\,\tilde{\rho}\left(\widetilde{\xi x}^{n-1}_{\lambda_{n-1}}, \xi\tilde{x}^{n}_{\lambda_n}, t\right)$$

where $s = \left(\frac{\tilde{k}}{1-\tilde{k}}\right)$

$$\tilde{\rho}\left(\xi\tilde{x}^{n}_{\lambda_n}, \widetilde{\xi x}^{n+1}_{\lambda_{n+1}}, t\right) \leq s^n\tilde{\rho}\left(\widetilde{\xi x}^{0}_{\lambda_0}, \widetilde{\xi x}^{1}_{\lambda_1}, t\right)$$

Sequence $\left\{\widetilde{\xi x}^{n}_{\lambda_n}\right\}$ is a Cauchy sequence in $\widetilde{X}$.

Given that $\tilde{X}$ is a complete soft parametric metric space, hence $\left\{\xi\tilde{x}^{n}_{\lambda_n}\right\}$ is converges.

Here is $\xi\tilde{x}^{*}_{\lambda} \in \tilde{X}$ such that $\widetilde{\xi x}^{n}_{\lambda_n} \to \widetilde{\xi x}^{*}_{\lambda},\ n \to \infty$.

Using continuity of (f, φ) ,

$$(f,\varphi)\widetilde{\xi x}^{*}_{\lambda} = (f,\varphi)(\lim_{n\to\infty}\widetilde{\xi x}^{n}_{\lambda_n}) = \lim_{n\to\infty}(f,\varphi)\widetilde{\xi x}^{n}_{\lambda_n}$$

$$= \lim_{n\to\infty}\widetilde{\xi x}^{n+1}_{\lambda_{n+1}} = \widetilde{\xi x}^{*}_{\lambda}.$$

$i.e. (f,\varphi)\widetilde{\xi x}^{*}_{\lambda} = \widetilde{\xi x}^{*}_{\lambda}$. Thus, (f, φ) is a fixed point in $\widetilde{X}$.

Uniqueness: Let $\widetilde{\xi y}^{*}_{\mu}$ is another fixed point of (f, φ) in $\tilde{X}$ such that $\widetilde{\xi x}^{*}_{\lambda} \neq \widetilde{\xi y}^{*}_{\mu}$,
$\tilde{\rho}\left((f,\varphi)\xi\tilde{x}^{*}_{\lambda}, (f,\varphi)\widetilde{\xi y}^{*}_{\mu}, t\right) \leq$

$$\tilde{k}\max\begin{bmatrix}\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right),\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,(f,\varphi)\widetilde{\xi x}_\lambda^*,t\right),\\ \tilde{\rho}\left(\xi\tilde{y}_\mu^*,(f,\varphi)\widetilde{\xi y}_\mu^*,t\right),\tilde{\rho}\left(\widetilde{\xi x}_\lambda^*,(f,\varphi)\widetilde{\xi y}_\mu^*,t\right),\\ \tilde{\rho}\left((f,\varphi)\widetilde{\xi x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right),\frac{\tilde{\rho}\left(\widetilde{\xi x}_\lambda^*,(f,\varphi)\widetilde{\xi x}_\lambda^*,t\right).\tilde{\rho}\left(\widetilde{\xi y}_\mu^*,(f,\varphi)\widetilde{\xi y}_\mu^*,t\right)}{1+\tilde{\rho}\left((f,\varphi)\widetilde{\xi x}_\lambda^*,(f,\varphi)\widetilde{\xi y}_\mu^*,t\right)}\end{bmatrix}$$

$$\leq k\max\begin{bmatrix}\tilde{\rho}\left(\widetilde{\xi x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right),\tilde{\rho}\left(\widetilde{\xi x}_\lambda^*,\widetilde{\xi x}_\lambda^*,t\right),\\ \tilde{\rho}\left(\widetilde{\xi y}_\mu^*,\widetilde{\xi y}_\mu^*,t\right),\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right),\\ \tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right),\frac{\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi x}_\lambda^*,t\right).\tilde{\rho}\left(\widetilde{\xi y}_\mu^*,\widetilde{\xi y}_\mu^*,t\right)}{1+\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right)}\end{bmatrix}$$

$$\tilde{\rho}\left((f,\varphi)\widetilde{\xi x}_\lambda^*,(f,\varphi)\widetilde{\xi y}_\mu^*,t\right)\leq\tilde{\alpha}\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right)$$

This is true only when $\tilde{\rho}\left(\xi\tilde{x}_\lambda^*,\widetilde{\xi y}_\mu^*,t\right)=0.$

$$\Rightarrow\widetilde{\xi x}_\lambda^*=\widetilde{\xi y}_\mu^*$$

So, it is unique.

. **Lemma:** Let $(\tilde{X},\tilde{\rho},E,\Omega,\Sigma)$be a complete soft parametric metric space for random operator and let (f,φ) a continuous mapping.

Let mapping $(f,\varphi):\left(\tilde{X},\tilde{\rho},E\right)\to\left(\tilde{X},\tilde{\rho},E\right)$ satisfies the following condition:

$$\tilde{\rho}\left((f,\varphi)\widetilde{\xi x}_\lambda,(f,\varphi)\widetilde{\xi y}_\mu,t\right)\leq\tilde{k}\left[\tilde{\rho}\left(\widetilde{\xi x}_\lambda,(f,\varphi)\xi\tilde{x}_\lambda,t\right)+\tilde{\rho}\left(\widetilde{\xi y}_\mu,(f,\varphi)\widetilde{\xi y}_\mu,t\right)\right]$$

$$+\tilde{l}\left[\tilde{\rho}\left(\widetilde{\xi x}_\lambda,(f,\varphi)\widetilde{\xi y}_\mu,t\right)+\tilde{\rho}\left((f,\varphi)\xi\tilde{x}_\lambda,\widetilde{\xi y}_\mu,t\right)\right]$$

For all $\xi\tilde{x}_\lambda,\widetilde{\xi y}_\mu\in\tilde{X},\xi\tilde{x}_\lambda\neq\widetilde{\xi y}_\mu,$

and for all $t>0,$ where $\tilde{k}+\tilde{l}<\frac{1}{2},\tilde{k},\tilde{l}\in\left[0,\frac{1}{2}\right).$

Then (f,φ) has a unique fixed point in $\tilde{X}$.

Proof; This can be proved easily using the concept of above theorem.

Future Scope; These results can be proved using the concept of integral type mappings and also results can be established for n dimensional soft metric spaces.

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