# The ELECTRE multi-attribute group decision making method based on interval-valued intuitionistic fuzzy sets

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**Abstract**

In this paper, based on the ELECTRE method and new ranking for the interval-valued intuitionistic fuzzy set (IVIFS), the IVIF ELECTRE method to solve multi-attribute group decision-making problems with interval-valued intuitionistic fuzzy input data is proposed, it is extending the intuitionistic fuzzy set (IF) ELECTRE method. This method firstly use AHP (Analytic hierarchy process) to find the weights of attribute, and use new ranking method for IVIFS and similarity measure between IVIFS to determine the weights of decision makers (DMs), then give the concordance set, midrange concordance set, weak concordance set and cosponging discordance set, midrange discordance set, weak discordance set. From this, the concordance matrix, discordance index, concordance dominance matrix and discordance dominance matrix are proposed. Finally, the ranking order of all the alternatives $A_i(i = 1, 2, \ldots, n)$ and the best alternative are obtained. A numerical example is taken to illustrate the feasibility and practicability of the proposed method.

*Keywords*: Interval-valued intuitionistic fuzzy sets; ELECTRE method; Multi-attribute group decision making

## 1 Introduction

Since the multi-attribute decision making (MADM) was introduced in 1960's, it has been a hot topic in decision making and systems engineering, and been proven as a useful tool due to its broad applications in a number of practical problems. But in some real-life situations, a decision maker (DM) may not be able to accurately express his/her preferences for alternatives due to that (1) the DM may not possess a precise or sufficient level of knowledge of the problem; (2) the DM is unable to discriminate explicitly the degree to which one alternative are better than others. In order to handle inexact and imprecise data, in 1965 Zadeh [38] introduced fuzzy set (FS) theory. In 1983 Atanassov [1,2] generalized FS to intuitionistic fuzzy set (IFS) by using two characteristic functions to express the degree of membership and the degree of non-membership of elements of the universal set. Since IFS tackled the drawback of the single membership value in FS theory, IFS has been widely applied to the multi-attribute decision making (MADM) [4,7,8,10-14,20,22,23,28] and multi-attribute group decision making (MAGDM) [18,19,21].

In 1989 Atanassov and Gargov [3] further generalized the IFS in the spirit of the ordinary interval-valued fuzzy set (IVFS) and defined the concept of interval-valued intuitionistic fuzzy set (IVIFS), which enhances greatly the representation ability of uncertainty than IFS. Similar to the IFS, IVIFSs were also

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used in the problems of MADM [6,15-17,28,32] and MAGDM [29,31,33,34]. In these researches, some are extension of classic decision making methods in IVIFS environment. For example, Li [15] developed the closeness coefficient-based nonlinear-programming method for interval-valued intuitionistic fuzzy MAD-M with incomplete preference information, Li [16] proposed the TOPSIS-based nonlinear-programming methodology for MADM with IVIFSs, Li [17] proposed the linear-programming method for MADM with IVIFSs. These decision methods under interval-valued intuitionistic fuzzy environments also generalize the classic decision making methods, such as TOPSIS and LINMAP. In [32], Wang et al. proposed a expect to apply ELECTRE and PROMETHEE motheds to MADM and MAGDM with IVIFS.

In this paper, based on the new ranking method of interval in [27] and similarity measure of IVIFSs in [35, 37], the IF ELECTRE [30] method is applied to MAGDM with IVIFS, and obtain IVIFS ELECTRE method for solving MAGDM problems under IVIF environments.

This paper is organized as follows. Section 2 briefly reviews the analytic hierarchy process (AHP). Section 3 and Section 4 introduce the new ranking method of intervals and similarity measure between IVIFSs, respectively. Section 5 formulates an MAGDM problem in which the evaluation of alternatives in each attribute is expressed by IVIF sets, and also develops an extended ELECTRE method. Section 6 demonstrates the feasibility and applicability of the proposed method by applying it to the MAGDM problem of the air-condition. Finally, Section 7 presents the conclusions.

## 2 Analytic hierarchy process (AHP)

AHP was introduced for the first time in 1980 by Thomas L. Saaty [24]. For years, AHP has been used in various fields such as social sciences, health planning and management. Many researchers have preferred to use AHP to find the weights of attribute [25,26]. Due to the fact that attribute weights in the decision-making problems are various, it is not correct to assign all of them as equalled [5]. To solve the problem of indicating the weights, some methods like AHP, eigenvector, entropy analysis, and weighted least square methods were used. For the calculation of attribute weight in AHP the following steps are used:

(i) Arrange the attribute in $n \times n$ square matrix form as rows and columns.

(ii) Using pairwise comparisons, the relative importance of one attribute over another can be expressed as follow:

If two attribute have equal importance in pairwise comparison enter 1; if one of them is moderately more important than the other enter 3 and for the other enter 1/3; if one of them is strongly more important enter 5 and for the other enter 1/5; if one of them is very strongly more important enter 7 and for the other enter 1/7, and if one of them is extremely important enter 9 and for the other enter 1/9. 2, 4, 6 and 8 can be entered as intermediate values. Thus, pairwise comparison matrix is obtained as a result of the pairwise comparisons. Note that all elements in the comparison matrix are positive, in other words $a_{ij} > 0$ $(i, j = 1, 2, \ldots, n)$.

(a) To find the maximum eigenvalue $\lambda$ of the comparison matrix.

(b) Calculate consistency index $CI = \frac{\lambda - n}{n-1}$ and consistency ratio $CR = \frac{CI}{RI}$, where RI is the random consistency index given by Saaty.(Table 1)

(c) If $CR \geq 0.1$, then adjusts elements $a_{ij}$ $(i, j = 1, 2, \ldots, n)$ of the comparison matrix, (a) and (b) choices are done iteratively until $CR < 0.1$.

(d) Compute eigenvector of the maximum eigenvalue of the comparison matrix.

(e) Normalized eigenvector.

Table1:Random consistency index RI.

| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| RI | 0 | 0 | 0.58 | 0.90 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.51 |

# 3 Ranking method for intervals

Let $x = [a, b] \subseteq [0, 1]$ and $y = [c, d] \subseteq [0, 1]$ be two intervals. Since the location relations between $x = [a, b]$ and $y = [c, d]$ include the following six cases, Wan and Dong [27] calculated the occurrence probability for the fuzzy(or random) event $x \geq y$, denoted by $P(x \geq y)$, under different cases.

Case1: $a < b \leq c < d$,

$$P(x \geq y) = 0. \tag{1}$$

Case2: $a \leq c < b < d$ or $a < c < b \leq d$,

$$P(x \geq y) = \frac{(b-c)^2}{2(b-a)(d-c)}. \tag{2}$$

Case3: $a \leq c < d < b$ or $a < c < d \leq b$ or $a \leq c < d \leq b$,

$$P(x \geq y) = \frac{2b-d-c}{2(b-a)}. \tag{3}$$

Case4: $c \leq a < b < d$ or $c < a < b \leq d$,

$$P(x \geq y) = \frac{b+a-2c}{2(d-c)}. \tag{4}$$

Case5: $c \leq a < d < b$ or $c < a < d \leq b$,

$$P(x \geq y) = \frac{2bd+2ac-2bc-a^2-d^2}{2(b-a)(d-c)}. \tag{5}$$

Case6: $c \leq d \leq a < b$ or $c < a < b \leq d$,

$$P(x \geq y) = 1. \tag{6}$$

In order to rank intervals $\tilde{a}_i = [a_i, b_i]$ $(i = 1, 2, \ldots, n)$, Wang and Dong [27] construct the matrix of possibility degree as $P = (P_{ij})_{n\times n}$, where $P_{ij} = P(\tilde{a}_i \geq \tilde{a}_j)$ $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, n)$. Then, the ranking vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is derived as follows:

$$\omega_i = (\sum_{j=1}^{n} P_{ij} + \frac{n}{2} - 1)/(n(n-1)) \ \ (i = 1, 2, \cdots, n). \tag{7}$$

The larger the value of $\omega_i$, the bigger the corresponding intervals $\tilde{a}_i = [a_i, b_i]$. In other words, for the two intervals $\tilde{a}_i = [a_i, b_i]$ and $\tilde{a}_j = [a_j, b_j]$, if $\omega_i \geq \omega_j$, then $[a_i, b_i] \geq [a_j, b_j]$.

# 4 Similarity measure between IVIFSs

**Definition 1.**[3] An IVIFS $A$ in the universe set of discourse $X$ is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \,|x \in X\},$$

where $\mu_A(x) \subseteq [0, 1]$ and $\nu_A(x) \subseteq [0, 1]$ denote respectively the membership degree interval and the non-membership degree interval of $x$ to $A$,with the condition:

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$\sup\mu_A(x)+\sup\nu_A(x)\leq 1, \forall x\in X$.

Since IVIFS is composed of two ordered interval pairs, Xu [31,32] called them interval-valued intuitionistic fuzzy numbers(IVIFNs) and simply denoted by $G=([a,b],[c,d])$, where $[a,b]\subseteq[0,1],[c,d]\subseteq[0,1]$ and $b+d\leq 1$.

**Definition 2.**[37] Let $G_i=([a_i,b_i],[c_i,d_i])$ $(i=1,2)$ be two IVIFNs, the normalized Hamming distance between $G_1$ and $G_2$ can be defined as:

$$d(G_1,G_2)=\frac{1}{4}(|a_1-a_2|+|b_1-b_2|+|c_1-c_2|+|d_1-d_2|+\left|\pi_1'-\pi_2'\right|+\left|\pi_1''-\pi_2''\right|), \tag{8}$$

where $\pi_{G_i}=[\pi_i',\pi_i'']=[1-b_i-d_i,1-a_i-c_i]$ $(i=1,2)$ is called the degree of indeterminacy or called the degree of hesitancy of the IVIFN $G_i$.

**Definition 3.**[35, 37] Let $G_i=([a_i,b_i],[c_i,d_i])$ $(i=1,2)$ be two IVIFNs, then

$$s(G_1,G_2)=\begin{cases}1, & if\ G_1=G_2=G_2^c,\\ \frac{d(G_1,G_2^c)}{d(G_1,G_2)+d(G_1,G_2^c)}, & otherwise\end{cases} \tag{9}$$

is called the degree of similarity between $G_1$ and $G_2$, where $G_2^c=([c_2,d_2],[a_2,b_2])$ is denoted as the complement of $G_2$.

**Definition 4.**[37] Let $A$ and $B$ be two IVIFSs in X, then

$$s(A,B)=\frac{1}{n}\sum_{j=1}^{n}s(G_j^A,G_j^B)=\frac{1}{n}\sum_{j=1}^{n}\frac{d(G_j^A,(G_j^B)^c)}{d(G_j^A,G_j^B)+d(G_j^A,(G_j^B)^c)} \tag{10}$$

is called the degree of similarity between A and B , where $G_j^A$ and $G_j^B$ are j-th IVIFNs of $A$ and $B$ , respectively.

**Definition 5.**[6, 27] Let $G_i$ $(i=1,2,\ldots,n)$ be a collection of the IVIFNs, where $G_i=([a_i,b_i],[c_i,d_i])$. If

$$Y_\omega(G_1,G_2,\cdots,G_n)=\frac{\sum\limits_{j=1}^{n}\omega_jG_j}{\sum\limits_{j=1}^{n}\omega_j}, \tag{11}$$

where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the weight vector, then the function $Y_\omega$ is called the weighted average operator for the IVIFNs. Particularly, if $\omega_j$ $(j=1,2,\ldots,n)$ are crisp values, then the weighted average operator $Y_\omega$ is calculated as follows:

$$Y_\omega(G_1,G_2,\cdots,G_n)=\frac{\sum\limits_{j=1}^{n}\omega_jG_j}{\sum\limits_{j=1}^{n}\omega_j}=\left(\left[\frac{\sum\limits_{j=1}^{n}\omega_ja_j}{\sum\limits_{j=1}^{n}\omega_j},\frac{\sum\limits_{j=1}^{n}\omega_jb_j}{\sum\limits_{j=1}^{n}\omega_j}\right],\left[\frac{\sum\limits_{j=1}^{n}\omega_jc_j}{\sum\limits_{j=1}^{n}\omega_j},\frac{\sum\limits_{j=1}^{n}\omega_jd_j}{\sum\limits_{j=1}^{n}\omega_j}\right]\right). \tag{12}$$

# 5 MAGDM problems and ELECTRE method with IVIFSs

## 5.1 Problems description for MAGDM with IVIFSs

Assume that there are $m$ alternatives $\{A_1,A_2,\ldots,A_m\}$ and $k$ experts $\{p_1,p_2,\ldots,p_k\}$, each alternative $A_i$ has $n$ attributes $\{a_1,a_2,\ldots,a_n\}$. For each alternative $A_i$, each expert gives evaluation on different attribute.

The multi-attribute group decision making (MAGDM) is choose the best one from all alternatives according to these evaluations. Assume that $G^t_{Mij} = [a^t_{ij}, b^t_{ij}]$ and $G^t_{Nij} = [c^t_{ij}, d^t_{ij}]$ are respectively the membership degree and non-membership degree of alternative $A_i \in A$ on an attribute $a_j$ given by DM $p_t$ to the fuzzy concept "excellent". In other words, the evaluation of $A_i$ on $a_j$ given by $p_t$ is an IVIFN as follows:

$$G^t_{ij} = (G^t_{Mij}, G^t_{Nij}), \tag{13}$$

where $[a^t_{ij}, b^t_{ij}] \subseteq [0, 1], [c^t_{ij}, d^t_{ij}] \subseteq [0, 1]$ and $b^t_{ij} + d^t_{ij} \leq 1$ $(1 \leq i \leq m, 1 \leq j \leq n, 1 \leq t \leq k)$.

## 5.2 Determination of the weights of DMs

Since the different DMs play different roles during the process of decision making, thus the importance of DMs should be taken into consideration. The weight vector of DMs is denoted by $z = (z_1, z_2, \ldots, z_k)^T$. In the following, an approach determined the weights of DMs is given.

Suppose that the evaluation of alternative $A_i$ given by DM $p_t$ on each attribute are respectively the IVIFNs $G^t_{i1}, G^t_{i2}, ..., G^t_{in}$. By Eq.(12), the individual overall attribute value of $A_i$ given by $p_t$ is obtained as follows:

$$E^t_i = ([a^t_i, b^t_i], [c^t_i, d^t_i]) = Y_\omega(G^t_{i1}, G^t_{i2}, \cdots G^t_{in}), \tag{14}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weight vector of attributes.

Let $E^t = (E^t_1, E^t_2, \ldots, E^t_m)$ and $E^u = (E^u_1, E^u_2, \ldots, E^u_m)$ are evaluation vectors of all alternatives given by DMs $p_t$ and $p_u$, respectively. Using Eqs.(8-10), the similarity degree $s_{tu}$ between $E^t$ and $E^u$ is obtained, and the similarity matrix $S$ is constructed as follows:

$$S = (s_{tu})_{k\times k}. \tag{15}$$

Obviously, $S$ is a non-negative symmetric matrix, by the Perron-Frobenius theorem [12], there exists the maximum module eigenvalue $\lambda > 0$, and the corresponding eigenvector $x = (x_1, x_2, \ldots, x_k)^T$ satisfies that $x_t > 0$ $(t = 1, 2, \ldots, k)$ and $\lambda x = Sx$.

Let $z = \lambda x = Sx$, then each component of $z$ is the weight of corresponding expert. The normalized vector $z$, the weight $z_t$ $(t = 1, 2, \ldots, k)$ of DM $p_t$ is obtained as follows:

$$z_t = \frac{x_t}{(x_1 + x_2 + \cdots + x_k)} \quad (t = 1, 2, \cdots, k). \tag{16}$$

## 5.3 ELECTRE methods based on IVIFS

Based on the idea of ELECTRE method, a new approach, named as IVIF ELECTRE, is formulated to solve a MCDM problem under interval-valued intuitionistic fuzzy environment. For each pair of alternatives $k$ and $l$ $(k, l = 1, 2, \ldots, m$ and $k \neq l)$, each attribute in the different alternatives can be divided into two distinct subsets. The concordance set $E_{kl}$ of $A_k$ and $A_l$ is composed of all attribute for which $A_k$ is preferred to $A_l$. In other words, $E_{kl} = \{j | x_{kj} \geq x_{lj}\}$, where $J = \{j | j = 1, 2, \ldots, n\}$, $x_{kj}$ and $x_{lj}$ denoted the evaluation of DM in the jth attribute to alternative $A_k$ and $A_l$, respectively. The complementary subset, which is the discordance set, is $F_{kl} = \{j | x_{kj} < x_{lj}\}$. In the proposed IVIF ELECTRE method, we can classify different types of concordance and discordance sets using the concepts of score function, accuracy function and hesitation degree, and use concordance and discordance sets to construct concordance and discordance matrices, respectively. The decision makers can choose the best alternative using the concepts of positive and negative ideal points.

Xu [31] and Xu and Chen [36] defined the score function $S(G)$ and accuracy function $H(G)$ for an IVIFN G=([a,b],[c,d]) as follows:

$$S(G) = \frac{1}{2}(a + b - c - d), \tag{17}$$

$$H(G) = \frac{1}{2}(a + b + c + d). \tag{18}$$

Here, we define the hesitation degree for an IVIFN G=([a,b],[c,d]) as follows:

$$\pi(G) = 1 - \frac{1}{2}(a + b + c + d). \tag{19}$$

From (18) and (19), easy to see that a higher accuracy degree $H(G)$ correlates with a lower hesitancy degree $\pi(G)$.

Considering the better alternative has the higher score degree or higher accuracy degree in cases where alternatives have the same score degree. A higher score degree refers to a larger membership degree or smaller non-membership degree, and a higher accuracy degree refers to a smaller hesitation degree. Based on this, using the above three functions to compare IVIF values of different alternatives. The concordance set can be classified as concordance set, midrange concordance set and weak concordance set. Similarly, The discordance sets can also be classified as the discordance set, midrange discordance set, and weak discordance set.

Next, the concordance set, midrange concordance set, weak concordance set, discordance set, midrange discordance set, weak discordance set are defined respectively as follows.

Let $G_{kj} = ([a_{kj}, b_{kj}], [c_{kj}, d_{kj}])$ and $G_{lj} = ([a_{lj}, b_{lj}], [c_{lj}, d_{lj}])$ denote the $j$th attribute value of alternative $A_k$ and $A_l$, respectively. The concordance set $C_{kl}$ is composed of all attribute for which $A_k$ is preferred to $A_l$,i.e.,

$$C_{kl} = \{j | [a_{kj}, b_{kj}] \geq [a_{lj}, b_{lj}], [c_{kj}, d_{kj}] < [c_{lj}, d_{lj}] \ \ and \ [\pi'_{kj}, \pi''_{kj}] < [\pi'_{lj}, \pi''_{lj}]\}, \tag{20}$$

where $J = \{j | j = 1, 2, \ldots, n\}$.

The midrange concordance set $C'_{kl}$ is defined as

$$C'_{kl} = \{j | [a_{kj}, b_{kj}] \geq [a_{lj}, b_{lj}], [c_{kj}, d_{kj}] < [c_{lj}, d_{lj}] \ \ and \ [\pi'_{kj}, \pi''_{kj}] \geq [\pi'_{lj}, \pi''_{lj}]\}. \tag{21}$$

The major difference between (20) and (21) is the hesitancy degree; the hesitancy degree at the $k$th alternative with respect to the $j$th attribute is higher than the $l$th alternative with respect to the $j$th attribute in the midrange concordance set. Thus, Eq. (20) is more concordant than (21).

The weak concordance set $C''_{kl}$ is defined as

$$C''_{kl} = \{j | [a_{kj}, b_{kj}] \geq [a_{lj}, b_{lj}] \ \ and \ \ [c_{kj}, d_{kj}] \geq [c_{lj}, d_{lj}]\}. \tag{22}$$

The degree of non-membership at the kth alternative with respect to the jth attribute is higher than the lth alternative with respect to the jth attribute in the weak concordance set; thus, Eq. (21) is more concordant than (22).

The discordance set is composed of all attribute for which $A_k$ is not preferred to $A_l$. The discordance set $D_{kl}$ is formulated as follows:

$$D_{kl} = \{j | [a_{kj}, b_{kj}] < [a_{lj}, b_{lj}], [c_{kj}, d_{kj}] \geq [c_{lj}, d_{lj}] \ \ and \ [\pi'_{kj}, \pi''_{kj}] \geq [\pi'_{lj}, \pi''_{lj}]\}, \tag{23}$$

The midrange discordance set $D'_{kl}$ is defined as

$$D'_{kl} = \{j | [a_{kj}, b_{kj}] < [a_{lj}, b_{lj}], [c_{kj}, d_{kj}] \geq [c_{lj}, d_{lj}] \ \ and \ [\pi'_{kj}, \pi''_{kj}] < [\pi'_{lj}, \pi''_{lj}]\}. \tag{24}$$

The weak discordance set $D''_{kl}$ is defined as

$$D''_{kl} = \{j|[a_{kj}, b_{kj}] < [a_{lj}, b_{lj}] \ \ and \ \ [c_{kj}, d_{kj}] < [c_{lj}, d_{lj}]\}. \tag{25}$$

The IVIF ELECTRE method is an integrated IVIFS and ELECTRE method. The relative value of the concordance set of the IVIF ELECTRE method is measured through the concordance index. The concordance index $e_{kl}$ between $A_k$ and $A_l$ is defined as:

$$e_{kl} = \min_{j\in C^*}\{w_{C^*} \times d(G_{kj}, G_{lj})\}, \tag{26}$$

where $d(G_{kj}, G_{lj})$ is defined in (8), denoted the distance between jth attribute of alternatives $A_k$ and $A_l$, and $w_{C^*}$ is equal to $w_C$, $w_{C'}$ or $w_{C''}$, which denoted the weight of the concordance, midrange concordance, and weak concordance sets, respectively.

The concordance matrix $E$ is defined as follows:

$$E = \begin{bmatrix} - & e_{12} & \cdots & \cdots & e_{1m} \\ e_{21} & - & e_{23} & \cdots & e_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ e_{(m-1)1} & \cdots & \cdots & - & e_{(m-1)m} \\ e_{m1} & e_{m2} & \cdots & e_{m(m-1)} & - \end{bmatrix}, \tag{27}$$

where the maximum value of $e_{kl}$ is denoted by $e^*$, which is the positive ideal point, and a higher value of $e_{kl}$ indicates that $A_k$ is preferred to $A_l$.

the discordance index is defined as follows:

$$h_{\mathrm{kl}} = \max_{j\in D^*}\{w_{D^*} \times d(G_{kj}, G_{lj})\}, \tag{28}$$

where $d(G_{kj}, G_{lj})$ is defined in (8), denoted the distance between jth attribute of alternatives $A_k$ and $A_l$, and $w_{D^*}$ is equal to $w_D$, $w_{D'}$ or $w_{D''}$, which denoted the weight of the discordance, midrange discordance, and weak discordance sets, respectively.

The discordance matrix $H$ is defined as follows:

$$H = \begin{bmatrix} - & h_{12} & \cdots & \cdots & h_{1m} \\ h_{21} & - & h_{23} & \cdots & h_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ h_{(m-1)1} & \cdots & \cdots & - & h_{(m-1)m} \\ h_{m1} & h_{m2} & \cdots & h_{m(m-1)} & - \end{bmatrix}, \tag{29}$$

where the maximum value of $h_{kl}$ is denoted by $h^*$, which is the negative ideal point, and a higher value of $H_{kl}$ indicates that $A_k$ is less favorable than $A_l$.

The concordance dominance matrix calculation process is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution, thus, the concordance dominance matrix $K$ is defined as follows:

$$K = \begin{bmatrix} - & k_{12} & \cdots & \cdots & k_{1m} \\ k_{21} & - & k_{23} & \cdots & k_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ k_{(m-1)1} & \cdots & \cdots & - & k_{(m-1)m} \\ k_{m1} & k_{m2} & \cdots & k_{m(m-1)} & - \end{bmatrix}, \tag{30}$$

where $k_{kl} = e^* - e_{kl}$, which refers to the separation of each alternative from the positive ideal solution. A higher value of $k_{kl}$ indicates that $A_k$ is less favorable than $A_l$.

The discordance dominance matrix calculation process is based on the concept that the chosen alternative should have the farthest distance from the negative ideal solution, thus, the discordance dominance matrix $L$ is defined as follows:

$$L = \begin{bmatrix} - & l_{12} & \cdots & \cdots & l_{1m} \\ l_{21} & - & l_{23} & \cdots & l_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ l_{(m-1)1} & \cdots & \cdots & - & l_{(m-1)m} \\ l_{m1} & l_{m2} & \cdots & l_{m(m-1)} & - \end{bmatrix}, \tag{31}$$

where $l_{kl} = h^* - h_{kl}$, which refers to the separation of each alternative from the negative ideal solution. A higher value of $l_{kl}$ indicates that $A_k$ is preferred to $A_l$.

In the aggregate dominance matrix determining process, the distance of each alternative to both positive and negative ideal points can be calculated to determine the ranking order of all alternatives. The aggregate dominance matrix R is defined as follows:

$$R = \begin{bmatrix} - & r_{12} & \cdots & \cdots & r_{1m} \\ r_{21} & - & r_{23} & \cdots & r_{2m} \\ \cdots & \cdots & - & \cdots & \cdots \\ r_{(m-1)1} & \cdots & \cdots & - & r_{(m-1)m} \\ r_{m1} & r_{m2} & \cdots & r_{m(m-1)} & - \end{bmatrix}, \tag{32}$$

where

$$r_{kl} = \frac{l_{kl}}{k_{kl} + l_{kl}},$$

$r_{kl}$ refers to the relative closeness to the ideal solution, with a range from 0 to 1. A higher value of $r_{kl}$ indicates that the alternative $A_k$ is simultaneously closer to the positive ideal point and farther from the negative ideal point than the alternative $A_l$, thus, it is a better alternative.

Let $\overline{T}_k = \frac{1}{m-1} \sum_{l=1, l \neq k}^{m} r_{kl},\ k = 1, 2, \cdots, m,$ (33)

and $\overline{T}_k$ is the final value of evaluation. All alternatives can be ranked according to $\overline{T}_k$. The best alternative $T^*$, which is simultaneously the shortest distance to the positive ideal point and the farthest distance from the negative ideal point, can be generated and defined as follows:

$$T^* = \max_{1 \le k \le m} \{\overline{T}_k\}, \tag{34}$$

where $A^*$ is the best alternative.

## 5.4 Group decision making method

In the following we shall utilize the AHP and interval-valued intuitionistic fuzzy weighted average operator $Y$ ( i.e. Eq. (12)) to propose a new MAGDM method with IVIFN information. The detailed steps are summarized as follows:

Step 1. DMs use IVIFSs to represent the evaluation information in the each attribute of alternatives;
Step 2. Use AHP to calculate the weight of attribute;
Step 3. Calculate the individual overall attribute value of each alternative by Eq.(14);
Step 4. Obtain the similarity matrix of the DMs according to Eq.(10);

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Step 5. Derive the weight value of each DM from Eq.(16);
Step 6. Using the weight of DM to integrate the same attribute value of different DMs of each alternative in terms of Eq.(14);
Step 7. By the possibility degree ranking method for intervals in Section 3, calculate the ranking vector of the membership degree interval, the non-membership degree interval and the hesitancy degree interval of between the difference alternatives on each attribute, respectively.
Step 8. Obtain the concordance, midrange concordance, weak concordance, discordance, midrange discordance and weak discordance set according to Eqs.(20)-(25), respectively;
Step 9. Compute the concordance matrix, discordance matrix, concordance dominance matrix, discordance dominance matrix and aggregate dominance matrix in terms of Eqs.(26)-(32);
Step 10. Obtain the ranking order of all alternatives and the best alternative according to Eqs.(33)-(34).

# 6 Numerical example

In this section, we use the air-condition system selection problem given by [27] to verify the feasibility of the proposed method. The problem is described as follows:

Suppose there exist three air-condition systems $\{A_1, A_2, A_3\}$, four attributes $a_1$ (economical), $a_2$(function), $a_3$(being operative) and $a_4$(longevity) are taken into consideration in the selection problem. Three experts (DMs) $\{p_1, p_2, p_3\}$ participate in the decision making. The membership degrees and non-membership degrees for the alternative $A_i$ on the attribute $a_j$ given by expert $p_t$ were listed in Tables 2 − 4.

Table 2: IVIFNs given by the expert $p_1$.

| Attribute | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $a_1$ | ([0.4, 0.8], [0.0, 0.1]) | ([0.5, 0.7],[0.1, 0.2]) | ([0.5, 0.7],[0.2, 0.3]) |
| $a_2$ | ([0.3, 0.6], [0.0, 0.2]) | ([0.3, 0.5],[0.2, 0.4]) | ([0.6, 0.8],[0.1, 0.2]) |
| $a_3$ | ([0.2, 0.7], [0.2, 0.3]) | ([0.4, 0.7],[0.0, 0.2]) | ([0.4, 0.7],[0.1, 0.2]) |
| $a_4$ | ([0.3, 0.4], [0.4, 0.5]) | ([0.1, 0.2],[0.7, 0.8]) | ([0.6, 0.8],[0.0, 0.2]) |

Table 3: IVIFNs given by the expert $p_2$.

| Attribute | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $a_1$ | ([0.5, 0.9], [0.0, 0.1]) | ([0.7, 0.8], [0.1, 0.2]) | ([0.5, 0.6], [0.1, 0.4]) |
| $a_2$ | ([0.4, 0.5], [0.3, 0.5]) | ([0.5, 0.6], [0.2, 0.3]) | ([0.6, 0.7], [0.1, 0.2]) |
| $a_3$ | ([0.5, 0.8], [0.0, 0.1]) | ([0.5, 0.8], [0.0, 0.2]) | ([0.4, 0.8], [0.1, 0.2]) |
| $a_4$ | ([0.4, 0.7], [0.1, 0.2]) | ([0.5, 0.6], [0.3, 0.4]) | ([0.2, 0.6], [0.2, 0.3]) |

Table 4: IVIFNs given by the expert $p_3$.

| Attribute | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $a_1$ | ([0.3, 0.9], [0.0, 0.1]) | ([0.3, 0.8], [0.1, 0.2]) | ([0.2, 0.6], [0.1, 0.2]) |
| $a_2$ | ([0.2, 0.5], [0.1, 0.4]) | ([0.5, 0.6], [0.1, 0.3]) | ([0.2, 0.6], [0.2, 0.3]) |
| $a_3$ | ([0.4, 0.7], [0.1, 0.2]) | ([0.2, 0.8], [0.0, 0.2]) | ([0.3, 0.6], [0.1, 0.3]) |
| $a_4$ | ([0.3, 0.6], [0.3, 0.4]) | ([0.3, 0.5], [0.2, 0.3]) | ([0.4, 0.7], [0.1, 0.2]) |

In the following, we will illustrate the decision making process.
(1) Calculation of weights of attributes

In order to find the weights of attributes, A commission, which is organized by sampling method, determined the importance of attribute by using AHP. A $4 \times 4$ size matrix is formed because 4 attribute are considered in this study. All the diagonal elements of the matrix will be 1, the elements of symmetrical position with respect to the diagonal are reciprocal, in other words, if $a_{ij}$ is $i$th row and $j$th column element of matrix, then its symmetrical position is filled using $a_{ji} = 1/a_{ij}$ formula.

The comparison matrix W is obtained as follows:

$$W = \begin{pmatrix} 1 & 2 & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & 1 & \frac{1}{3} & \frac{1}{6} \\ 3 & 3 & 1 & \frac{1}{3} \\ 4 & 6 & 3 & 1 \end{pmatrix}.$$

By computing the eigenvalues and the eigenvectors of $W$, we obtained that the maximum eigenvalue of $W$ was 4.0875, the corresponding eigenvector was $\omega = (0.1905, 0.1230, 0.4046, 0.8849)^T$, consistency index $CI$=0.0292 and consistency ratio $CR = 0.0324 < 0.1$.

Normalized eigenvectors, the four attributes weights are obtained as follows:
$\omega_1 = 0.1213$, $\omega_2 = 0.0765$, $\omega_3 = 0.2517$, $\omega_4 = 0.5505$.

(2) Calculate the individual overall attribute value of each alternative

By Eq.(14), the individual overall attribute value of each alternative can be obtained as in Table 5.

Table 5: The individual overall attribute values of the alternatives for weight vector of attributes.

| $E_i^t$ | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $p_1$ | ([0.2870,0.5393],[0.2705,0.3782]) | ([0.2393,0.4095],[0.4128,0.5456]) | ([0.5375,0.7627],[0.0571,0.2121]) |
| $p_2$ | ([0.4373,0.7341],[0.0780,0.1857]) | ([0.5242,0.6746],[0.1926,0.3178]) | ([0.3173,0.6580],[0.1551,0.2793]) |
| $p_3$ | ([0.3175,0.6539],[0.1980,0.3133]) | ([0.2901,0.6196],[0.1299,0.2627]) | ([0.3353,0.6551],[0.1077,0.2328]) |

(3) Calculation of the similarity matrix and the weight vector of DMs

The similarity matrix for the DMs is constructed by Eq.(10) as follows:

$$S = \begin{pmatrix} 1 & 0.5415 & 0.6059 \\ 0.5415 & 1 & 0.7577 \\ 0.6059 & 0.7577 & 1 \end{pmatrix}.$$

Because the maximum eigenvalue of $S$ is 2.2746, the corresponding eigenvector is $x = (0.5373, 0.5878, 0.6048)^T$, the expert′s weights are obtained from Eq.(16) as follows: $z_1 = 0.3106$, $z_2 = 0.3398$, $z_3 = 0.3496$.

(4) Integrate the attribute value of different DMs

By Eq.(14), the attribute value of different DMs are respectively integrated as in Table 6.

Table 6: The attribute value of different DMs in the different alternatives and different attributes.

| | $A_1$ | $A_2$ | $A_3$ |
|---|---|---|---|
| $a_1$ | ([0.3990,0.8689],[0,0.1]) | ([0.4980,0.7689],[0.1,0.2]) | ([0.3951,0.6311],[0.1311,0.2990]) |
| $a_2$ | ([0.2990,0.5311],[0.1369,0.3719]) | ([0.4379,0.5689],[0.1650,0.3311]) | ([0.4602,0.6961],[0.1350,0.2350]) |
| $a_3$ | ([0.3719,0.7340],[0.0971,0.1971]) | ([0.3641,0.7689],[0,0.2]) | ([0.3650,0.6990],[0.1,0.2350]) |
| $a_4$ | ([0.3340,0.5719],[0.2631,0.3631]) | ([0.3058,0.4408],[0.3893,0.4893]) | ([0.3942,0.6971],[0.1029,0.2340]) |

(5) Calculate the ranking vector

The ranking vector of the membership degree interval, the non-membership degree interval and the hesitancy degree interval of between the difference alternatives on each attribute is calculated by Eqs.(1-7), respectively, as in Table 7.

Table 7: The attribute value of different DMs in the different alternatives and different attributes.

| | membership degree interval | | non-membership degree interval | | hesitancy degree interval | |
|---|---|---|---|---|---|---|
| $a_1$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ |
| | 0.5006 | 0.4994 | 0.25 | 0.75 | 0.5873 | 0.4127 |
| | $A_1$ | $A_3$ | $A_1$ | $A_3$ | $A_1$ | $A_3$ |
| | 0.6286 | 0.3714 | 0.25 | 0.75 | 0.5388 | 0.4612 |
| | $A_2$ | $A_3$ | $A_2$ | $A_3$ | $A_2$ | $A_3$ |
| | 0.6808 | 0.3192 | 0.3207 | 0.6793 | 0.4341 | 0.5659 |
| $a_2$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ |
| | 0.3214 | 0.6786 | 0.5135 | 0.4865 | 0.5878 | 0.4122 |
| | $A_1$ | $A_3$ | $A_1$ | $A_3$ | $A_1$ | $A_3$ |
| | 0.27295 | 0.72705 | 0.6477 | 0.3523 | 0.59905 | 0.40095 |
| | $A_2$ | $A_3$ | $A_2$ | $A_3$ | $A_2$ | $A_3$ |
| | 0.34565 | 0.65435 | 0.6764 | 0.3236 | 0.5173 | 0.4827 |
| $a_3$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ |
| | 0.48325 | 0.51675 | 0.6177 | 0.3823 | 0.4723 | 0.5277 |
| | $A_1$ | $A_3$ | $A_1$ | $A_3$ | $A_1$ | $A_3$ |
| | 0.52875 | 0.47125 | 0.4246 | 0.5754 | 0.4995 | 0.5005 |
| | $A_2$ | $A_3$ | $A_2$ | $A_3$ | $A_2$ | $A_3$ |
| | 0.54255 | 0.45745 | 0.3426 | 0.6574 | 0.5273 | 0.4727 |
| $a_4$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ | $A_1$ | $A_2$ |
| | 0.66115 | 0.33885 | 0.25 | 0.75 | 0.56895 | 0.43105 |
| | $A_1$ | $A_3$ | $A_1$ | $A_3$ | $A_1$ | $A_3$ |
| | 0.35955 | 0.64045 | 0.75 | 0.25 | 0.44015 | 0.55985 |
| | $A_2$ | $A_3$ | $A_2$ | $A_3$ | $A_2$ | $A_3$ |
| | 0.2633 | 0.7367 | 0.75 | 0.25 | 0.365 | 0.635 |

(6) Determine the concordance, midrange concordance, weak concordance, discordance, midrange discordance and weak discordance set

Applying Eqs.(20-25) and Table 7, the concordance, midrange concordance, weak concordance, discordance, midrange discordance and weak discordance set is calculated, respectively, as follows:

$$C = \begin{pmatrix} - & - & 3 \\ 2 & - & 1 \\ 2 & 2 & - \end{pmatrix}, \quad C' = \begin{pmatrix} - & 1,4 & 1 \\ 3 & - & 3 \\ 4 & 4 & - \end{pmatrix}, \quad C'' = \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \end{pmatrix},$$

$$D = \begin{pmatrix} - & 2 & 2 \\ - & - & 2 \\ 3 & 1 & - \end{pmatrix}, \quad D' = \begin{pmatrix} - & 3 & 4 \\ 1,4 & - & 4 \\ 1 & 3 & - \end{pmatrix}, \quad D'' = \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \end{pmatrix}.$$

For example, $c_{13} = \{3\}$, which is in the 1st (horizontal) row and the 3rd (vertical) column of the concordance set, is "3." $c_{12} = \{-\}$, which is in the 1st row and 2nd column of the concordance set, is "empty," and so forth.

(7) Compute the concordance matrix, discordance matrix, concordance dominance matrix, discordance dominance matrix and aggregate dominance matrix

We give the relative weights as: $[\omega_C, \omega_{C'}, \omega_{C''}, \omega_D, \omega_{D'}, \omega_{D''}] = [1, \frac{2}{3}, \frac{1}{3}, 1, \frac{2}{3}, \frac{1}{3}]$. By Eqs.(26)-(32), the concordance matrix, discordance matrix, concordance dominance matrix, discordance dominance matrix and aggregate dominance matrix are obtained, respectively, as follows:

$$E = \begin{pmatrix} - & 0.08575 & 0.02235 \\ 0.04759 & - & 0.05697 \\ 0.09643 & 0.07862 & - \end{pmatrix}, \quad H = \begin{pmatrix} - & 0.1039 & 0.16309 \\ 0.09967 & - & 0.18088 \\ 0.12298 & 0.1204 & - \end{pmatrix},$$

$$K = \begin{pmatrix} - & 0.01068 & 0.07408 \\ 0.04884 & - & 0.03946 \\ 0 & 0.01781 & - \end{pmatrix}, \quad L = \begin{pmatrix} - & 0.07698 & 0.01779 \\ 0.08121 & - & 0 \\ 0.0579 & 0.06048 & - \end{pmatrix}.$$

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$$R = \begin{pmatrix} - & 0.8782 & 0.1936 \\ 0.6246 & - & 0 \\ 1 & 0.7725 & - \end{pmatrix}$$

(8) Compute the ranking order of all alternatives and obtain the best alternative
Applying Eq.(33),

$$\overline{T} = \begin{pmatrix} 0.5359 \\ 0.3123 \\ 0.88625 \end{pmatrix}$$

The optimal ranking order of alternatives is given by $A_3 > A_1 > A_2$. The best alternative is $A_3$. The ranking order given by [27] is identical. The best air-condition system is $A_3$.

This example shows the effectiveness of the ranking method proposed in this paper.

# 7 Conclusion

Regarding the MAGDM problem, the IVIF theory provides a useful and convenient way to reflect the ambiguous nature of subjective judgments and assessments. In this paper, firstly, using the normalized Hamming distance between IVIFS to construct similarity matrix and obtain the wights of DMs. Then, using possibility degree of IVIF to calculate the ranking vector. Based on this, the concordance and discordance sets, concordance and discordance matrices etc. are obtained. Finally, by computing the ranking order of all alternatives, decision makers can choose the best alternative, the example verify the correctness of the method.

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