A Note On Fuzzy Supra P-Spaces

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ABSTRACT
In this paper, several characterizations of fuzzy supra P-spaces are given. Also inter relation between fuzzy supra P-spaces and other fuzzy supra topological spaces are established.

Keywords: Fuzzy supra F_σ-set, Fuzzy supra G_δ-set, Fuzzy supra nowhere dense set, Fuzzy supra dense set, Fuzzy supra P-Space, Fuzzy supra globally disconnected space, Fuzzy supra hyperconnected space, Fuzzy supra basically disconnected Space.

1. INTRODUCTION

In the year 1965, L. A.Zadeh [13] was first introduced the concept of fuzzy sets and fuzzy set operations in his classical paper. In 1968, the theory of fuzzy topological spaces was introduced and developed by C.L.Chang [4].

In 1983, Mashhour.A.S.et.al., [6] introduced and studied the concept of Supra topological spaces. In 1987, Abd El-monsef et.at [1] introduced the concepts of fuzzy supra topological as a natural generalization of the notion of supra topological spaces.

In 1972, Mishra.A.K [7] introduced the concepts of P-spaces as a generalization of ω_α-additive spaces of Sikorski [11] and Cohen.L.W and Goffman.C [5]. The concept of P-spaces in fuzzy setting was introduced by Thangaraj.G and Balasubramanian.G [12].

In this paper several characterizations of fuzzy supra P-spaces are obtained. Inter relations between fuzzy supra P-spaces and other fuzzy supra topological spaces are given. It is establish that a condition for a fuzzy supra strongly irresolvable and fuzzy supra globally disconnected space to become a fuzzy supra P-space and a condition for a fuzzy supra globally disconnected and fuzzy supra GID-space to become a fuzzy supra P-space. Also shows that fuzzy supra residual sets are fuzzy supra open sets and fuzzy supra first category sets are fuzzy supra closed sets in fuzzy supra globally disconnected and fuzzy supra P-spaces.

2. Preliminaries

Definition 2.1 [1].

A collection T^* of fuzzy sets in a set U is called fuzzy supra topology on U if the following conditions are satisfied:

1) **0** and **1** belongs to T^*.
2) $g_\chi \in T^*$ for each $\chi \in \Lambda$ implies $(\vee_{\chi \in \Lambda} g_\chi) \in T^*$.

The pair (U,T^*) is called a fuzzy supra topological space. The elements of T^* are called fuzzy supra open sets and the complement of a fuzzy supra open set is called fuzzy supra closed set.

Definition: 2.2 [9]

Let (U,T^*) be a fuzzy supra topological space and μ be a fuzzy set in U, then the fuzzy supra closure and fuzzy supra interior of μ defined respectively as

$cl^*(\mu) = \wedge \{ g / g$ is a fuzzy supra closed set in U and $\mu \le g\}$
$int^*(\mu) = \vee \{ g / g$ is a fuzzy supra open set in U and $g \le \mu\}$

Definition: 2.3 [9]
Let (U,T) be a fuzzy topological space and T^* be a fuzzy supra topology on U. We call T^* a fuzzy supra topology associated with T if $T \le T^*$.
Remark: 2.4 [1]
(1). Every fuzzy topological space is a fuzzy supra topological space.
(2). If (U,T*) is an associated fuzzy supra topological space with the fuzzy topological space (U, T), then every fuzzy open(closed) set in the fuzzy topological space (U,T) is fuzzy supra open(closed) set in the fuzzy supra topological space (U,T*).
Lemma 2.5 [4]
For a fuzzy set μ in a fuzzy topological space U,
(i) 1 – int(μ) = cl(1– μ),
(ii) 1 – cl(μ) = int(1 – μ).
Definition 2.6 [8]
A fuzzy supra open set μ in fuzzy supra topological space (U,T^*) is called fuzzy supra F_σ-set in (U,T^*) if $\mu = \vee_{i=1}^{\infty} (\mu_i)$, where $1 - \mu_i \in T^*$ for $i \in I$,
Definition 2.7 [8]
A fuzzy supra open set μ in fuzzy supra topological space (U,T^*) is called fuzzy supra G_δ-set in (U,T^*) if $\mu = \wedge_{i=1}^{\infty}(\mu_i)$, where $\mu_i \in T^*$ for $i \in I$.
Definition 2.8 [8]
A fuzzy set μ in a fuzzy supra topological space (U,T^*) is called a fuzzy supra dense set if there exists no fuzzy supra closed set β in (U,T^*) such that $\mu<\beta<1$. That is, $cl^*(\mu)=1$, in (U,T^*).
Definition 2.9 [3]
A fuzzy set μ in fuzzy supra topological space (U,T^*) is called a fuzzy supra nowhere dense set if there exists no non-zero fuzzy supra open set δ in (U,T^*) such that $\delta < cl^*(\mu)$. That is, $int^*cl^*(\mu)=0$, in (U,T^*).
Definition 2.10 [3]
A fuzzy set μ in a fuzzy supra topological space (U,T^*) is called a fuzzy supra first category set if $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are fuzzy supra nowhere dense set in (U,T^*). Any other fuzzy set in (U,T^*) is said to be fuzzy supra second category space.
Definition 2.11 [2]
A fuzzy set μ in a fuzzy supra topological space (U,T*) is called a fuzzy supra boundary of μ is defined as $Bd(\mu) = cl^*(\mu) \wedge cl^*(1-\mu)$. Obviously Bd(μ) is a fuzzy supra closed set in (U,T*).
Definition 2.12 [3]
A fuzzy set μ in a fuzzy supra topological space (U,T*) is called a fuzzy supra simply-open set if Bd(μ) is a fuzzy supra no-where dense set in (U,T*). That is, μ is an fuzzy supra simply-open set in (U,T*) if $int^*cl^*[cl(\mu) \wedge cl^*(1- \mu)] = 0$ in (U,T*).
Definition 2.13 [2]
A fuzzy set μ in a fuzzy supra topological space (U,T*) is called a fuzzy supra semi-open set in (U,T*) if $\mu \le cl^*int^*(\mu)$; fuzzy supra semi-closed set in (U,T*) if $int^*cl^*(\mu) \le \mu$.
Definition 2.14 [3]
A fuzzy set μ in a fuzzy supra topological space (U,T*) is called a fuzzy supra somewhere dense set if there exists a non-zero fuzzy open set μ in (U,T*) such that $\mu < cl(\mu)$. That is, $int^*cl^*(\mu) \neq 0$, in (U,T*)

Theorem 2.1 [3]
If $cl^*(\mu) = 1$ for a fuzzy set μ defined on U in a fuzzy strongly irresolvable space (U,T*), then $cl^*int^*(\mu) = 1$ in (U,T*).

Theorem 2.2 [3]
Let (U,T*) be a fuzzy topological space. Then, the following are equivalent:
(1) (U,T*) is a fuzzy GID-space.
(2) Each fuzzy dense and fuzzy G_δ-set in (U,T*) is fuzzy semi-open in (U,T*)

Theorem 2.3 [3]
If (U,T*) is a fuzzy perfectly disconnected space, then (U,T*) is a fuzzy extremally disconnected space.

Theorem 2.4 [3]
If μ is a fuzzy residual set in a fuzzy globally disconnected space (U,T*), then μ is a fuzzy G_δ-set in (U,T*).

Theorem 2.5 [3]
Let (U,T*) be a fuzzy supra topological space. Then the following are equivalent:
(i). (U,T*) is an fuzzy supra Baire space.
(ii). int*(λ) = 0, for every fuzzy supra first category set λ in (U,T*)
(iii). cl*(μ) = 1, for every fuzzy supra residual set μ in (U,T*).

Theorem 2.6 [3]
If cl*(μ) is a fuzzy supra open set for a fuzzy supra set μ defined on U in a fuzzy supra strongly irresolvable space (U,T*), then μ is a fuzzy supra semi-open set in (U,T*).

3. Fuzzy Supra P-Spaces

Definition 3.1 [10]
A fuzzy supra topological space (U,T*) is called a fuzzy supra P-space if each fuzzy supra G_δ-set in (U,T*) is fuzzy supra open in (U,T*).

Proposition 3.2
If (μ_i)'s (i=1 to ∞) are fuzzy supra sets defined on U in a fuzzy supra P –space (U,T*), then $\vee_{i=1}^{\infty}[Bd(\mu_i)]$ is a fuzzy supra closed set in (U,T*).
Proof.
Let (μ_i)'s (i=1 to ∞) be fuzzy supra sets defined in (U,T*). Then, $Bd(\mu_i) = cl^*(\mu_i) \wedge cl^*(1-\mu_i)$ and $Bd(\mu_i)$ is a fuzzy supra closed set in (U,T*). Now $\vee_{i=1}^{\infty}[Bd(\mu_i)]$ is a fuzzy supra F_σ-set in (U,T*). Since (U,T*) is a fuzzy supra P-space, $\vee_{i=1}^{\infty}[Bd(\mu_i)]$ is a fuzzy supra closed set in (U,T*).

Proposition 3.3
If $int^*(\vee_{i=1}^{\infty}[Bd(\mu_i)]) = 0$, where (μ_i)'s (i = 1 to ∞) are fuzzy supra sets defined on U in a fuzzy supra P-space (U,T*), then (μ_i)'s are fuzzy supra simply open sets in (U,T*).
Proof.
Let (μ_i)'s (i=1 to ∞) be fuzzy supra sets defined in (U,T*). Since (U,T*) is a fuzzy supra P-space, by Proposition 3.2, $\vee_{i=1}^{\infty}[Bd(\mu_i)]$ is a fuzzy supra closed set in (U,T*).
Then, $cl^*(\vee_{i=1}^{\infty}[Bd(\mu_i)]) = \vee_{i=1}^{\infty}[Bd(\mu_i)]$ in (U,T*) and this implies that $int^*cl^*(\vee_{i=1}^{\infty}[Bd(\mu_i)]) = int^*(\vee_{i=1}^{\infty}[Bd(\mu_i)])$.
By hypothesis, $int^*(\vee_{i=1}^{\infty}[Bd(\mu_i)]) = 0$ and then $int^*cl^*(\vee_{i=1}^{\infty}[Bd(\mu_i)]) = 0$. Now $\vee_{i=1}^{\infty} int^*cl^*[Bd(\mu_i)] \leq int^*cl^*(\vee_{i=1}^{\infty}[Bd(\mu_i)])$, implies that $\vee_{i=1}^{\infty} int^*cl^*[Bd(\mu_i)] \leq 0$. That is, $\vee_{i=1}^{\infty} int^*cl^*[Bd(\mu_i)] = 0$ and this implies that $int^*cl^*[Bd(\mu_i)]=0$, in (U,T*). Hence (μ_i)'s are fuzzy supra simply open sets in (U,T*).

Proposition 3.4
If μ and α are fuzzy supra G_δ-sets in a fuzzy supra P-space (U,T*) such that $\mu \wedge \alpha = 0$, then (i). $cl^*(\mu) \neq 1$ and $cl^*(\alpha) \neq 1$, in (U,T*).
(ii). $1-\mu$ and $1-\alpha$ are fuzzy supra somewhere dense sets in (U,T*) and $(1-\mu) \vee (1-\alpha) = 1$ in (U,T*).
Proof.
(i). Let μ and α be fuzzy supra G_δ-sets in (U,T*) such that $\mu \wedge \alpha = 0$. Since (U,T*) is a fuzzy supra P-space, μ and α are fuzzy supra open sets in (U,T*). Now $\mu \wedge \alpha = 0$, implies that $\mu \leq 1-\alpha$ and then $int^*(\mu) \leq int^*(1-\alpha)$. By Lemma 2.5, $int^*(1-\alpha) = 1 - cl^*(\alpha)$, in (U,T*). Then, $\mu \leq 1 - cl^*(\alpha)$ and $cl^*(\alpha) \leq 1-\mu$. Since $1-\mu$ is a fuzzy supra closed set in (U,T*), $cl^*(\alpha) \neq 1$, in (U,T*). Also $\mu \leq 1-\alpha$, implies that $cl^*(\mu) \leq cl^*(1-\alpha)$ and $cl^*(\mu) \leq 1 - int^*(\alpha) = 1-\alpha$, in (U,T*). Since $1-\alpha$ is a fuzzy supra closed set in (U,T*), $cl^*(\mu) \neq 1$ in (U,T*).
(ii). By (i), $cl^*(\mu) \neq 1$ and then $1 - cl^*(\mu) \neq 0$. This implies that $int^*(1-\mu) \neq 0$. Since $int^*(1-\mu) \leq int^*cl^*(1-\mu)$, $int^*cl^*(1-\mu) \neq 0$ and thus $1-\mu$ is a fuzzy supra somewhere dense set in (U,T*). Now $cl^*(\alpha) \neq 1$ and then $1 - cl^*(\alpha) \neq 0$. This implies, by Lemma 2.5, that $int^*(1-\alpha) \neq 0$. Since $int^*(1-\alpha) \leq int^*cl^*(1-\alpha)$, $int^*cl^*(1-\alpha) \neq 0$ and thus $1-\alpha$ is a fuzzy supra somewhere dense set in (U,T*). Hence $1-\mu$ and $1-\alpha$ are fuzzy supra somewhere dense sets in (U,T*). Now $(1-\mu) \vee (1-\alpha) = 1 - (\mu \wedge \alpha) = 1 - 0 = 1$. Hence $(1-\mu) \vee (1-\alpha) = 1$, where $1-\mu$ and $1-\alpha$ are fuzzy supra somewhere dense sets in (U,T*).

Proposition 3.5
If μ and α are fuzzy supra G_δ-sets in a fuzzy supra P -space (U,T*), such that μ∧α = 0, then μ and α are fuzzy supra cs dense sets in (U,T*).
Proof.
Let μ and α be fuzzy supra G_δ-sets in (U,T*) such that μ ∧ α = 0. Since (U,T*) is a fuzzy supra P -space, by Proposition 3.4(ii), 1 − μ and 1 − α are fuzzy supra somewhere dense sets in (U,T*) and then μ and α are fuzzy supra cs dense sets in (U,T*).

4. Fuzzy Supra P-Spaces and Other Fuzzy supra Topological Spaces

Definition 4.1
A fuzzy supra topological space (U,T*) is called a fuzzy supra basically disconnected space, if the closure of every fuzzy supra open F_σ-set of (U,T*) is fuzzy supra open in (U,T*).
Proposition 4.2.
If a fuzzy supra topological space (U,T*) is a fuzzy supra P-space, then (X,T) is a fuzzy supra basically disconnected space.
Proof.
Let μ be a fuzzy supra open F_σ-set in (U,T*). Then, 1 − μ is a fuzzy supra closed G_δ-set in (U,T*). Since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ-set 1−μ is a fuzzy supra open set and thus int*(1 − μ) = 1 − μ, in (U,T*). By Lemma 2.5, int*(1 − μ) = 1− cl*(μ) and then1 − cl*(μ) = 1 − μ. This implies that cl*(μ) = μ and μϵT*, implies that cl*(μ) is fuzzy supra open in (U,T*). Hence (U,T*) is a fuzzy supra basically disconnected space.
Definition 4.3
A fuzzy supra topological space (U,T*) is called fuzzy supra strongly irresolvable space, if for every fuzzy supra dense set μ in (U,T*), cl*int*(μ)= 1. That is., cl*(μ) = 1 implies that cl*int*(μ)= 1 in (U,T*).
Definition 4.4
A fuzzy supra topological space (U,T*) is called fuzzy supra globally disconnected space, if each fuzzy supra semi-open set in (U,T*) is a fuzzy supra open set in (U,T*).
The following proposition gives a condition for a fuzzy supra strongly irresolvable and fuzzy supra globally disconnected space to become a fuzzy supra P-space.

Proposition 4.5
If each fuzzy supra G_δ-set is a fuzzy supra dense set in a fuzzy supra strongly irresolvableand fuzzy supra globally disconnected space (U,T*), then (U,T*) is a fuzzy supra P -space.
Proof.
Let μ be a fuzzy supra G_δ-set in (U,T*). By hypothesis, μ is a fuzzy supra dense set in (U,T*). That is.. cl*(μ) = 1. Since (U,T*) is a fuzzy supra strongly irresolvable space, by Theorem 2.1, cl*(μ) = 1 implies that cl*int*(μ) = 1, in (U,T*). Then, μ ≤ cl*int*(μ) and thus μ is a fuzzy supra semi-open set in (U,T*). Since (U,T*) is a fuzzy supra globally disconnected space, the fuzzy supra semi-open set μ is fuzzy supra open in (U,T*). Thus, the fuzzy supra G_δ-set is fuzzy supra open in (U,T*), implies that (U,T*) is a fuzzy supra P -space.
Definition 4.6
A fuzzy supra topological space (U,T*) is called fuzzy supra GID space, if for each fuzzy supra dense and fuzzy supra G_δ-set μ in (U,T*), cl*int*(μ) = 1, in (U,T*).
The following proposition gives a condition for a fuzzy supra globally disconnected and fuzzy supra GID-space to become a fuzzy supra P-space.

Proposition 4.7
If each fuzzy supra G_δ-set is a fuzzy supra dense set in a fuzzy supra globally disconnected and fuzzy supra GID-space (U,T*), then (U,T*) is a fuzzy supra P -space.
Proof.
Let μ be a fuzzy supra G_δ-set in (U,T*). By hypothesis, μ is a fuzzy supra dense set in (U,T*). Thus, μ is a fuzzy supra dense and fuzzy supra G_δ-set in (U,T*). Since (U,T*) is the fuzzy supra GID-space, by Theorem 2.2, μ is a fuzzy supra semi-open set in (U,T*). Also since (U,T*) is a fuzzy supra globally disconnected space, the fuzzy supra semi-open set μ is fuzzy supra open in (U,T*). Thus, the fuzzy supra G_δ-set μ is fuzzy supra open in (U,T*), implies that (U,T*) is a fuzzy supra P -space.

Definition 4.8
A fuzzy supra topological space (U,T*) is called a fuzzy supra F-space, if for any two fuzzy supra F_σ-sets η and μ in U with η ≤ 1 − μ, there exists fuzzy G_δ-sets α and β in U such that η ≤ α, μ ≤ β and α ≤ 1 − β.

Proposition 4.9.

If the fuzzy supra topological space (U,T*) is a fuzzy supra F-space and fuzzy supra P-space and α and β are fuzzy supra closed sets such that $\alpha \wedge \beta = 0$, in (U,T*), then there exist fuzzy supra open sets γ and η in (U,T*) such that $\alpha \leq \gamma$ and $\beta \leq \eta$ and $\gamma \leq 1-\eta$.

Proof.

Suppose that α and β are fuzzy supra closed sets such that $\alpha \wedge \beta = 0$, in (U,T*). Now $\alpha \wedge \beta = 0$, implies that $\alpha \leq 1-\beta$ in (U,T*). Consider the fuzzy supra sets μ and α defined as follows: $\mu = \alpha \vee (\vee_{i=1}^{\infty}(\theta_i))$, $\alpha = \beta \vee (\vee_{i=1}^{\infty}(\theta_i)$, where $(\theta_i))$'s are fuzzy supra closed sets in (U,T*). Then, μ and α are fuzzy supra F_σ-sets in (U,T*) and $\alpha \leq \mu$ and $\beta \leq \alpha$. This implies that $\alpha \leq \mu$ and $1-\alpha \leq 1-\beta$. If $\mu \leq 1-\alpha$, in the fuzzy supra F -space (U,T*), then there exist fuzzy supra G_δ-sets γ and η in (U,T*) such that $\mu \leq \gamma$, $\alpha \leq \eta$ and $\gamma \leq 1-\eta$. Since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ-sets γ and η are fuzzy supra open sets in (U,T*). Then, $\alpha \leq \mu \leq \gamma$ and $\beta \leq \alpha \leq \eta$ and $\gamma \leq 1-\eta$. Thus, for the fuzzy supra closed sets α and β in (U,T*) such that $\alpha \wedge \beta = 0$, there exist fuzzy supra open sets γ and η in (U,T*) such that $\alpha \leq \gamma$ and $\beta \leq \eta$ and $\gamma \leq 1-\eta$.

Definition 4.10 [3]

A fuzzy supra topological space (U,T*) is called fuzzy supra extremally disconnected space, iff closure of every fuzzy supra open set is fuzzy supra open set in (U,T*). That is., cl*(μ)= μ where $\mu \in T^*$.

Proposition 4.11

If μ is a fuzzy supra G_δ-set in a fuzzy supra extremally disconnected and fuzzy supra P -space (U,T*), then cl*(μ) is a fuzzy supra open set in (U,T*).

Proof.

Let μ be a fuzzy supra G_δ-set in (U,T*). Since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ-set μ is a fuzzy supra open set in (U,T*). Also since (U,T*) is a fuzzy supra extremally disconnected space, cl*(μ) is a fuzzy supra open set in (U,T*).

Definition 4.12 [3]

A fuzzy supra topological space (U,T*) is called fuzzy supra perfectly disconnected space if for any two non-zero fuzzy supra sets λ and μ defined on U with $\lambda \leq 1-\mu$, cl*(λ)$\leq$ 1- cl*(μ), in (U,T*).

Proposition 4.13

If μ is a fuzzy supra G_δ-set in a fuzzy supra perfectly disconnected and fuzzy supra P -space (U,T*), then cl*(μ) is a fuzzy supra open set in (U,T*).

Proof.

Let μ be a fuzzy supra G_δ-set in (U,T*). Since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ- set μ is a fuzzy supra open set in (U,T*). Also since (U,T*) is a fuzzy supra perfectly disconnected space, by Theorem 2.3, (U,T*) is a fuzzy supra extremally disconnected space and thus (U,T*) is a fuzzy supra extremally disconnected and fuzzy supra P -space. Then, by Proposition 4.11, cl*(μ) is a fuzzy supra open set in (U,T*).

The following propositions show that fuzzy supra residual sets are fuzzy supra open sets and fuzzy suprafirst category sets are fuzzy supra closed sets in fuzzy supra globally disconnected and fuzzy supra P-spaces.

Proposition 4.14

If μ is a fuzzy supra residual set in a fuzzy supra globally disconnected and fuzzy supra P -space (U,T*), then μ is a fuzzy supra open set in (U,T*).

Proof.

Let μ be a fuzzy supra residual set in (U,T*). Since (U,T*) is a fuzzy supra globally disconnected space, by Theorem 2.4, μ is a fuzzy supra G_δ-set in (U,T*). Also since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ-set μ is a fuzzy supra open set in (U,T*). Thus, the fuzzy supra residual set in a fuzzy supra globally disconnected and fuzzy supra P-space (U,T*) is a fuzzy supra open set in(U,T*).

Proposition 4.15

If μ is a fuzzy supra first category set in a fuzzy supra globally disconnected andfuzzy supra P -space (U,T*), then μ is a fuzzy supra closed set in (U,T*).

Proof.

Let μ be a fuzzy supra first category set in (U,T*). Then, $1-\mu$ is a fuzzy supra residual set in (U,T*). Since (U,T*) is a fuzzy supra globally disconnected and fuzzy supra P-space, by Proposition 4.14, $1-\mu$ is a fuzzy supra open set in (U,T*) and hence μ is a fuzzy supra closed set in (U,T*).

Definition 4.16 [3]

A fuzzy supra topological space (U,T*) is called fuzzy supra Baire space, if $int^*[\vee_{i=1}^{\infty}(\mu_i)]=0$, where (μ_i)'s are fuzzy supra nowhere dense sets in (U,T*).

Proposition 4.17

If μ is a fuzzy supra first category set in a fuzzy supra globally disconnected, fuzzy supra Baire and fuzzy supra P -space (X,T), then μ is a fuzzy supra nowhere dense set in (U,T*).

Proof.

Let μ be a fuzzy supra first category set in (U,T*). Since (U,T*) is a fuzzy supra globally disconnected and fuzzy supra P -space, by Proposition 4.15, μ is a fuzzy supra closed set in (U,T*). Also since μ is a fuzzy supra first category set in a fuzzy supra Baire space (U,T*), by Theorem 2.5, int*(μ) = 0, in (U,T*). This implies that int*cl*(μ) = int*(μ) = 0, in (U,T*). Hence μ is a fuzzy supra nowhere dense set in (U,T*).

Definition 4.18

A fuzzy supra topological space (U,T*) is called fuzzy supra D-Baire space, if every fuzzy supra first category set in (U,T*) is a fuzzy supra nowhere dense set in (U,T*).

Proposition 4 . 1 9

If (U,T*) is a fuzzy supra globally disconnected, fuzzy supra Baire and fuzzy supra P -space, then (U,T*) is a fuzzy supra D-Baire space.

Proof.

Let μ be a fuzzy supra first category set in (U,T*). Since (U,T*) is a fuzzy supra globally disconnected, fuzzy supra Baire and fuzzy supra P -space, by Proposition 4.17, μ is a fuzzy supra nowheredense set in (U,T*) and hence (U,T*) is a fuzzy supra D-Baire space.

Remark 4.20

The converse of the above proposition need not be true. That is., a fuzzy supra D-Baire space need not be a fuzzy supra globally disconnected space.

Definition 4.21[3]

A fuzzy supra topological space (U,T*) is called fuzzy supra hyperconnected space if every non-null fuzzy supra open subset of (U,T*) is fuzzy supra dense set in (U,T*).

Proposition 4.22

If the fuzzy supra topological space (U,T*) is a fuzzy supra hyperconnected space, then (U,T*) is not a fuzzy supra D-Baire space.

Proof.

Let (μ_i)'s (i = 1 to ∞) be fuzzy supra open sets in (U,T*). Since (U,T*) is a fuzzy supra hyperconnected space, (μ_i)'s are fuzzy supra dense sets in (U,T*). Then, $cl^*(\mu_i) = 1$, in (U,T*). Now $int^*cl^*(1-\mu_i) = 1-cl^*int^*(\mu_i) = 1-cl^*(\mu_i) = 1-1 = 0$ and thus $(1-\mu_i)$'s are fuzzy supra nowhere dense sets in (U,T*).

Let $\alpha = \vee_{i=1}^{\infty}(1-\mu_i)$ and then α is a fuzzy supra first category set in (U,T*). Now $int^*cl^*(\alpha) = int^*cl^*(\vee_{i=1}^{\infty}(1-\mu_i)) = int^*cl^*(1-\wedge_{i=1}^{\infty}(\mu_i)) = 1-cl^*int^*(\wedge_{i=1}^{\infty}(\mu_i)) \geq 1-cl^*(\wedge_{i=1}^{\infty}(\mu_i)) \geq 1-cl^*(\mu_i) = 1-1 = 0$ and $int^*cl^*(\alpha) \geq 0$. Thus α is not a fuzzy supra nowhere dense set in (U,T*). Hence (U,T*) is not a fuzzy supra D-Baire space.

The following proposition shows that a fuzzy supra hyperconnected and fuzzy supra P-space is a fuzzy supra D-Baire space.

Proposition 4.23

If the fuzzy supra topological space (U,T*) is a fuzzy supra hyperconnected and fuzzy supra P -space, then (U,T*) is a fuzzy supra D-Baire space.

Proof.

Let μ be a fuzzy supra first category set in (U,T*). Then, $\mu = \vee_{i=1}^{\infty}(\mu_i)$, where (μ_i)'s are fuzzy supra nowhere dense sets in (U,T*). Now $1 - cl^*(\mu_i)$ is a fuzzy supra open set in (U,T*). Let $\alpha = \wedge_{i=1}^{\infty}[1-cl^*(\mu_i)]$ is a fuzzy supra G_δ-set in (U,T*). Now $\alpha = \wedge_{i=1}^{\infty}[1-cl^*(\mu_i)] = 1-[\vee_{i=1}^{\infty}cl^*(\mu_i)] \leq 1-\vee_{i=1}^{\infty}(\mu_i) = 1-\mu$. Thus $\alpha \leq 1-\mu$. Since (U,T*) is a fuzzy supra P -space, the fuzzy supra G_δ-set α is a fuzzy supra open set in (U,T*) and then $int^*cl^*(\mu) \leq int^*cl^*(1-\alpha) = 1-cl^*int^*(\alpha) = 1-cl^*(\alpha)$. Since

(U,T*) is a fuzzy supra hyperconnected space, for the fuzzy supra open set α, cl*(α) = 1 and then int*cl*(μ) ≤ 1 − 1 = 0. That is., int*cl*(μ) = 0. Thus, μ is a fuzzy supra nowhere dense set in (U,T*) and hence (U,T*) is a fuzzy supra D-Baire space.

Remark 4.24

The converse of the above proposition need not be true. That is, a fuzzy supra D-Baire space need not be a fuzzy supra hyperconnected space.

Proposition 4.25

If μ is a fuzzy supra G_δ-set in a fuzzy supra extremally disconnected, fuzzy supra strongly irresolvable and fuzzy supra P-space (U,T*), then

(i). μ is a fuzzy supra semi-open set in (U,T*).

(ii). cl*(μ) is a fuzzy supra regular closed set in (U,T*).

Proof.

(i). Let μ be a fuzzy supra G_δ-set in (U,T*). Since (U,T*) is a fuzzy supra extremally disconnected and fuzzy supra P -space, by Proposition 4.11, cl*(μ) is a fuzzy supra open set in (U,T*). Since (U,T*) is a fuzzy supra strongly irresolvable space, by Theorem 2.6, μ is a fuzzy supra semi-open set in (U,T*).

(ii). By (i), μ ≤ cl*int*(μ) in (U,T*). Then, cl*(μ) ≤ cl*[cl*int*(μ)] = cl* int*(μ) ≤ cl*int*cl*(μ). Also cl*[int*cl*(μ)] ≤ cl*[cl*(μ)] = cl*(μ). Thus, cl*[int* cl*(μ)] = cl*(μ) and hence cl*(μ) is a fuzzy supra regular closed set in (U,T*).

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CONFLICT OF INTEREST

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