Thermoconvective Instability in A Porous Maxwell Ferromagnetic Fluid

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ABSTRACT
The study considers a porous Maxwell ferromagnetic fluid layer which is assumed to be heated from below.The boundaries are infinite and free-free. AMaxwell model is implicated on this fluid layer under realistic boundary conditions and the Galerkin approach is followed to solve the concerned eigenvalue problemfor which the Rayleigh number is obtained in stationary and oscillatory convection. At stationary state, it is noticed that the Maxwell ferromagnetic fluid behaves similar to a Newtonian ferromagnetic fluid. The effects of concerned parameters are analysed and the results have been depicted both analytically and graphically. It has been noticed that the presence of porous medium delays the convection as compared to the continuous medium. A comparative study between stationary and oscillatory convection results that the oscillatory convection sets in earlier.

Keywords: Convection, Maxwell model, ferromagnetic fluid, porous medium.

1. INTRODUCTION

Ferrromagnetic fluids are well-known fluids and proves to be of great importance in today's techonological world due to their large applications as they are colloidal liquids composed of ferromagnetic nano substances. It was since 1960's when they came to existence andsoon several researchers have contributed to their development. It was Finlayson [1] who first worked on the convective instability problemin a ferroliquid placed within free boundaries by using heating and found the exact solution for the concerned problem. Past research studies shows that they play a crucial role in one's life as they are responsiblefor daily magnetism. Chandrasekhar [2] worked on convection problems in Newtonian type fluids and gave a descriptive account regarding his study. Rosensweig [3,4] worked specially on "ferrohydrodynamics" and wrote his monograph on it.

Following this, many researchers like,Lalas and Carmi [5], Shliomis [6], Sunil *etal.*[7-10] worked on ferromagnetic fluids and tried to obtain some more information on them. Further, Jasmine [11] also found the topic interesting and studied ferrofluids with the application of magnetic field. Prakash*et al.*[12]carried out the convection problem in ferrofluid layer by taking viscosity as magnetic field dependent. Siddheshwar*et al.* [13] discussed some concept regardingfinite-amplitude ferro-convection and electro-convection.Pundir*et al.* [14, 15] introduced the concept of couple stress in ferromagnetic fluids. A study on Rayleigh–Bénard convection was given by Meghana and Pranesh [16]for a ferromagnetic fluid layer by introducing couple-stress and discussed four types of rotation modulation.

The convection studies of non-Newtonian fluids within porous mediumdraws the attention of researchers since from the very beginning due to its importance in diverse areas of real life. Significantly,Vaidyanathan *et al.* [17]examined ferroliquids for theSoret-driven effect on ferro thermohaline convection.Sekar *et al.* [18] worked in an anisotropic porous medium and performed a study on thermohaline convection regarding ferrofluidwith Soret driven effect.

The very first viscoelastic model has been initiated by Maxwell and is still in focus. Narayana *et al.* [19] worked on porous media effectively and carried out a study on binary Maxwell fluid by analysing linear and nonlinear stability. Chand and Kumar [20] also worked on thermal rotatory Maxwell fluid with porous medium. Gaikwad and Kamble [21] put through a convection problem in porous layer of Maxwell fluid to examine the effects of cross diffusion on it. Mahajan *et al.* [22] has also done a significant work in area of porous media. He considered a linear approach of penetrative convection in ferroliquid via internal heating. Awasthi *et al.* [23] discussed a triply diffusive convective study in a Maxwell fluid in porous media taking concept of internal heat source.Pundir *et al.* [24] considered a study on thermally heated Maxwell ferromagnetic fluid layer within porous medium. Another study by Pundir *et al.* [25] in porous media considered a double-diffusive convection on a rotatory nanofluid including couple-stress. Recently, Pundir *et al.* [26] worked on a double-diffusive convection problem of a Darcy couple-stress nanofluid with magnetic field.

The present work accounts a problem of thermoconvective instabilityina porous ferromagnetic fluid using Maxwell model to perform some comparative studies which have not been considered yet. Therefore, an attempt has been made to fill this gap.This research work is an extension of the study given by Pundir *et al.* [24]. In this study, an incompressible Maxwell ferromagnetic fluid of depth d is allowed to be kept between two free boundaries. The medium is taken to be Darcy porous. Theobjective of this study is to check the effects of time relaxation parameter and magnetization parameterson the system that govern the concerned problem. Both the stationary convection and the oscillatory convection cases have been taken into consideration. The problem follows the normal mode approach and the Galerkin method to find out the stationary and oscillatory thermal Rayleigh number. Various comparative studies have also been performed to obseve the behavior of convection under different conditions.

2. Mathematical Formulation

An infinite, horizontalMaxwell ferromagnetic fluid layerhas been considered. The layer is incompressible and having depth equal to d .The layer is constricted within two free boundaries. The medium is taken to be porous. The corresponding layer is allowed to heat from downwards.The value of temperature corresponding to bottom and top surfaceis T_0 and T_1 respectively. A temperature gradient (uniform)is sustained (which is $\beta = \left|\frac{dT}{dz}\right|$). The gravity is given as $\vec{g} = (0,0,-g)$.

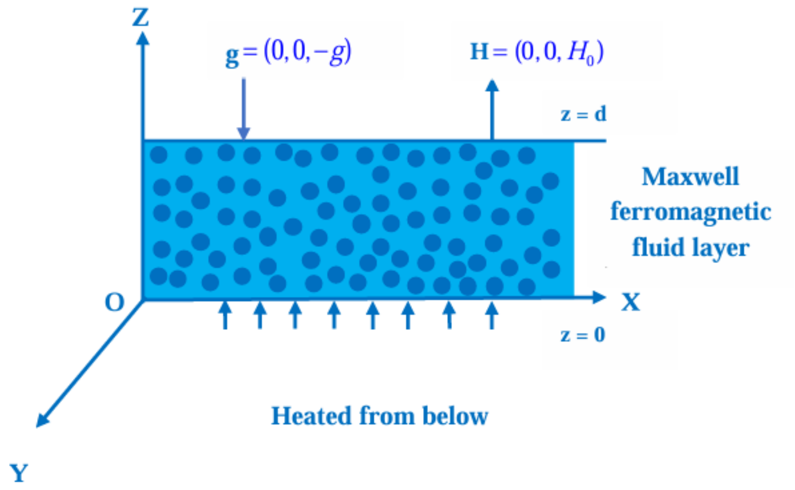

Figure 1: Physical Configuration

The equations governing the physical model are as follows:

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial \vec{q}}{\partial t} = \left(1+\lambda\frac{\partial}{\partial t}\right)\left(-\nabla p + \rho_0\left(1-\alpha\left(T_1-T_0\right)\right)\vec{g} + \nabla\cdot\left(\vec{H}\cdot\vec{B}\right)\right) - \frac{\mu}{k_1}\vec{q} \tag{2}$$

$$\varepsilon\left[\rho_0 C_{V,H} - \mu_0 \overline{H}\cdot\left(\frac{\partial \overline{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + (1-\varepsilon)\rho_s c_s\frac{\partial T}{\partial t} + \mu_0 T\left(\frac{\partial \overline{M}}{\partial T}\right)_{V,H}\cdot\frac{d\overline{H}}{dT} = K_1\nabla^2 T \tag{3}$$

where $\vec{q}$ is velocity, p is pressure, k_1 is permeability, ε is porosity, λ is relaxation time parameter, μ is viscosity, $\vec{H}$ is magnetic field, $\vec{g} = (0,0,-g)$ is gravity, μ_0 is magnetic permeability, T is temperature, $C_{V,H}$ is heat capacity at constant volume and magnetic field, α is the coefficient of volume expansion, $\overline{M}$ is magnetization, K_1 is thermal conductivity and ρ is the fluid density and ρ_0 is the fluid density at temperature T_0.

The Maxwell's equations are given by

$$\nabla \cdot \overrightarrow{B} = 0, \tag{4}$$
$$\nabla \times \overrightarrow{H} = 0, \tag{5}$$

where the magnetic induction is given by
$$\overrightarrow{B} = \mu_0 \left(\overrightarrow{H} + \overrightarrow{M} \right), \tag{6}$$
The magnetization is defined by the equation:
$$\overrightarrow{M} = \left(\frac{\overrightarrow{H}}{H} \right) M\left(H, T\right), \tag{7}$$
The linearized magnetic equation of state is given by
$$M = M_0 + \chi\left(H - H_0\right) - K_2\left(T - T_0\right), \tag{8}$$
where M_0 is the magnetization when magnetic field is H_0 and temperature T_0, $\chi = \left(\frac{\partial \overrightarrow{M}}{\partial \overrightarrow{H}} \right)_{H_0, T_0}$ is the magnetic susceptibility and $K_2 = -\left(\frac{\partial \overrightarrow{M}}{\partial T} \right)_{H_0, T_0}$ is the pyromagnetic coefficient.

3. Basic State
The solution of equations (1) to (7) in the basic state is given by
$$\vec{q} = \overrightarrow{q_b} = 0,\ \rho = \rho_b\left(z\right),\ \mathrm{p} = \mathrm{p}_b\left(z\right),\ T = T_b\left(z\right) = T_0 - \beta z,\ \beta = \frac{T_0 - T_1}{d},$$
$$\overrightarrow{H_b} = \left(H_0 - \frac{K_2 \beta z}{1+\chi} \right)\vec{k},\ \overrightarrow{M_b} = \left(M_0 + \frac{K_2 \beta z}{1+\chi} \right)\vec{k},\ H_0 + M_0 = H_0^{ext} \tag{9}$$

4. Perturbed State
Following Finlayson[1], the perturbations in the basic state are given by;
$$\vec{q} = \overrightarrow{q_b} + \overrightarrow{q'},\ \rho = \rho_b\left(z\right) + \rho',\ p = p_b\left(z\right) + p', T = T_b\left(z\right) + \theta',$$
$$\overrightarrow{H} = \overrightarrow{H_b}\left(z\right) + \overrightarrow{H'},\ \overrightarrow{M} = \overrightarrow{M_b}\left(z\right) + \overrightarrow{M'} \tag{10}$$

where $\overrightarrow{q'} = \left(u', v', w'\right),\ \rho',\ p',\ \theta',\ \overrightarrow{H'},\ \overrightarrow{M'}$ denotes velocity, density, pressure, temperature, magnetic field intensity, and magnetization in the perturbed state. Using (10), the perturbation equations in linear formbecome:
$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0, \tag{11}$$
$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial u'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(-\frac{\partial p'}{\partial x} + \mu_0\left(H_0 + M_0\right)\frac{\partial H_1'}{\partial z} \right) - \frac{\mu}{k_1}u' \tag{12}$$
$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial v'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(-\frac{\partial p'}{\partial y} + \mu_0\left(H_0 + M_0\right)\frac{\partial H_2'}{\partial z} \right) - \frac{\mu}{k_1}v' \tag{13}$$
$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial w'}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right)\left(\begin{array}{c} -\frac{\partial p'}{\partial z} + \rho_0 \alpha \theta' g + \mu_0\left(H_0 + M_0\right)\frac{\partial H_3'}{\partial z} \\ -\mu_0 K_2 \beta H_3' + \frac{\mu_0 K_2^2 \beta \theta'}{1+\chi} \end{array} \right) - \frac{\mu}{k_1}w' \tag{14}$$

$$\rho C_1 \frac{\partial \theta'}{\partial t} - \mu_0 T_0 K_2 \frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = K_1 \nabla^2 \theta' + \left(\rho C_2 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{1+\chi}\right) w' \tag{15}$$

where

$$\rho C_1 = \varepsilon \rho_0 C_{V,H} + \varepsilon \mu_0 K_2 H_0 + (1-\varepsilon)\rho_s C_s, \tag{16}$$

and

$$\rho C_2 = \varepsilon \rho_0 C_{V,H} + \varepsilon \mu_0 K_2 H_0, \tag{17}$$

$$\frac{\partial}{\partial x}\left(H_1' + M_1'\right) + \frac{\partial}{\partial y}\left(H_2' + M_2'\right) + \frac{\partial}{\partial z}\left(H_3' + M_3'\right) = 0,\ \overrightarrow{H'} = \nabla \phi', \tag{18}$$

where ϕ' is the perturbed magnetic potential, and

$$H_3' + M_3' = (1+\chi) H_3' - K_2 \theta',\ H_1' + M_1' = \left(1 + \frac{M_0}{H_0}\right) H_i',\ (i = 1, 2). \tag{19}$$

where we have assumed $K_2 \beta d << (1+\chi) H_{0.}$

Eliminating $u',\ v',\ p'$ between (12), (13) and (14) and using (11), we get

$$\frac{\rho_0}{\varepsilon}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}\nabla^2 w' = \left(1 + \lambda \frac{\partial}{\partial t}\right)\begin{pmatrix} \rho_0 \alpha g \nabla_1^2 \theta' - \mu_0 K_2 \beta \frac{\partial}{\partial z}\nabla_1^2 \phi' \\ + \frac{\mu_0 K_2^2 \beta \nabla_1^2 \theta'}{1+\chi} \end{pmatrix} - \frac{\mu}{k_1} w' \tag{20}$$

From (18) and (19) we obtain,

$$(1+\chi)\frac{\partial^2 \phi'}{\partial z^2} + \left(1 + \frac{M_0}{H_0}\right)\nabla_1^2 \phi' - K_2 \frac{\partial \theta'}{\partial z} = 0 \tag{21}$$

5. Normal Mode Method

Analysing the disturbances intonormal mode, the perturbed quantities take the form

$$(w',\ \theta',\ \varphi')(x, y, z, t) = \left[W(z),\ \Theta(z),\ \Phi(z)\right]\exp\left[i\left(k_x x + k_y y\right) + nt\right] \tag{22}$$

Using (22), in (20), (15) and (21) and non dimensionalising the variables by keeping

$$z^* = \frac{z}{d},\ W^* = \frac{d}{\nu}\mathrm{W},\ a = kd,\ \mathrm{t}^* = \frac{\nu t}{d^2},\ D^* = Dd,\ \theta^* = \frac{K_1 a R^{1/2}}{(\rho C_2)\beta \nu d}\Theta,$$

$$\phi^* = \frac{(1+\chi) K_1 a R^{1/2}}{K_2 (\rho C_2)\beta \nu d^2}\Phi,\ p_l = \frac{k_1}{d^2},\ \nu = \frac{\mu}{\rho_0},\ \mathrm{P}_r' = \frac{\nu(\rho C_2)}{K_1},\ \mathrm{P}_r = \frac{\nu(\rho C_1)}{K_1},$$

$$R = \frac{g\alpha\beta d^4 (\rho C_2)}{\nu K_1},\ M_1 = \frac{\mu_0 K_2^2 \beta}{(1+\chi)\alpha \rho_0 g},\ \mathrm{M}_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_2},$$

$$\mathrm{M}_3 = \frac{1 + \frac{M_0}{H_0}}{1+\chi},\ \omega = \frac{nd^2}{\nu},\ \lambda_0 = \frac{\lambda \nu}{d^2}, \tag{23}$$

where $M_1,\ M_2,\ M_3$ are magnetization parameters and λ_0 is time relaxation constant.

For simplification, removing the *,we obtain

$$\left[\frac{\omega}{\varepsilon} + \frac{1}{p_l}(1 + \lambda_0 \omega)^{-1}\right]\left(D^2 - a^2\right)W + aR^{1/2}(1 + M_1)\theta - aR^{1/2} M_1 D\phi = 0 \tag{24}$$

$$(1 - M_2) aR^{1/2} W + \left(D^2 - a^2 - P_r \omega\right)\theta + P_r' M_2 \omega D\phi = 0 \tag{25}$$

$$D\theta - \left(D^2 - a^2 M_3\right)\phi = 0 \tag{26}$$

6. Solution Methodology

Applying Galerkin method, the above system of equations (24) – (26) subject to the boundary conditions

$$W = 0 = \theta = D^2 w = D\phi \quad at \ z = 0 \ and \ z = 1 \tag{27}$$

is written in the form

$$W = W_0 \sin \pi z,\ \theta = \theta_0 \sin \pi z,\ \phi = -\frac{\phi_0}{\pi} \cos \pi z \tag{28}$$

where $W_0,\ \theta_0,\ \phi_0$ are constants.

Utilising (28) into (24) – (26), we obtain

$$-\left[\frac{\omega}{\varepsilon} + \frac{1}{p_l}\left(1+\lambda_0 \omega\right)^{-1}\right]\left(\pi^2 + a^2\right) W_0 + aR^{1/2}\left(1+M_1\right)\theta_0 - aR^{1/2} M_1 \phi_0 = 0 \tag{29}$$

$$\left(1-M_2\right) aR^{1/2} W_0 - \left(\pi^2 + a^2 + \mathrm{P}_r\, \omega\right)\theta_0 + \mathrm{P}_r{}'\, M_2 \omega \phi_0 = 0 \tag{30}$$

$$-\pi^2 \theta_0 + \left(\pi^2 + a^2 M_3\right)\phi_0 = 0 \tag{31}$$

For the non-trivial solutions of the above system, the determinant of the coefficients of $W_0,\ \theta_0,\ \phi_0$ in equations (29), (30) and (31) has to vanish.

$$\begin{pmatrix} -\left[\frac{\omega}{\varepsilon} + \frac{1}{p_l}\left(1+\lambda_0\omega\right)^{-1}\right]\left(\pi^2 + a^2\right) & aR^{1/2}(1+M_1) & -aR^{1/2} M_1 \\ (1-M_2) aR^{1/2} & -(\pi^2 + a^2 + \Pr\omega) & \Pr' M_2 \omega \\ 0 & -\pi^2 & (\pi^2 + a^2 M_3) \end{pmatrix} \begin{pmatrix} W_0 \\ \theta_0 \\ \phi_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{32}$$

7. Stationary Convection

For stationary convection, putting $\omega = 0$ and $M_2 \cong 0$ in (32), the stationary Rayleigh number becomes

$$R_{st.} = \frac{\left(\pi^2 + a^2\right)^2 \left(\pi^2 + a^2 M_3\right)}{p_l\, \mathrm{a}^2 [\pi^2 + a^2 (1+M_1) M_3]} \tag{33}$$

The expression (33) shows that $R_{st.}$ is independent of the viscoelastic parameter and is similar tothe Rayleigh Number of Newtonian ferromagnetic fluidsin porous medium.

In absence of M_3, eq. (33) reduces to

$$R_{st.} = \frac{\left(\pi^2 + a^2\right)^2}{p_l\, \mathrm{a}^2} \tag{34}$$

which is the same as the Rayleighnumber for ordinary fluids in porous medium.

To examine the role of permeability,buoyancy magnetization, non-buoyancy magnetization, we check the behavior of $\frac{dR_{st.}}{dp_l}, \frac{dR_{st.}}{dM_1}, \frac{dR_{st.}}{dM_3}$ analytically.

$$\frac{dR_{st.}}{dp_l} = -\frac{\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)}{p_l^2a^2\left[\pi^2+a^2\left(1+M_1\right)M_3\right]} \tag{35}$$

$$\frac{dR_{st.}}{dM_1} = -\frac{\left(\pi^2+a^2\right)^2M_3\left(\pi^2+a^2M_3\right)}{p_l\left[\pi^2+a^2\left(1+M_1\right)M_3\right]^2} \tag{36}$$

$$\frac{dR_{st.}}{dM_3} = -\frac{\left(\pi^2+a^2\right)^2\left(1+M_1\right)\left(\pi^2+a^2M_3\right)}{p_l\left[\pi^2+a^2\left(1+M_1\right)M_3\right]^2}+\frac{\left(\pi^2+a^2\right)^2}{p_l\left[\pi^2+a^2\left(1+M_1\right)M_3\right]} \tag{37}$$

It is clearly found from the above derivatives that all of the three, p_l, M_1, M_3 always have destabilizing effect on the system.

8. Oscillatory Convection

Putting $\omega = i\omega$ in (32), it reduces to

$$R = \Delta_1 + i\omega\Delta_2 \tag{38}$$

where

$$\Delta_1 = \frac{\begin{array}{c}-p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\right)P_r\left(\pi^2+a^2M_3\right)\omega^2+p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\right)\pi^2P_r'M_2\omega^2\\+\varepsilon\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\varepsilon\left(\pi^2+a^2\right)P_r\left(\pi^2+a^2M_3\right)\lambda_0\omega^2\\-\varepsilon\left(\pi^2+a^2\right)\pi^2P_r'M_2\lambda_0\omega^2\end{array}}{a^2\left(1-M_2\right)\varepsilon p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\left(1+M_1\right)M_3\right)} \tag{39}$$

and

$$\Delta_2 = \frac{\begin{array}{c}p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)-\lambda_0\varepsilon\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)\\+\varepsilon\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r-\varepsilon\left(\pi^2+a^2\right)\pi^2P_r'M_2\end{array}}{a^2\left(1-M_2\right)\varepsilon p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\left(1+M_1\right)M_3\right)} \tag{40}$$

For oscillatory convection $\omega \neq 0$, we have $\Delta_2 = 0$, which yields the frequency of oscillation as

$$\omega^2 = \frac{\begin{array}{c}-p_l\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\lambda_0\varepsilon\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)\\-\varepsilon\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r+\varepsilon\left(\pi^2+a^2\right)\pi^2P_r'M_2\end{array}}{\lambda_0^2p_l\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)} \tag{41}$$

The existence of oscillatory instability depends on value of ω^2. If there is no positive value of ω^2 then there is no chance of oscillatory instability. From equation (41), ω^2 does not admit any positive value if

$$p_l\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\varepsilon\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r > \lambda_0\varepsilon\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\varepsilon\left(\pi^2+a^2\right)\pi^2P_r'M_2$$

If there exist positive values of ω^2, then the thermal oscillatory Rayleigh number is given by

$$R_{osc.} = \frac{\begin{array}{l} -p_l\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)P_r\omega^2\left(1+\lambda_0^2\omega^2\right)+p_l\left(\pi^2+a^2\right)\pi^2P_r'M_2\omega^2\left(1+\lambda_0^2\omega^2\right) \\ +\varepsilon\left(\pi^2+a^2\right)^2\left(\pi^2+a^2M_3\right)+\varepsilon\left(\pi^2+a^2\right)\left(\pi^2+a^2M_3\right)\lambda_0P_r\omega^2 \\ -\varepsilon\left(\pi^2+a^2\right)\pi^2P_r'M_2\lambda_0\omega^2 \end{array}}{a^2\left(1-M_2\right)\varepsilon p_l\left(1+\lambda_0^2\omega^2\right)\left(\pi^2+a^2\left(1+M_1\right)M_3\right)} \tag{42}$$

where ω^2 is given by equation (41).

9. RESULTS AND DISCUSSION

To discuss the effect of the parameters governing the problem on the system, we have plotted the graph of $R_{st.}$ vs. a for fixed values of the parameter.The value of M_1 is assumed to be 1000 [1]. The value of M_2 will be considered negligiblei.e., 10^{-6}. The Darcy model has been used which takes the value of permeability p_l in the range of 0.001to 0.009 [7-10] and M_3 is taken to have value from 1 to 25 [18] and cannot be less than 1. The value of P_r is taken to be 0.01 [17]. The value of porosity ε lies between 0 and 1. The value of relaxation time constant λ_0 can be between 0.001 to 0.01. For this problem, we have taken fixed values of parameters as $p_l = 0.002, M_1 = 1000, M_2 = 10^{-6}, M_3 = 1, \varepsilon = 0.4, \lambda_0 = 0.005, P_r = 0.01, P_r' = 0.005$.

Figures 2-5 shows the curves for stationary convection. Figure 2 depicts the curve between $R_{st.}$ and a for $M_1 = 1000, M_3 = 1$ and $p_l = 0.002, 0.003, 0.004, 0.005$. Figure 3 depicts the curve between $R_{st.}$ and a for $p_l = 0.002, M_3 = 1$ and $M_1 = 1000, 1500, 2000, 2500$. Figure 4 depicts the curve between $R_{st.}$ and a for $p_l = 0.002, M_1 = 1000$ and $M_3 = 1, 2, 3, 4$. On increasing the value of p_l, M_1, M_3, the value of $R_{st.}$ decreases, thus making the system destabilizing. Figure 5 shows the comparative study in continuous and porous medium. The curves shows clearly that the Rayleigh number $R_{st.}$ falls largely in continuous medium and results to early convection.

Figures6-8 shows the curves for oscillatory convection. Figure 6 depicts the curve between $R_{\text{osc.}}$ and a for $M_1 = 1000, M_2 = 10^{-6}, M_3 = 1, \varepsilon = 0.4, \lambda_0 = 0.005, P_r = 0.01, P_r' = 0.005$ and $p_l = 0.002, 0.003, 0.004$. Figure 7 depicts the curve between $R_{\text{osc.}}$ and a for $p_l = 0.002, M_1 = 1000, M_2 = 10^{-6}, M_3 = 1, \lambda_0 = 0.005, P_r = 0.01, P_r' = 0.005$ and $\varepsilon = 0.4, 0.5, 0.6$. Figure 8 depicts the curve between $R_{\text{osc.}}$ and a for $p_l = 0.002, M_1 = 1000, M_2 = 10^{-6}, M_3 = 1, \varepsilon = 0.4, P_r = 0.01, P_r' = 0.005$ and for relaxation time constant $\lambda_0 = 0.005, 0.006, 0.007$. On increasing the value of p_l, $R_{\text{osc.}}$ decreases and thereby destabilizing the system whereas increasing ε and λ_0 increases $R_{\text{osc.}}$, thus making the system stabilizing.

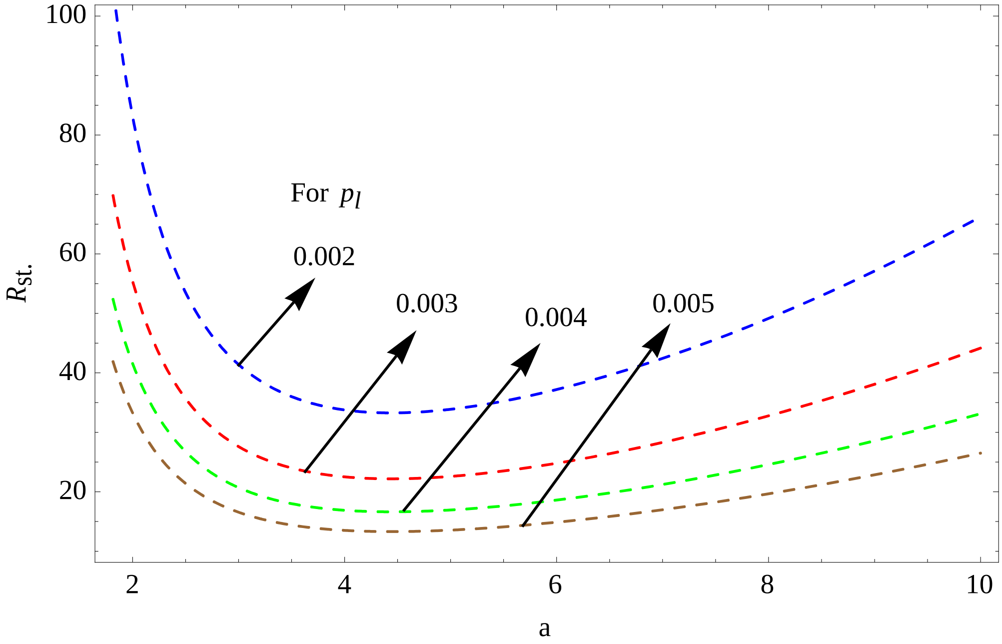

Figure 2: Variation of stationary Rayleigh number $R_{st.}$ with wavenumber a for different values of permeability p_l

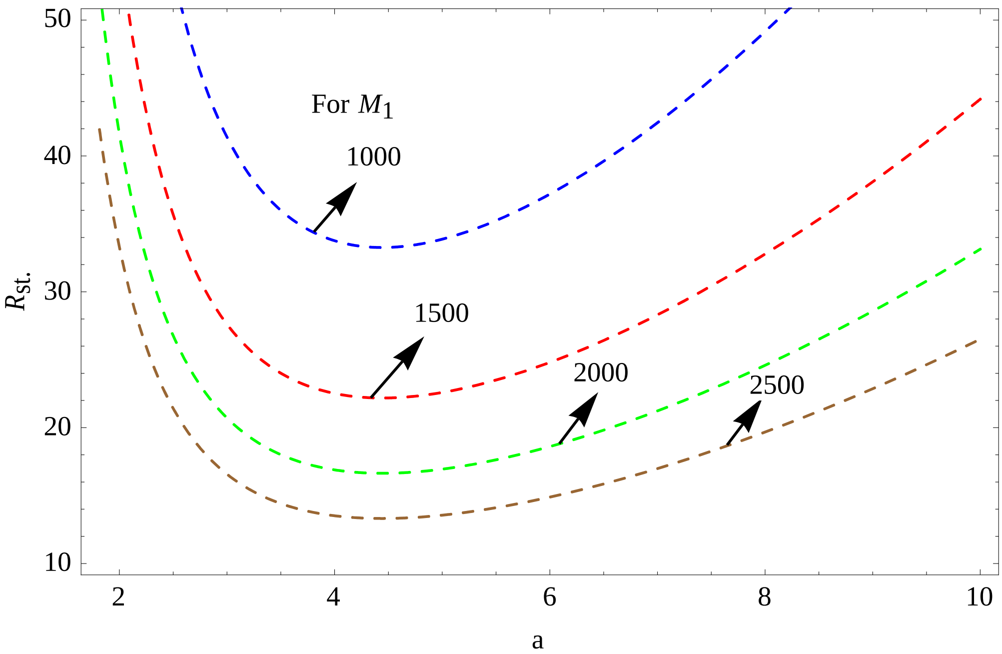

Figure 3: Variation of stationary Rayleigh number $R_{st.}$ with wavenumber a for different values of M_1

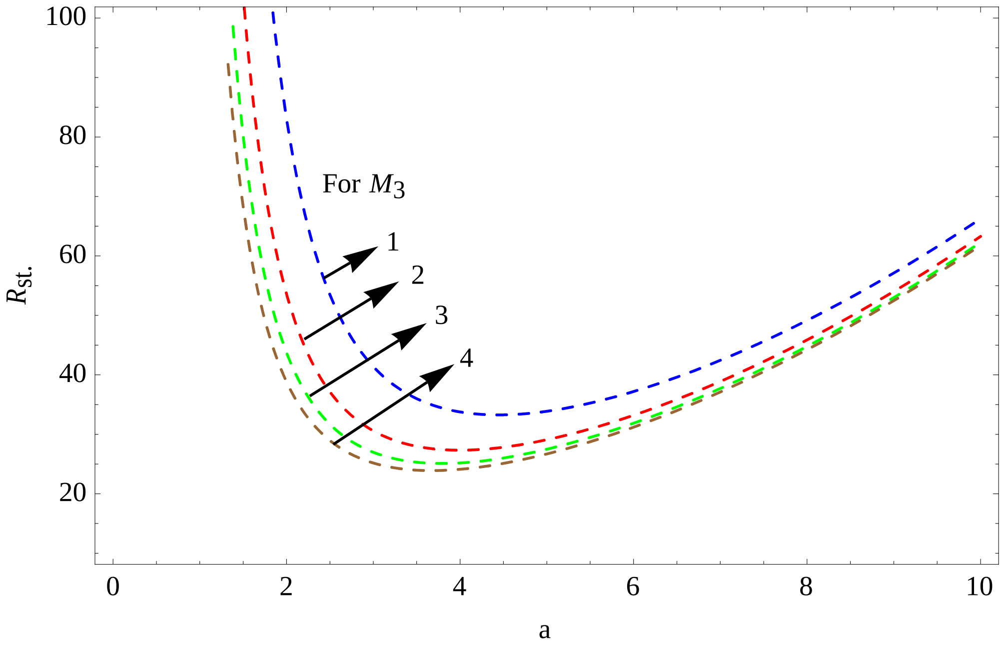

Figure 4: Variation of stationary Rayleigh number $R_{st.}$ with wavenumber a for different values of M_3

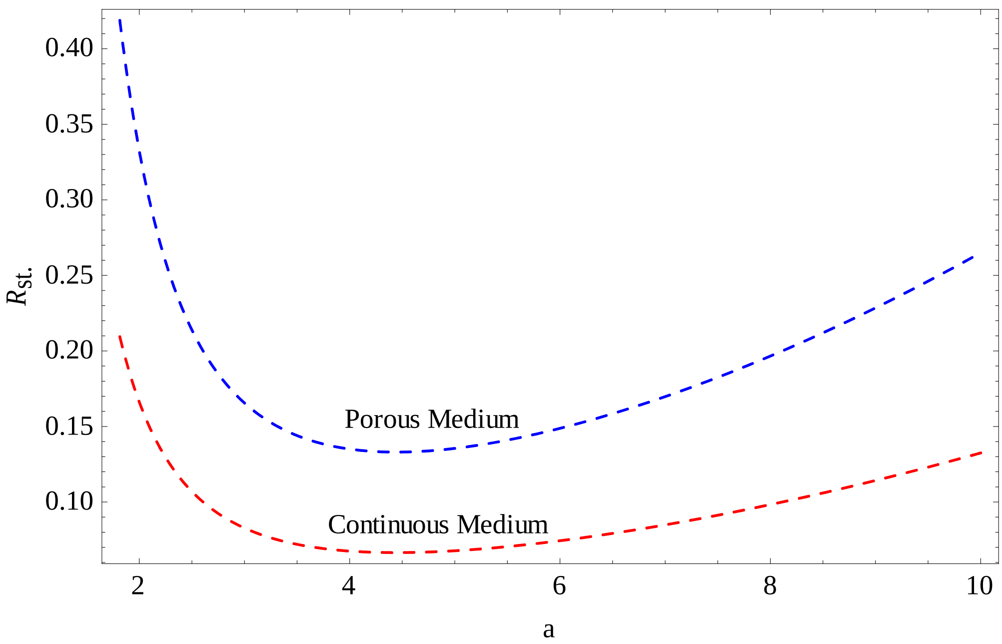

Figure 5: A Comparative study of continuous and porous medium

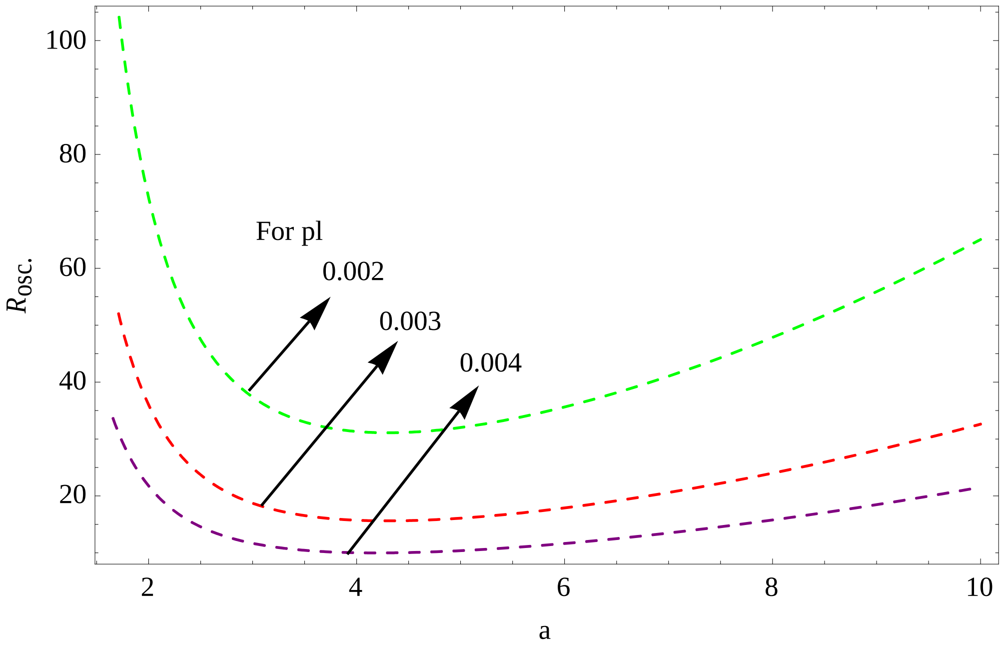

Figure 6: Variation of oscillatory Rayleigh number $R_{osc.}$ with wavenumber a for different values of permeability p_l

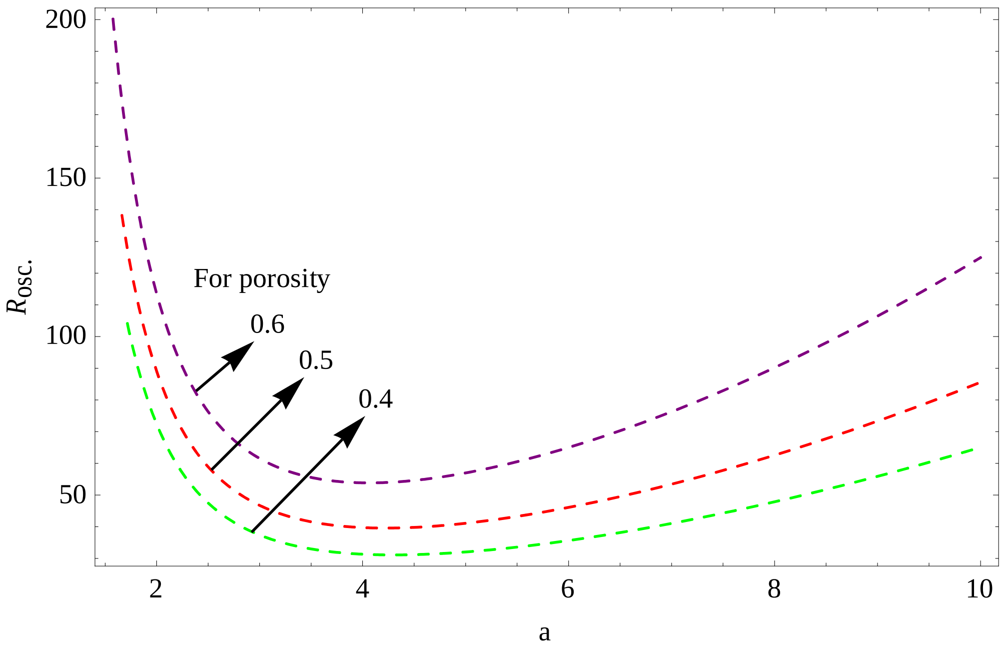

Figure 7: Variation of oscillatory Rayleigh number $R_{osc.}$ with wavenumber a for different values of porosity ε

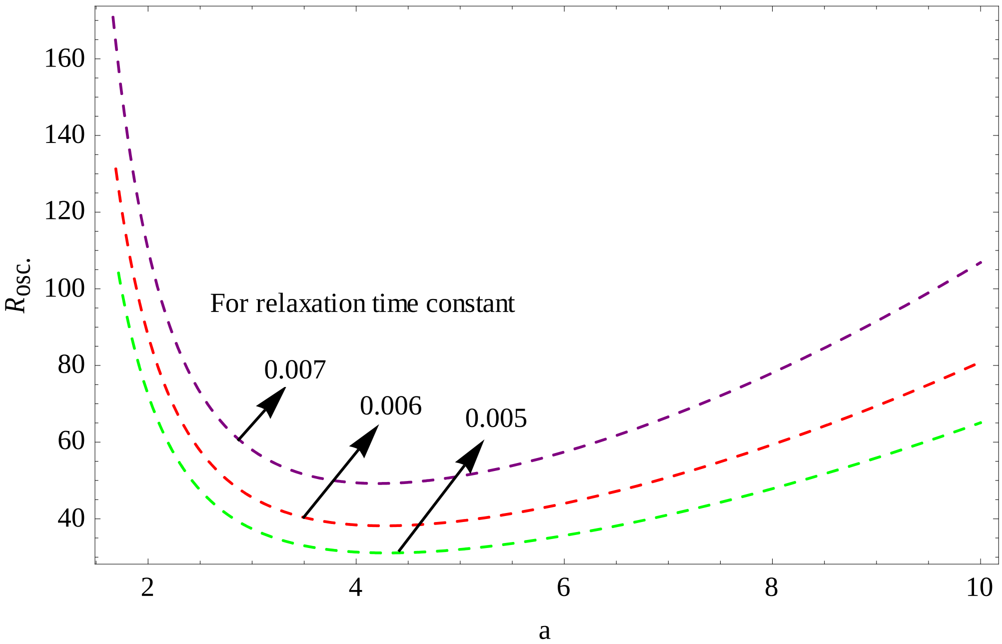

Figure 8: Variation of oscillatory Rayleigh number $R_{osc.}$ with wavenumber a for different values of λ_0

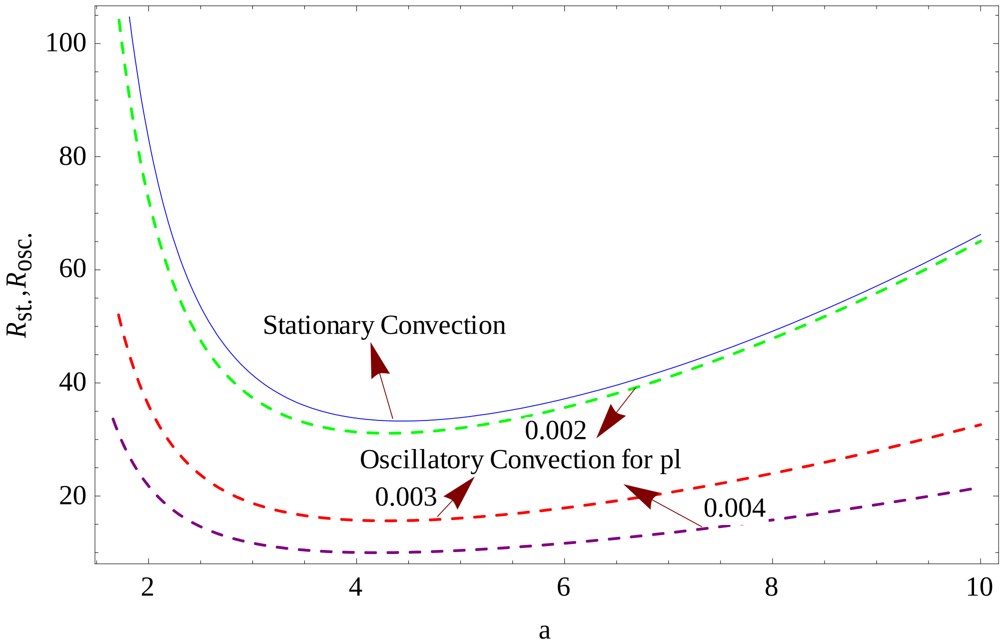

Figure 9: Effect of permeability p_l on stationary and oscillatory convection

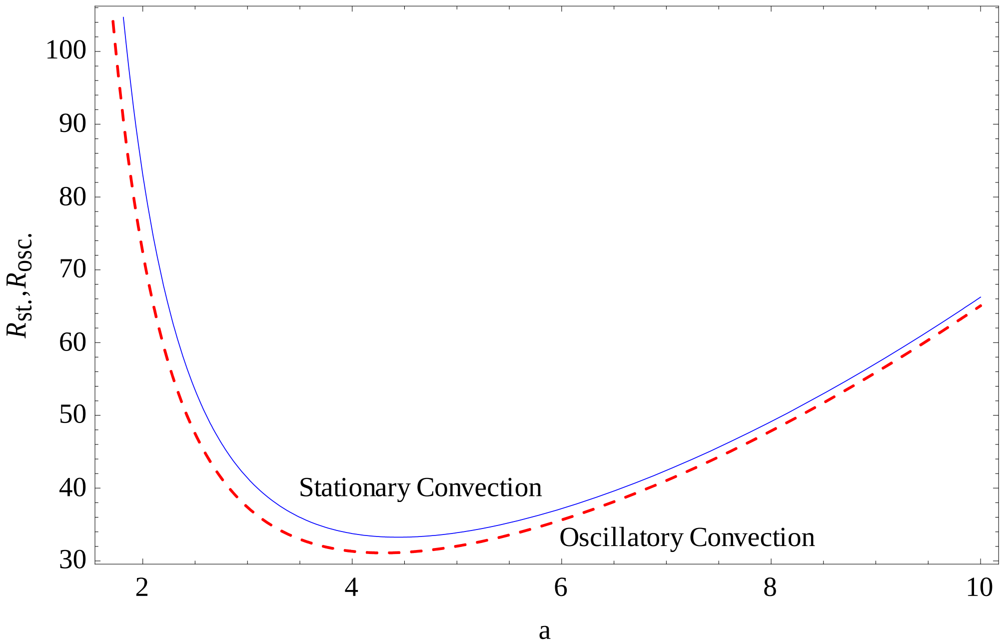

Figure 10: Comparative study of stationary and oscillatory convection

Figure 9 shows the effect of p_l on stationary and oscillatory convection. It is noticed from the curves that for same value of p_l (i.e., 0.002), the oscillatory convection sets earlier as compared to the stationary one. Further, it is also noticed that as the value of p_l increases, the value of $R_{\text{osc.}}$ decreases for the oscillatory convection, thus making the onset of convection easier. Figure 10 shows the comparative study of stationary and oscillatory convection. It is noticed that the oscillatory convection commences earlier.

10. CONCLUSION

In this study, a heated porous layer of Maxwell ferromagnetic fluid has been considered using the normal mode approach. To obtain the solution of porous Maxwell ferromagnetic fluid layer which is kept within free boundaries, a Galerkin procedure is used and thermal Rayleigh number is obtained for both the stationary and oscillatory cases. The effects of time relaxation constant λ_0, buoyancy magnetization parameter M_1, non-buoyancy magnetization parameter M_3 and medium permeability parameter p_l have been examined. The observations are as follows:

(i)It is observed that for the stationary convection, the Maxwell ferromagnetic fluid converts to a Newtonian ferromagnetic fluid.

(ii) It is found that all the three parameters M_1, M_3 and p_l have destabilizing effect on the system in stationary mode.

(iii) In stationary convection, a comparative study with and without porous medium shows that the convection sets earlier in continuous medium as compared to the porous.

(iv) In oscillatory state, the porosity parameter ε stabilizes the system whereas the permeability p_l destabilizes the system.

(v) The effect of relaxation time constant stabilizes the system.

(vi) The comparative study of stationary and oscillatory convection accounts that the presence of oscillatory modes sets the convection earlier.

(vii) The condition for non-existence of oscillatory instability is achieved.

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