Neural network based fractional order sliding mode tracking control of nonholonomic mobile robots

Naveen Kumar[a,b], Km Shelly Chaudhary[a,*]

[a]National Institute of Technology Kurukshetra, Kurukshetra 136119, Haryana, India,
[b] Mahatma Jyotiba Phule Rohilkhand University Bareilly, Bareilly 243006, Uttar Pradesh, India,
[*]Meerut College Meerut, Meerut 250002, Uttar Pradesh, India
Corresponding author:shelly_62000004@nitkkr.ac.in(KS.Chaudhary)

Abstract. In this study, the position tracking control problem of a nonholonomic mobile robots with system uncertainties and external disturbances is examined. In the design approach, a fractional-order sliding surface is presented that offers asymptotic stability of the system states towards their equilibrium points. A fractional order sliding mode controller is developed based on the presented sliding surface in order to handle system uncertainties and external disturbances in a robust manner. A radial basis function neural network is used to approximate the nonlinearities of the dynamic structure. The weighted matrices of neural networks are updated in an online mode. The controller's adaptive bound portion is used to manage neural network reconstruction error and provide upper bounds on disturbances and uncertainty. Using the Lyapunov technique and Barbalat's Lemma, the asymptotic stability of the control system is evaluated. Moreover, a numerical simulation study is carried out to illustrate the effectiveness of the proposed control approach by comparing the results with the existing control approaches.

Keywords: Nonholonomic mobile robots, Fractional order sliding surface; Sliding mode control; Neural networks

1 Introduction

Because of their wide applications in the field of medical profession, industries, military operations, and many other areas [1,2,3], trajectory tracking control of nonholonomic mobile robots has become a very intriguing study area in recent years. Nonholonomic mobile robots are the mechatronic structures that are extremely nonlinear, coupled, and time-varying. Because of these nonlinearities, uncertainties, and external disruptions, there are several practical challenges in managing them. To address these challenges, various classical control schemes such as Model-based controllers, PID controllers, Back-stepping based controllers, Sliding mode controllers, Adaptive controllers, etc. [4,5,6] have been presented in the literature to control these systems.

Among these, sliding mode controllers (SMC) [7] are the most commonly used controllers because of their inherent capacity to resist uncertainty and external disturbances. The intrinsic adaptability attribute of the sliding mode control scheme is that when the system is operated on the sliding manifold, it functions independently of the system dynamics. In sliding mode controller, a sliding syrface is utilized to assure the convergence of tracking errors toward zero. For superior controller performance, linear and nonlinear sliding surfaces are now utilized in SMCs. Using linear sliding surface, Linear sliding mode controllers [8] (LSMC) have been presented in the study. LSMC investigates the asymptotic convergence of the trajectory tracking error even when the finite-time trajectory tracking error cannot be solved by these controllers. Terminal sliding mode controllers (TSMC) [9] have been presented in the literary texts to solve this issue. In TSMC, a non-linear sliding manifold is employed instead of a linear sliding manifold. These controllers guarantee tracking error convergence in a finite amount of time, but occasionally they pose singularity problems that result in unboundedly high control input values. The Non-singular Terminal Sliding Mode Controller (NTSMC) [10], which restricts the non-linear sliding manifold's parameters, has been proposed as a modified controller to handle this problem. The singularity problem is solved in NTSMC, although it has a slow convergence rate at the equilibrium point due to the presence of the term $e^{r/s}, r > s$ in the sliding manifold, resulting in a reduction in the convergence rate's magnitude away from the equilibrium.

For the enhanced and precise performance of controllers, different combinations of sliding mode controllers with Fractional Calculus [11,12,13] have been presented in the literature. Because of their greater order convergence speed, fractional-order controllers outperform integer-order controllers [14,15,16]. The study on integrating the fractional-order derivative [17] with SMC begins with applying the fractional order derivative to LSMC , which is known as the

J. COMPUTATIONAL ANALYSIS AND APPLICATIONS, VOL. 33, NO. 1, 2024, COPYRIGHT 2024 EUDOXUS PRESS, LLC

fractional-order sliding mode controller (FoSMC) [18]. These controllers give superior tracking performance as compared to simple sliding mode controllers. The reason for this is that the fractional-order system's mathematical solution has a faster order convergence speed than the integer-order system. As fractional order sliding mode controllers are very efficient controller but the presence of uncertainties and disturbances in the dynamic structure of the manipulator causes many real-time difficulties. So, the employment of intelligent approaches such as neural networks [19,20,21] and fuzzy logics [22,23] improves the controller's suitability for real-world deployments. In the article [24], the design of a fractional-order sliding mode controller with a time-varying sliding surface is presented for trajectory tracking problem of robot manipulators. In this paper authors prove asymptotic convergence of tracking errors towards their system states. Authors of the article [25] present a fractional adaptation law for sliding mode control scheme for multi-input multi-output nonlinear dynamic system. In the article [26] authors present a coupled fractional-order sliding mode control scheme using obstacle avoidance for the control of a four-wheeled steerable mobile robot. A new fractional-order global sliding mode control scheme for nonholonomic mobile robot systems under external disturbances is presented in article [27]. While many studies have been conducted on the position tracking problem of dynamic systems under the influence of external disturbances and system uncertainties, relatively few of these studies combine intelligent techniques with the advantageous features of fractional-order sliding mode controllers for the control of nonholonomic mobile robots. So, the novelty of the presented work lies on the combination of fractional order sliding surface and the presented contoller that enhances the performance of the dynamical system in a robust manner.

In this paper to enhance the performance of the controller, a neural network based fractional-order sliding mode controller is presented for the position control problem of nonholonomic mobile robots under the influence of uncertainties and disturbances. The radial basis function neural network (RBFNN) is utlized in the developed controller to resemble the nonlinearity of the dynamic structure, and the exponential reaching rule is utilized when the system is independent of its general dynamics. The designed controller's adaptive compensator part handles the neural networks reconstruction error and upper bounds on disturbances. The Lyapunov stability criterion and Barbalat's lemma are used to examine the asymptotic convergence of tracking errors towards their equilibrium states. Moreover, simulation studies are performed to validate the proposed controller's performance in a comparative manner

The main contribution of the presented work is as follows:

1. A new combination of fractional order sliding surface with neural network based fractional order sliding mode controller is presented.
2. The position tracking problem for nonholonomic mobile manipulators under the influence of system uncertainties and external disturbances is discussed.
3. The stability and asymptotic convergence of tracking errors is examined using Lyapunov stability criterion and Barbalat's lemma.
4. Simulation studies are used to compare the performance of the proposed controller to that of existing controllers.

The remaining part of the paper is divided as follows. Sections 2 offer a dynamic model for a nonholonomic mobile robot. Section 3 presents the controller design, while section 4 contains the stability analysis. Section 5 offers a simulation study, and section 6 concludes the article.

2 Dynamics of nonholonomic mobile robots

The dynamics equation for 3-dof nonholonomic mobile robots with generalized cordinates $q = [x, y, \theta]^T$ satisfies the Euler-Lagrange equation is given by:

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + F(\dot{q}) + T_d = B(q)\tau + A^T(q)\lambda \tag{1}$$

where $M(q) \in R^{3\times3}$ be inertial matrix, $V(q, \dot{q}) \in R^{3\times3}$ be centripetal-coriolis matrix, $F(\dot{q}) \in R^{3\times1}$ be friction vector, $T_d \in R^{3\times1}$ be unknown bounded disturbance, $B(q) \in R^{3\times2}$ be input transformation matrix, $\tau \in R^{3\times1}$ be control input, $A^T(q) \in R^{3\times1}$ be constraint associated matrix and $\lambda \in R$ be Langranges multiplier.

J. COMPUTATIONAL ANALYSIS AND APPLICATIONS, VOL. 33, NO. 1, 2024, COPYRIGHT 2024 EUDOXUS PRESS, LLC

With $M(q) = \begin{bmatrix} m & 0 & m_1 \\ 0 & m & -m_2 \\ m_1 & -m_2 & I \end{bmatrix}, V_m(q,\dot{q}) = \begin{bmatrix} 0 & 0 & -m_2\dot{\theta} \\ 0 & 0 & m_1\dot{\theta} \\ 0 & 0 & 0 \end{bmatrix}, A^T(q) = \begin{bmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{bmatrix}, B(q) = \begin{bmatrix} \cos\theta/r & \cos\theta/r \\ \sin\theta/r & \sin\theta/r \\ b/r & -b/r \end{bmatrix}, m_1 = mh\sin\theta, m_2 = mh\cos\theta$

where m is total mass of nonholonomic mobile base, I is moment of inertia of mobile base.

Let the mobile robot system is subject to the following nonholonomic kinematic constraint.

$$A(q)\dot{q} = 0 \tag{2}$$

These constraints are limitations on the dynamic equation of mobile robots to the manifold $\Im_B$ as $\Im_B = \{(q,\dot{q})|B(q)\dot{q} = 0\}$. From equation (2), we can get the full rank matrix $P(q) \in R^{3\times 2}$ as:

$$P^T(q)A^T(q) = 0 \tag{3}$$

From constraints given in equations (2) and (3), we have a new vector $\dot{v} \in R^2$ satisfies the following condition

$$\dot{q} = P(q)\dot{v} \tag{4}$$

Differentiating equation (4), we have

$$\ddot{q} = P(q)\ddot{v} + \dot{P}(q)\dot{v} \tag{5}$$

Putting equation (4) and (5) in equation (1) and multiplying the obtained equation by P^T we get

$$\bar{M}_f\ddot{v} + \bar{V}_f\dot{v} + \bar{F}_f + \bar{\tau}_{fd} = P^T\tau \tag{6}$$

where $\bar{M}_f = P^T M(q)P, \bar{V}_f = P^T M(q)\dot{P} + P^T V_m(q,\dot{q})P, \bar{F}_f = P^T F(\dot{q})$, $\bar{\tau}_{fd} = P^T T_d$.

Let the dynamics equation (6) of nonholonomic mobile robots satisfy the following properties and assumptions.

Property 1 The Inertial matrix $\bar{M}_f$ is symmetric, bounded positive-definite and invertible,.

Property 2 The term $A = (\dot{\bar{M}}_f - 2\bar{V}_f)$ satisfies skew-symmetric property $i.e.$ $x^T Ax = 0 \quad \forall x \in R^n$.

Assumption 1 $\bar{F}_f \leq a_1 + a_2\|\dot{v}\|$ for arbitrary positive constants a_1, a_2 .
Assumption 2 $\|\bar{\tau}_{fd}\| \leq a_3$ for arbitrary positive constant a_3.
Assumption 3 If $v = [y, \theta]^T \in R^2$ is uniformally bounded and continuous, then all the jacobian matrices are also uniformally bounded and continuous.

3 Controller Structure

3.1 Fractional-order sliding surface

The proposed fractional-order sliding surface is given as

$$S(t) = D^{\alpha+1}\eta(t) + \dot{\eta}(t) + \lambda\eta(t) \tag{7}$$

where $\alpha \in (0,1)$, $\eta(t) = v_d(t) - v(t)$ denotes position tracking error, $v_d(t) \in R^2$ denotes desired trajectory, $\lambda = diag[\lambda_1, \lambda_2] \in R^{2\times 2}$ with $\lambda_1, \lambda_2 > 0$, and $S(t) = [S_1(t), S_2(t)]^T \in R^2$ be sliding variable. The j^{th} element of the proposed sliding surface is written as

$$S_j(t) = D^{\alpha+1}\eta_j(t) + \dot{\eta}_j(t) + \lambda\eta_j(t) \tag{8}$$

where $j = 1, 2$
On differentiating equation (8), we have

$$\dot{S}_j(t) = D^{\alpha+2}\eta_j(t) + \ddot{\eta}_j(t) + \lambda\dot{\eta}_j(t) \tag{9}$$

The reduced dynamics equation for nonholonomic mobile robots in terms of sliding variable $S(t) \in R^2$ can be written as

$$\bar{M}_f\dot{S} = -\bar{V}_f S - P^T\tau + f(y) + \bar{\tau}_{fd} + \bar{F}_f(\dot{v}) \tag{10}$$

where, $f(y) = \bar{M}_f[D^{\alpha+2}\eta(t) + \ddot{v}_d + \lambda\dot{\eta}(t)] + \bar{V}_f(v, \dot{v})[\dot{v}_d + D^{\alpha+1}\eta(t) + \lambda\eta(t)]$ be non-linear dynamics part comprises of two factors as $f(y) = \hat{f}(y) + \bar{f}(y)$ in which $\hat{f}(y)$ is known dynamic part of the system and $\bar{f}(y)$ is uncertain part of the dynamic system. For approximating this non-linear function $f(y)$, radial basis function neural networks (RBFNN) has been utilized. The input vector y during approximation of non-linear function $f(y)$ by RBFNN is choosen as $y = [\eta^T, \dot{\eta}^T, D^{\alpha+1}\eta^T, D^{\alpha+2}\eta^T]^T$.

3.2 RBFNN

Due to the adaptive nature of RBFNN [28], it is utilized to reproduce the non-linear part of the manipulator's dynamics. Let the function approximation on a simply connected compact set of the continuous function $f(y)$ be

$$f(y) = W^T \xi(y) + \epsilon(y) \tag{11}$$

where, $W \in R^{N \times b}$ demonstrates weight matrix, it will update online in an adaptive manner, $\xi(\cdot)\colon R \to R^N$ denotes predefine basis array, $\epsilon(y)\colon R \to R^b$ denotes reconstruction error, N denotes the no. of nodes used in the structure of neural-networks. So, we have $\|\epsilon(y)\| < \epsilon_N$ for some $\epsilon_N > 0$.
For larger values of N, $\epsilon(y)$ may be reduced to very small value.
In the structure of RBFNN, the Gaussian function $\xi(y)$ [29], has been used which is given as

$$\xi_i(y) = exp(\frac{-\|y - c_i\|^2}{\sigma_i^2}), \quad i = 1, 2...N. \tag{12}$$

Putting the value of function $f(y)$ from (11) into (10). then, the reduced error dynamical equation be given by

$$\bar{M}_f \dot{S} = -\bar{V}_f S - P^T \tau + W^T \xi(y) + \epsilon(y) + \bar{\tau}_{fd} + \bar{F}_f(\dot{v}) \tag{13}$$

3.3 Adaptive bound

From assumptions 1,2 and the upper bound ϵ_N, we have

$$\|\bar{\tau}_{fd} + \bar{F}_f(\dot{v}) + \epsilon(y)\| \leq a_1 + a_2\|\dot{v}\| + a_3 + \epsilon_N \tag{14}$$

As an adaptive bound, define $\mu = a_1 + a_2\|\dot{v}\| + a_3 + \epsilon_N$

$$\mu = \begin{bmatrix} 1 \ \|\dot{v}\| \ 1 \ 1 \end{bmatrix} \begin{bmatrix} a_1 \ a_2 \ a_3 \ \epsilon_N \end{bmatrix}^T = H^T(\|\dot{v}\|)\phi \tag{15}$$

where $H \in R^m$ is known vector function and $\phi \in R^m$ be the parameter vector.
To compensate the influence of friction, reconstruction error, and disturbances, the adaptive compensator is chosen as

$$\chi = \frac{\hat{\mu}^2 S}{\hat{\mu}\|S\| + \delta} \tag{16}$$

J. COMPUTATIONAL ANALYSIS AND APPLICATIONS, VOL. 33, NO. 1, 2024, COPYRIGHT 2024 EUDOXUS PRESS, LLC

where $\dot{\delta} = -\beta\delta$, $\delta(0) > 0$, $\beta > 0$ and $\hat{\mu} = H^T\hat{\phi}$.
The control input law is offered as follows to reach the reference trajectory

$$P^T\tau = \hat{W}^T\xi(y) + K_1 S + K_2 sign(S) + \chi \tag{17}$$

with K_1, K_2 as gain matrices and $\tau = [\tau_1, \tau_2]^T \in R^2$
Using equation (17), the reduced dynamics equation in form of sliding variable $S(t)$ can be given as

$$\bar{M}_f\dot{S} = -\bar{V}_f S + \tilde{W}^T\xi(y) - K_1 S - K_2 sign(S) + \epsilon(y) + \bar{\tau}_{fd} + \bar{F}_f(\dot{v}) - \chi \tag{18}$$

where $\tilde{W} = W - \hat{W}$

4 Stability analysis

4.1 Asymptotical convergence of tracking error and boundedness of signals

If we select the update laws for varying parameters as:

$$\dot{\hat{W}} = \Lambda_w\xi(y)S^T \tag{19}$$

$$\dot{\hat{\phi}} = \Lambda_\phi H\|S\| \tag{20}$$

where $\Lambda_w = \Lambda_w^T \in R^{N\times N}$ and $\Lambda_\phi = \Lambda_\phi^T \in R^{m\times m}$ are positive-definite matrices. Then, the trajectory tracking error asymptotically converges to zero along with the boundedness of signals.
Proof: Let the Lyapunov function be

$$L = \frac{1}{2}S^T\bar{M}_f S + \frac{1}{2}tr(\tilde{W}^T\Lambda_w^{-1}\tilde{W}) + \frac{1}{2}tr(\tilde{\phi}^T\Lambda_\phi^{-1}\tilde{\phi}) + \frac{\delta}{\beta} \tag{21}$$

where $\tilde{W} = W - \hat{W}$ and $\tilde{\phi} = \phi - \hat{\phi}$.
Differentiating equation (21), we get

$$\dot{L} = \frac{1}{2}S^T\dot{\bar{M}}_f S + S^T\bar{M}_f\dot{S} + tr(\tilde{W}^T\Lambda_w^{-1}\dot{\tilde{W}}) + tr(\tilde{\phi}^T\Lambda_\phi^{-1}\dot{\tilde{\phi}}) + \frac{\dot{\delta}}{\beta} \tag{22}$$

Putting equation (18) into equation (22) with $\dot{\tilde{W}} = -\dot{\hat{W}}, \dot{\tilde{\phi}} = -\dot{\hat{\phi}}$, and $\dot{\delta} = -\beta\delta$, we get

$$\dot{L} = \frac{1}{2}S^T(\dot{\bar{M}}_f - 2\bar{V}_f)S + S^T\tilde{W}^T\xi(y) - S^T(K_1S + K_2sign(S)) + S^T(\bar{F}_f(\dot{v}) + \epsilon(y) + \bar{\tau}_{fd}) - \frac{\hat{\mu}^2 S}{\hat{\mu}\|S\| + \delta}) - tr(\tilde{W}^T\Lambda_w^{-1}\dot{\hat{W}}) - tr(\tilde{\phi}^T\Lambda_\phi^{-1}\dot{\hat{\phi}}) - \delta \tag{23}$$

From equations (19), (20), and property 2, equation (23) can be written as

$$\dot{L} = -S^T(K_1S + K_2sign(S)) + S^T(\bar{F}_f(\dot{v}) + \epsilon(y) + \bar{\tau}_{fd}) - \frac{\hat{\mu}^2\|S\|^2}{\hat{\mu}\|S\| + \delta} - \tilde{\phi}^T H\|S\| - \delta \tag{24}$$

Using adaptive bound μ, we get

$$S^T(\bar{F}_f(\dot{v}) + \epsilon(y) + \bar{\tau}_{fd}) \leq H^T(\hat{\phi} + \tilde{\phi})\|S\| \tag{25}$$

From (25), we have equation (24) as

$$\dot{L} \leq -S^TK_1S - S^TK_2sign(S) - \frac{(H^T\hat{\phi})^2\|S\|^2}{H^T\hat{\phi}\|S\| + \delta} + (H^T\hat{\phi})\|S\| - \delta \tag{26}$$

$$\dot{L} \leq -S^TK_1S - \frac{\delta(H^T\hat{\phi})\|S\|}{H^T\hat{\phi}\|S\| + \delta} - \delta = -S^TK_1S - \frac{\delta^2}{H^T\hat{\phi}\|S\| + \delta} \tag{27}$$

$$\dot{L} \leq -S^TK_1S \leq -K_{min}\|S\|^2 \tag{28}$$

where K_{min} be the min. eigenvalue of matrix K_1.
So, it is concluded that $L_1(S(0), \tilde{W}, \tilde{\phi})$ and $L_1(S(t), \tilde{W}, \tilde{\phi})$ are both bounded functions with $L_1(S(t), \tilde{W}, \tilde{\phi})$ as non-increasing function . Thus, it has been shown that $S(t)$, $\tilde{W}$, and $\tilde{\phi}$ are all bounded. As $S(t)$ is function of location and velocity tracking error, so bounded value of $S(t)$ leads to the boundedness of these tracking errors.
Differentiating equation (28), we have $\ddot{L} \leq -2S^TK_1\dot{S}$. As $S(t)$ and

$\dot{S}(t)$ (Equation (18)) are both bounded implies $\ddot{L}_1$ is also bounded, that means $\dot{L}_1$ is uniformly continuous. Using to Barbalat's lemma, the position tracking errors approaches to zero in an asymptotic manner.

5 Simulation

To show the effecient performance of the designed controller, a simulation study is carried out on a nonholonomic mobile robot. The dynamic structure and parameters used in this study for position tracking problem of nonholonomic mobile robot is given in [30]. The non-holonomic constraint applied on mobile robot system is considered as: $-\dot{x}\sin(\theta) + \dot{y}\cos(\theta) = 0$. The simulation study on nonholonomic mobile robot is carried out using Matlab. ODE45 Matlab solver is utilized to solve ordinary differential equation. For calculating fractional order derivative, definition of Grunwald-Letnikov(GL) derivative [31] has been used.

To show the effectiveness and robustness of the proposed control scheme, the performance of the proposed control scheme is compared with the existing controller given in article [32], proposed controller by taking adaptive compensator is equal to zero and with desired trajectory. Figures 1-6 show how well the suggested control technique for a nonholonomic mobile robot system works. Figures 1 and 2 compare the location and velocity tracking errors of the proposed controller. These data demonstrate that the trajectory tracking errors for the proposed control method converge rapidly when compared to the existing controller. The location tracking performance is displayed in Figures 3 and 4. In the second case *i.e.* proposed controller with $\Delta = 0$, due to the presence of the disturbances and reconstruction error, there is some fluctuations during the tracking of reference trajectory but the performance of the proposed controller is very smooth that shows the robustness of the proposed controll approach. These Figures shows that the dynamic system tracks the desired trajectory very efficiently for the proposed case as compare to the other two cases. In Figures 5 and 6, velocity tracking performance is given that shows in the initial phase, the velocity of the mobile robot system fluctuates but after a very small duration, it

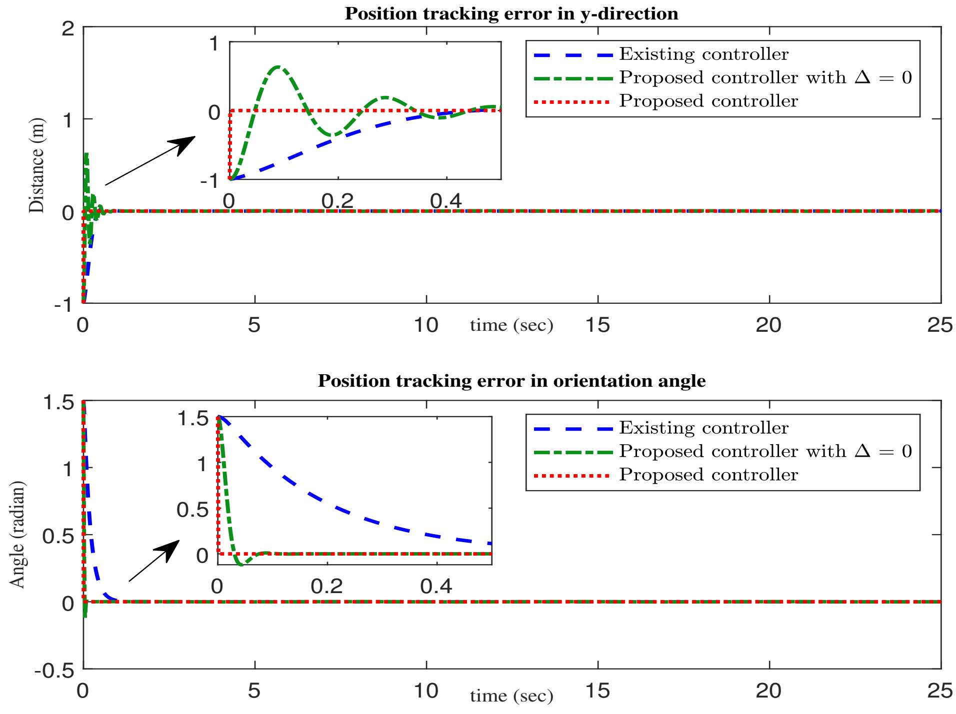

Fig. 1. Position tracking errors

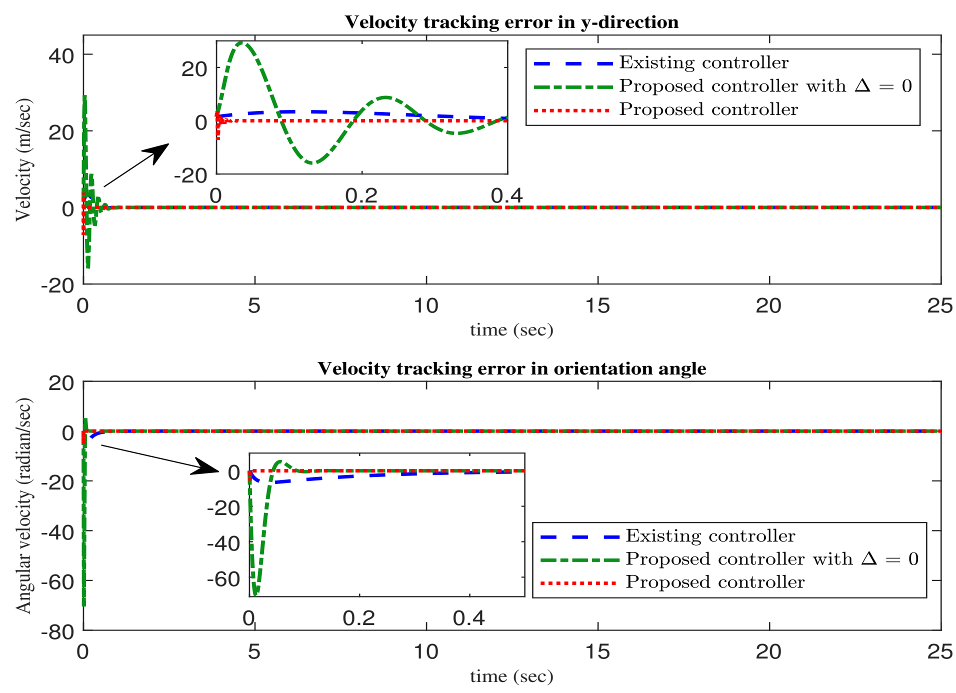

Fig. 2. Velocity tracking errors

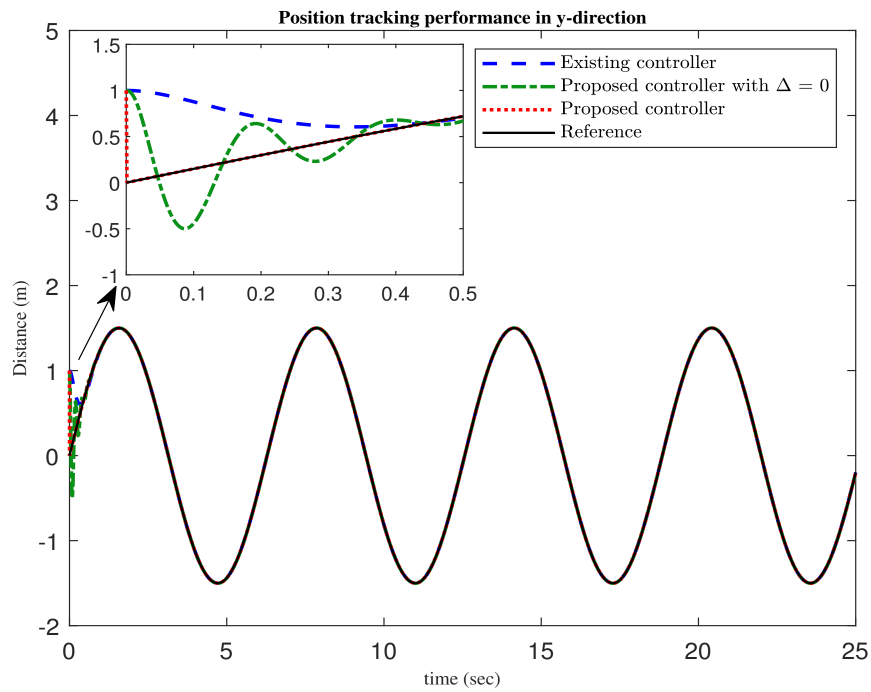

Fig. 3. Position tracking performance in y-direction

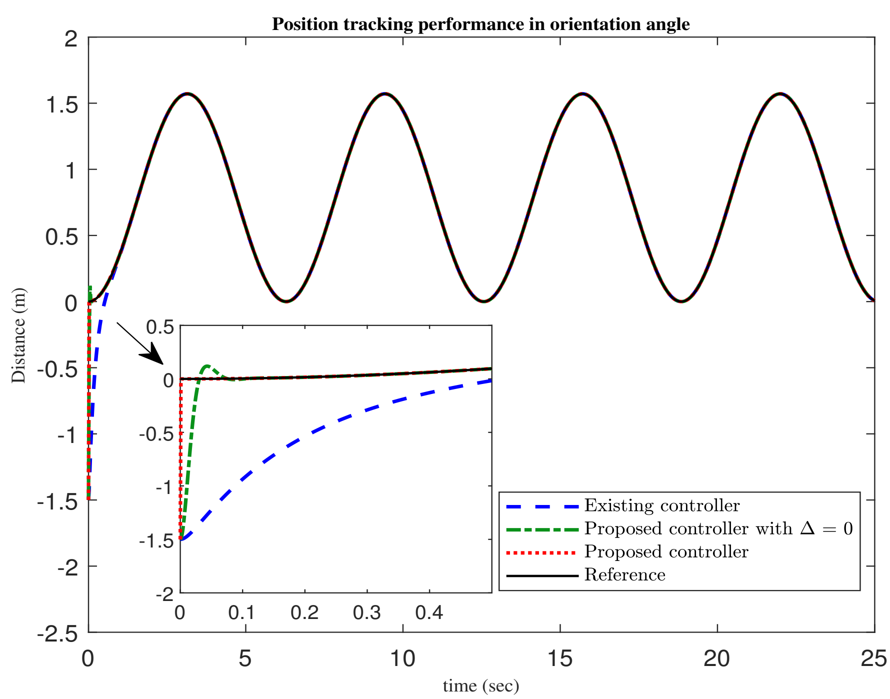

Fig. 4. Position tracking performance of orientation angle

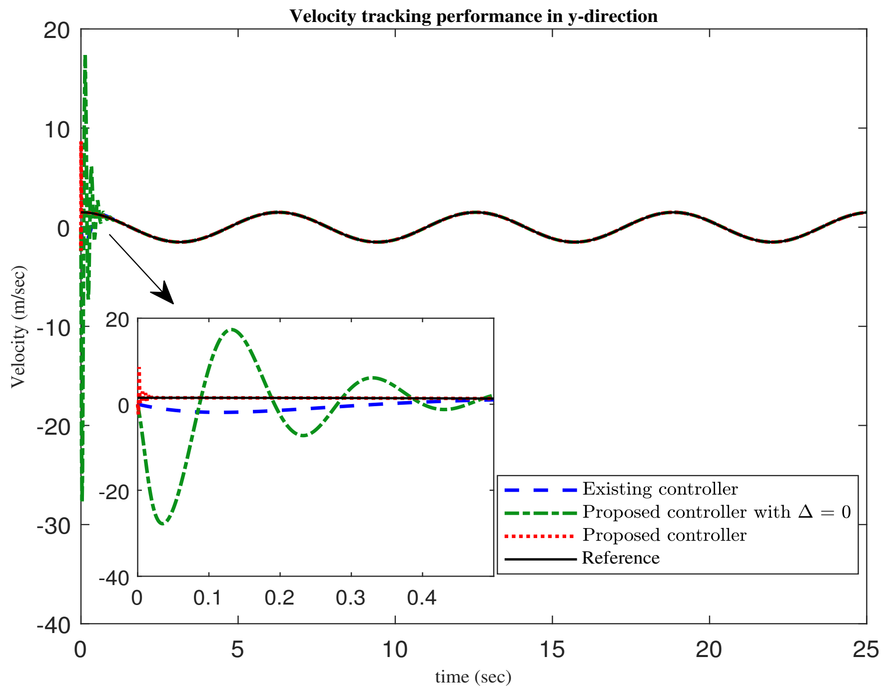

Fig. 5. Velocity tracking performance in y-direction

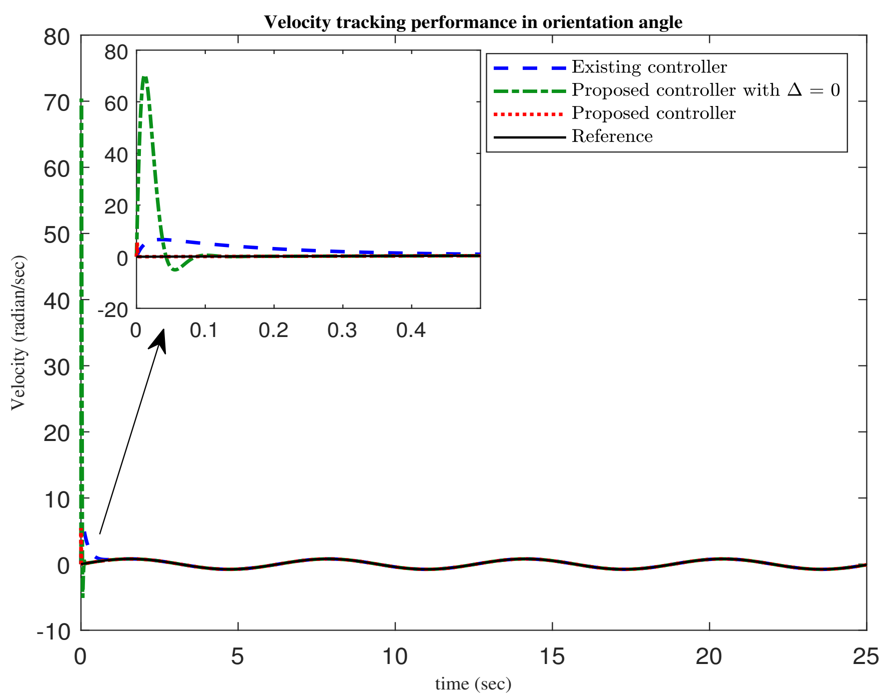

Fig. 6. Velocity tracking performance of orientation angle

tracks the reference velocity very smoothly for the proposed approach while for other two cases, the trajectory achieve after some time. From these figures, We get to the conclusion that the proposed controller precisely and quickly tracks the reference trajectory in a robust manner.

Further to compare the performance of controllers statistically, L^2 norm error analysis is presented in tabular form by comparing these parameters with existing controllers. Formula used for L^2 norm is given as

$$L^2[\eta] = \sqrt{\frac{1}{t_f - t_0} \int_{t_0}^{t_f} \|\eta(t)\|^2 dt} \tag{29}$$

Table 1. L^2-norm of position tracking error

Controllers	$L^2[\eta_1]$	$L^2[\eta_2]$
Existing controller [32]	0.1548	0.2180
Proposed controller with $\varDelta = 0$	0.0997	0.0696
Proposed Controller	0.0100	0.0150

A lower value of $L^2[\eta]$ shows a lower tracking error, which demonstrates the effectiveness of the control strategy.

6 Conclusions

In this article, a neural network based fractional-order sliding mode controller is designed for the trajectory tracking problem of non-holonomic mobile robots. In the designed controller, RBFNN is used for approximation of the nonlinear part of dynamic structure, and an exponential reaching law is adopted. An adaptive compensator makes up for reconstruction error and disturbance upper limits. In order to analyze the convergence of tracking errors asymptotically, the Lyapunov stability criterion and Barbalat's lemma are used. To show the effectiveness and robustness of the presented controller, a simulation study is carried out in a comparative manner. It can be evident from the simulated data and statistical analysis that the efficiency of the proposed controller is enhanced. Further this control approach can be implemented to another dynamical systems such as mobile manipulator systems, cart-pendulum systems, constrained reconfigurable dynamical systems ect.

References

1. X. Wu, Y. Wang, X. Dang, Robust adaptive sliding-mode control of condenser-cleaning mobile manipulator using fuzzy wavelet neural network, Fuzzy Sets and Systems 235 (2014) 62-82, doi:10.1016/j.fss.2013.07.009
2. H. Xie, J. Zheng, R. Chai, H.T. Nguyen, Robust tracking control of a differential drive wheeled mobile robot using fast nonsingular terminal sliding mode, Computers and Electrical Engineering 96 (2021) 107488. doi:10.1016/j.compeleceng.2021.107488
3. W. Dong, W. Huo,Tracking control of wheeled mobile robots with unknown dynamics, Proceedings 1999 IEEE International Conference on Robotics and Automation 4 (1999) 2645-2650. doi:10.1109/ROBOT.1999.773997
4. N. Chen, F. Song, G. Li, X. Sun, C. Ai, An adaptive sliding mode backstepping control for the mobile manipulator with nonholonomic constraints, Communications in Nonlinear Science and Numerical Simulation 18 (10) (2013) 2885-2899. doi:10.1016/j.cnsns.2013.02.002
5. M. Begnini, D.W. Bertol, N.A. Martins, A robust adaptive fuzzy variable structure tracking control for the wheeled mobile robot: Simulation and experimental results, Control Engineering Practice 64 (2017) 27-43. doi:10.1016/j.conengprac.2017.04.006
6. K.S. Chaudhary, N. Kumar, Fractional order fast terminal sliding mode control scheme for tracking control of robot manipulators. ISA transactions. 2023 Nov 1;142:57-69. doi:10.1016/j.isatra.2023.08.008
7. M. Defoort, T. Floquet, A. Kokosy, W. Perruquetti, Sliding-mode formation control for cooperative autonomous mobile robots, IEEE Transactions on Industrial Electronics 55 (11) (2008). doi:10.1109/TIE.2008.2002717
8. Gambhire, S. J., Kishore, D. R., Londhe, P. S., Pawar, S. N.: Review of sliding mode based control techniques for control system applications. International Journal of dynamics and control, 9, 363-378 (2021). doi:10.1007/s40435-020-00638-7
9. L. Zuo, R. Cui, W. Yan, Terminal sliding mode-based cooperative tracking control for non-linear dynamic systems, Transactions of the Institute of Measurement and Control 39 (7) (2017) 1081-1087. doi:10.1177/01423312166292
10. M.H. Korayem, R. Shiri, S.R. Nekoo, Z. Fazilati, Non-singular terminal sliding mode control design for wheeled mobile manipulator, Industrial Robot: An International Journal 44 (4) (2017) 501-511. doi:10.1108/IR-10-2016-0263
11. J. Singh, A. Gupta, D. Kumar, Computational Analysis of the Fractional Riccati Differential Equation with Prabhakar-type Memory. Mathematics. 2023 Jan 27;11(3):644. https://doi.org/10.3390/math11030644
12. J. Singh, A. Gupta, D. Baleanu, Fractional Dynamics and Analysis of Coupled Schrödinger-KdV Equation With Caputo-Katugampola Type Memory. Journal of Computational and Nonlinear Dynamics. 2023 Sep 1;18(9). https://doi.org/10.1115/1.4062391

13. J. Singh, Babu R, V.P. Dubey, D. Kumar, A homotopy based computational scheme for local fractional Helmholtz and Laplace equations. Journal of Computational Analysis and Applications. 2023 Jan 1;31(1).
14. J. Singh, R. Agrawal, K.S. Nisar. A new forecasting behavior of fractional model of atmospheric dynamics of carbon dioxide gas. Partial Differential Equations in Applied Mathematics. 2023 Nov 28:100595. https://doi.org/10.1016/j.padiff.2023.100595
15. V.P. Dubey, J. Singh, S. Dubey, D. Kumar. Analysis of Cauchy Problems and Diffusion Equations Associated with the Hilfer–Prabhakar Fractional Derivative via Kharrat–Toma Transform. Fractal and Fractional. 2023 May 20;7(5):413. https://doi.org/10.3390/fractalfract7050413
16. L. Mohan, A. Prakash, Stability and numerical analysis of fractional BBM-Burger equation and fractional diffusion-wave equation with Caputo derivative. Optical and Quantum Electronics 2024; 1-25. https://doi.org/10.1007/s11082-023-05608-9
17. A. Prakash, L. Mohan, Application of Caputo fractional operator to analyse the fractional model of Brain Tumour via modified technique, International Journal of Applied and Computational Mathematics 9, 1-33 (2023). 10.1007/s40819-023-01591-7
18. Y. Xie, X. Zhang, W. Meng, S. Zheng, L. Jiang, J. Meng, S. Wang, Coupled fractional-order sliding mode control and obstacle avoidance of a four-wheeled steerable mobile robot, ISA transactions 108 (2021) 282-294. doi:10.1016/j.isatra.2020.08.025
19. Lewis, F. W., Jagannathan, S., Yesildirak, A.: Neural network control of robot manipulators and non-linear systems. CRC press (1998).
20. Yin, F. C., Ji, Q. Z., Wen, C. W.: An adaptive terminal sliding mode control of stone-carving robotic manipulators based on radial basis function neural network. Applied Intelligence, 1-18 (2022). doi:10.1007/s10489-022-03445-z
21. Sonker, S., Devi, N.: Approximation of Signals by $E_l^1 E_l^1$ Product Summability Means of Fourier–Laguerre Expansion (2023). 10.1007/978-981-19-9906-2_5
22. Gao, Y., Er, M. J., Yang, S.: Adaptive control of robot manipulators using fuzzy neural networks. IEEE Transactions on Industrial Electronics, 48(6), 1274-1278 (2001). doi:10.1109/41.969410
23. Sonker, S., Moond, P.: Rate of Convergence of parametrically generalized bivariate Baskakov-Stancu operators (2023). 10.2298/FIL2327197S
24. Eray, O., Tokat, S.: The design of a fractional-order sliding mode controller with a time-varying sliding surface. Transactions of the Institute of Measurement and Control, 42(16), 3196-3215 (2020). doi:10.1177/01423312209446
25. Efe, M. O., Kasnakoglu, C.: A fractional adaptation law for sliding mode control. International Journal of Adaptive Control and Signal Processing, 22(10), 968-986 (2008). doi:0.1002/acs.1062
26. Xie, Y., Zhang, X., Meng, W., Zheng, S., Jiang, L., Meng, J., Wang, S.: Coupled fractional-order sliding mode control and obstacle avoidance of a four-wheeled steerable mobile robot. ISA transactions, 108, 282-294 (2021). https://doi.org/10.1016/j.isatra.2020.08.025
27. Labbadi, M., Boubaker, S., Djemai, M., Mekni, S. K., Bekrar, A.: Fixed-time fractional-order global sliding mode control for nonholonomic mobile robot systems under external disturbances. Fractal and Fractional, 6(4), 177 (2022). https://doi.org/10.3390/fractalfract6040177
28. Park, J., Sandberg, I. W.: Universal approximation using radial-basis-function networks. Neural computation, 3(2), 246-257 (1991). doi:10.1162/neco.1991.3.2.246

29. Feng, G., Chak, C. K.: Robot tracking in task space using neural networks. In Proceedings of 1994 IEEE International Conference on Neural Networks (ICNN'94) (Vol. 5, pp. 2854-2858). IEEE (1994). doi:10.1109/ICNN.1994.374684
30. Xie, H., Zheng, J., Sun, Z., Wang, H., Chai, R.: Finite-time tracking control for nonholonomic wheeled mobile robot using adaptive fast nonsingular terminal sliding mode. Nonlinear Dynamics, 110(2), 1437-1453 (2022). https://doi.org/10.1007/s11071-022-07682-2
31. Podlubny, I.: Fractional differential equations, mathematics in science and engineering (1999).
32. Boukattaya, M., Damak, T., & Jallouli, M.: Robust adaptive sliding mode control for mobile manipulators. aa, 2(2), 2 (2017).