Square Difference Edge Cordial Labeling Of Some Special Graphs

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ABSTRACT
A graph G with p vertices and q edges is said to admit a square difference edge cordial labeling if there is a bijectionf : V(G) →{1,2,..., p}such that for each edge e =uv, the induced mapf^*: E(G) →{0,1} is defined by$f^*(uv)$ = 1 if$|f(u)^2 - f(v)^2|$ is odd, otherwise 0 and$|e_f(0)-e_f(1)| \leq 1$ where$e_f(0)$ = edges with label zero and$e_f(1)$ = edges with label one. If a graph admits square difference edge cordial labeling, then it is said to be square difference edge cordial graph. In this paper, it is investigated that the square graphP_n^2, the shadow graph $D_2(P_n)$, the duplication of apex vertex of bistar$B_{n,n}$, the lotus inside a circleLC_n, the bistar$B_{n,n}$, the one point union of cycle C_3 with the star graph $K_{1,n}$(n is even), the graph $[P_n: S_2]$, the comb graphP_n^+, the jewel graphJ_n, the graph $(K_n-e)_n$and the total graph $T(P_n)$ are square difference edge cordial graph.

Keywords: Square difference edge cordial labeling, Square difference edge cordial graph.

1. INTRODUCTION

All graphs considered in this paper are finite, simple and undirected graphs. The symbolV(G) and E(G) denotes the vertex set and the edge set of a graph G. The cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called (p,q) graph. The concept of graph labeling was introduced by Rosa [13] in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Cordial labeling was first introduced in 1987 by Cahit [4]. Ponraj et.al.,[9] found a new labeling called difference cordial labeling in 2013. Ponraj et.al.,[10,11,12] studied the difference cordial labeling behaviour of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web and some more standard graphs. Shiama [14]profounded an idea of square difference labeling in 2012. Alfred Leo et.al.,[2] initiated the divided square difference cordial labeling in 2018. In this paper, a new labeling called the square difference edge cordial labeling is introduced. Further notations and terminologies are followed from Harary [5] and Bondy and Murty [3]. The following definitions are used in the present study.

Definition 1.1 [11] A mapping f : V(G) →{0,1}is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f. The induced edge labeling f^*: E(G) →{0,1} is given by $f^*(e = uv) = | f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ be the number of vertices of G having 0 and 1 respectively under f and $e_f(0)$, $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 1.2 [11]A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and$|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 1.3 [15] A square difference labeling of a graph G is a bijection f :V(G) →{0,1,2,3,4,...,p−1} such that the induced function f^*: E(G)→N defined by $f^*(uv) = |f(u)^2 - f(v)^2|$ for every uv∈E(G) are all distinct. A graph that admits square difference labeling is called a square difference graph.

Definition 1.4A graph G with p vertices and q edges is said to admit a square difference edgecordial labeling if there is a bijection f :V(G) →{1,2,..., p} such that for each edge e =uv, the induced map f^*: E(G) →{0,1} is defined by $f^*(uv)$ = 1 if$| f(u)^2 - f(v)^2|$ is odd, otherwise 0 and$|e_f(0)-e_f(1)| \leq 1$ where $e_f(0)$ = edges with label zero and $e_f(1)$ = edges with label one. If a graph admits square difference edge cordial labeling, then it is said to be square differenceedge cordial graph.

Definition 1.5 [16]A complete bipartite graph $K_{1,n}$is called a **star** and it has n+1 vertices and nedges. It is also denoted by S_n.
Definition 1.6 [6]The**bistar**$B_{n,n}$is the graph obtained by joining the center (apex) verticesof the two copies of $K_{1,n}$ by an edge. The vertex set $B_{n,n}$is $V(B_{n,n}) =\{u,v,u_i,v_i/1 \leq i \leq n\}$, where u,v are apex vertices and u_i,v_i are pendant vertices. The edge set of $B_{n,n}$is $E(B_{n,n}) =\{uv,uu_i,vv_i/1 \leq i \leq n\}$. So,$|V(B_{n,n})| = 2n+2$ and$|E(B_{n,n})| = 2n+1$.
Definition 1.7 [8] The graph **$[P_n : S_2]$** is obtained from a path**P_n**by joining every vertex ofa path to a root of a star S_2 by an edge.
Definition 1.8 [7]The graph obtained by joining n pendant edges at one vertex of the cycleC_3 is called the one point union of cycle $C_3$with a star graph $K_{1,n.}$
Definition 1.9 [6]The duplication of a vertex v of a graph G produces a new graph **G'**by adding vertex **v'**with $N(v') = N(v)$. In other words, a vertex **v'** is said to be a duplication of v if all the vertices which are adjacent to v are then adjacent to **v'**.
Definition 1.10 [17] The graph **$(K_n -e)_n$**is the one edge union of $(K_n -e)$.
Definition 1.11 [15]The total graph of a graph G is the graph whose vertex set $V(G)\cup E(G)$and two vertices are adjacent whenever they are either adjacent or incident in G. The total graphof G is denoted by T(G).
Definition 1.12 [18]For a simple connected graph G, the square graph G is denoted by $G^2$and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2ifthey are at a distance 1 or 2 apart in G.
Definition 1.13 [1]The jewel graphJ_n is obtained from a 4-cycle with vertices x,y,u,v by joiningx and y with a prime edge and by appending an edge from u and v which meets at the common vertices v_i,$1 \leq i \leq n$. The prime edge in a jewel graph is defined to be the edge joining the vertices x and y.
Definition 1.14 [18] The shadow graph$D_2(G)$ of a connected graph G is constructed by taking the two copies of G which may be G'andG'' and each vertex $u'$is joined with G' to the neighbours of the corresponding vertex v'in G''.
Definition 1.15 [11]The lotus inside a circleLC_n is a graph obtained from the cycle $c_n:u_1u_2...u_nu_1$ and a star $K_{1,n}$ with the central vertex v_0 and the end vertices $v_1,v_2,...,v_n$ by joiningeach v_i to $u_i$and u_{i+1}(mod n).
Definition 1.16 [8]The graph obtained by joining a single pendant edge to each vertex of a path is called **comb**$P_n^+$where n denotes the number of vertices in the path.

2. Main results

In this section, it is shown that the comb graphP_n^+, the graph $[P_n: S_2]$, the one point union of the cycle C_3 with the star graph $K_{1,n}$ (n is even), the shadow graph $D_2(P_n)$, the duplication of apex vertex of bistar$B_{n,n}$, the square graph P_n^2, the lotus inside a circleLC_n, the bistar$B_{n,n}$, the graph $(K_n -e)_n$, the total graph $T(P_n)$ and the jewel graph$J_n$are square difference edge cordial graph.

Theorem 2.1 The comb graph $P_n^+$is a square difference edge cordial graph.
Proof.Let G be the comb graphP_n^+.
Let $V(G) =\{u_i,v_i/1 \leq i \leq n\}$ and $E(G) =\{u_iu_{i+1} /1 \leq i \leq n-1\}\cup\{u_iv_i/1 \leq i \leq n\}$.
So,$|V(G)| = 2n$ and$|E(G)|= 2n-1$.
A function$f : V(G) \rightarrow\{1,2,3,...,2n\}$ is defined as follows:
$f(u_i) = 2i-1;1 \leq i \leq n$,
$f(v_i) = 2i;1 \leq i \leq n$.
The induced edge labels are
$f^*(u_iu_{i+1}) = 0;1 \leq i \leq n-1$,
$f^*(u_iv_i) = 1;1 \leq i \leq n$.
Thus,$|e_f(0)-e_f(1)| \leq 1$.Hence the comb graphP_n^+admits a square difference edge cordial labeling. Therefore the comb graph $P_n^+$is a square difference edge cordial graph.

Example 2.4 Let the comb graph P_6^+be considered. The square difference edge cordial labeling pattern is shown in Figure 2.1.

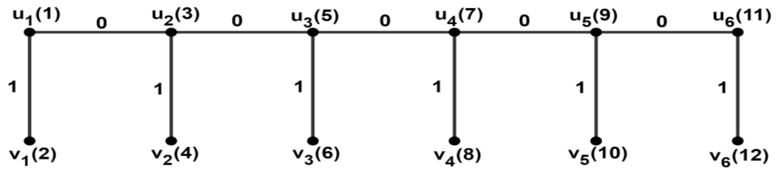

Figure 1: Square difference edge cordial graph of the comb graphP_6^+

Clearly,$|e_f(0)|$= 5 and$|e_f(1)|$= 6. Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the comb graphP_6^+ admits a square difference edge cordial labeling. Therefore the comb graph$P_6^+$is a square difference edge cordial graph

Theorem 2.3 The graph [P $_n$: S_2] is a square difference edge cordial graph.
Proof. Let G be the graph [P $_n$: S_2].
LetV(G) ={u_i, v_i, v_i', v_i'' /1 ≤i ≤n} and
E(G) ={$u_i u_{i+1}$ /1 ≤i ≤n−1}∪ {$u_i v_i, v_i v_i', v_i v_i''$ /1 ≤i ≤n}.
So, |V(G)| = 4n and |E(G)| = 4n−1.
A functionf : V(G) →{1,2,3,...,4n} is defined as follows:
f(u_i) = 2i−1;1 ≤i ≤n,
f(v_i) = 2i;1 ≤i ≤n,
f(v_i') =2(n+i)−1;1 ≤i ≤n,
f(v_i'') = 2(n+i);1 ≤i ≤n.
The induced edge labels are
$f^*(u_i u_{i+1})$ = 0;1 ≤i ≤n−1,
f^* $(u_i v_i)$ = 1;1 ≤i ≤n,
f^* $(v_i v_i')$ = 1;1 ≤i ≤n,
f^* $(v_i v_i'')$ = 0;1 ≤i ≤n.
Thus,$|e_f(0) - e_f(1)| \leq 1$.Hence the graph [P_n: S_2] is a square difference edge cordial graph.

Example 2.4 Let the graph [P_5: S_2] be considered. The square difference edge cordial labeling pattern is shown in Figure 2.2.

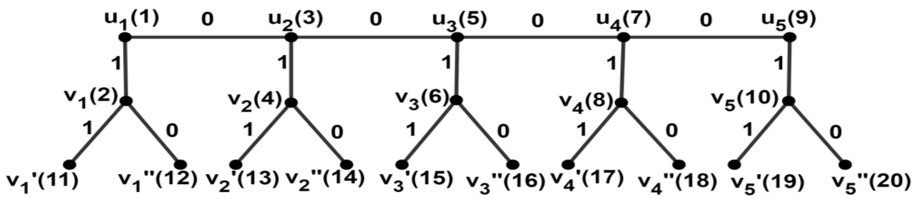

Figure 2: Square difference edge cordial graph of [P_5: S_2]

Clearly,$|e_f(0)|$ = 9 and$|e_f(1)|$= 10. Thus, $|e_f(0) - e_f(1)| \leq 1$. Hence the graph [P_5: S_2] admits
a square difference edge cordial labeling. Therefore the graph [P_5: S_2] is a square difference edge cordial graph.

Theorem 2.5 The shadow graph $D_2(P_n)$ is a square difference edge cordial graph.
Proof. Let G be the shadow graph$D_2(P_n)$.
LetV(G) ={u_i, v_i /1 ≤i ≤n} and E(G) ={$u_i u_{i+1}, u_i v_{i+1}, u_{i+1} v_i, v_i v_{i+1}$ /1 ≤i ≤n−1}.
So,|V(G)| = 2n and|E(G)| = 4n−4.
A functionf : V(G) →{1,2,3,...,2n} is defined as follows:
f(u_i) = 2i−1;1 ≤i ≤n,
f(v_i) = 2i;1 ≤i ≤n.
The induced edge labels are
$f^*(u_i u_{i+1})$ = 0;1 ≤i ≤n−1,
$f^*(v_i v_{i+1})$ = 0;1 ≤i ≤n−1,

$f^*(u_i v_{i+1})$ = 1;1 ≤i ≤n−1,
$f^*(u_{i+1} v_i)$ = 1;1 ≤i ≤n−1.
Thus,|e_f(0)−e_f(1)| ≤1. Hence the graph $D_2(P_n)$is a square difference edge cordial graph.

Example 2.6 Let the shadow graph$D_2(P_8)$ be considered. The square difference edge cordial labeling pattern is shown in Figure 2.3.

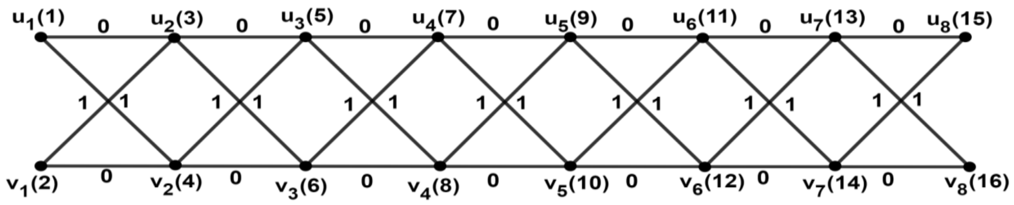

Figure 3: Square difference edge cordial graph of$D_2(P_8)$

Clearly,|e_f(0)|= 14 and|e_f(1)|= 14. Thus,|e_f(0) −e_f(1)| ≤1. Hence the shadow graph $D_2(P_8)$ admits a square difference edge cordial labeling. Therefore the shadow graph$D_2(P_8)$ is a square difference edge cordial graph.

Theorem 2.7 The square graph $P_n^2$is a square difference edge cordial graph.
Proof. Let G be the square graphP_n^2.
Let V(G) ={v_i /1 ≤i ≤n} and E(G) ={$v_i v_{i+1}$/1 ≤i ≤n−1}∪{$v_i v_{i+2}$/1 ≤i ≤n−2}.
So,|V(G)| = n and|E(G)|= 2n−3.
A functionf : V(G) →{1,2,3,...,n} is defined as follows:
f(v_i) = i;1 ≤i ≤n.
The induced edge labels are
$f^*(v_i v_{i+1})$ = 1;1 ≤i ≤n−1,
$f^*(v_i v_{i+2})$ = 0;1 ≤i ≤n−2.
Thus,|e_f(0)−e_f(1)| ≤1. Hence the square graph P_n^2 is a square difference edge cordial graph.

Example 2.8 Let the square graph P_9^2be considered. The square difference edge cordial labeling pattern is shown in Figure 2.4.

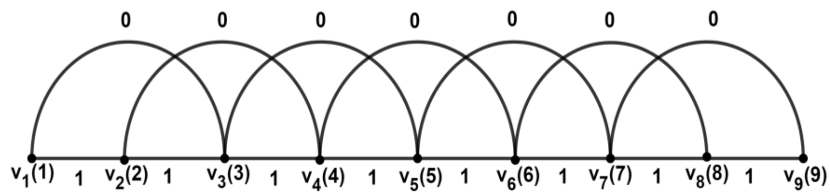

Figure 4: Square difference edge cordial graph of square graphP_9^2

Clearly,|e_f(0)| = 7 and|e_f(1)|= 8. Thus,|e_f(0)−e_f(1)| ≤1. Hence the square graphP_9^2admits a square difference edge cordial labeling. Therefore the square graph$P_9^2$is a square difference edge cordial graph.

Theorem 2.9 The total graph $T(P_n)$ is a square difference edge cordial graph.
Proof. Let G be the total graph$T(P_n)$.
LetV(G) ={u_i / 1 ≤i ≤ n} ∪{v_i /1 ≤i ≤ n−1} and
E(G) ={$u_i u_{i+1}$,$u_i v_i$,$u_{i+1} v_i$ / 1 ≤i ≤n−1}∪{$v_i v_{i+1}$ /1 ≤i ≤ n−2}.
So,|V(G)|= 2n−1 and|E(G)| = 4n−5.
A functionf : V(G) →{1,2,3,...,2n−1}is defined as follows:
f(u_i) = 2i−1;1 ≤i ≤n,
f(v_i) = 2i;1 ≤i ≤n−1.

The induced edge labels are
$f^*(u_i u_{i+1})$ = 0;1 ≤i ≤n−1,
$f^*(v_i v_{i+1})$ = 0;1 ≤i ≤n−2,
$f^*(u_i v_i)$ = 1;1 ≤i ≤n−1,
$f^*(u_{i+1} v_i)$ = 1;1 ≤i ≤n−1.
Thus, $|e_f(0)-e_f(1)| \leq 1$. Hence the total graphT(P_n) is a square difference edge cordial graph.

Example 2.10 Let the total graphT(P_7) be considered. The square difference edge cordial labeling pattern is shown in Figure 2.5.

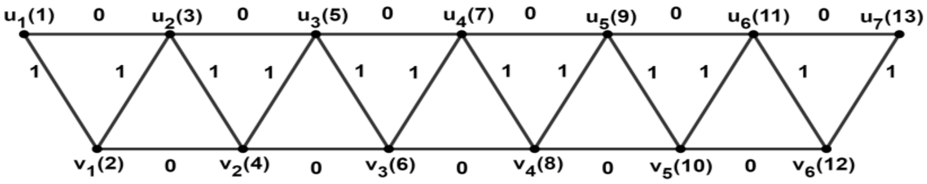

Figure 5: Square difference edge cordial graph of the total graphT(P_7)

Clearly,$|e_f(0)|$ = 11 and $|e_f(1)|$ = 12. Thus, $|e_f(0) - e_f(1)| \leq 1$. Hence the total graphT(P_7) admits a square difference edge cordial labeling. Therefore the total graphT(P_7) is a square difference edge cordial graph.

Theorem 2.11 The one point union of the cycle C_3 with the star graph $K_{1,n}$is a square difference edge cordial graph.
Proof. Let G be the one point union of the cycle C_3 with the star graph $K_{1,n.}$
LetV(G) ={u, v, w, v_i /1 ≤i ≤n} and
E(G) ={ uv, vw, wu, wv_i /1 ≤i ≤n}
So,|V(G)|= n+3 and|E(G)|= n+3.
A functionf : V(G) →{1,2,3,...,n+3} is defined as follows:
f(u) = 1,
f(v) = 2,
f(w) = 3,
f(v_i) = 3+i;1 ≤i ≤n.
The induced edge labels are
f^* (uv) = 1,
f^* (vw) = 1,
f^* (wu) = 0,
For 1 ≤i ≤ n,

$$f^*(wv_i) = \begin{cases} 1 & if\ i\ is\ odd \\ 0 & if\ i\ is\ even \end{cases}$$

Thus,$|e_f(0)-e_f(1)| \leq 1$. Hence the be the one point union of the cycle C_3 with the star graph $K_{1,n}$ is a square difference edge cordial graph.
Example 2.12 Let the one point union of cycle C_3 with the star graph $K_{1,8}$ be considered. The square difference edge cordial labeling pattern is shown in Figure 2.6.

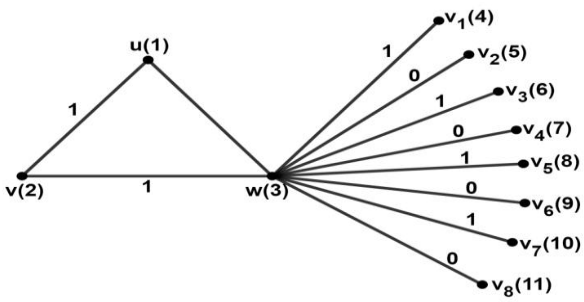

Figure 6: Square difference edge cordial graph of one point union of C_3 with $K_{1,8}$

Clearly, $|e_f(0)|$= 5 and $|e_f(1)|$= 6. Thus, $|e_f(0)-e_f(1)| \leq 1$. Hence the one point union of the cycle C_3 with the star graph $K_{1,8}$admits a square difference edge cordial labeling. Therefore the one point union of the cycle C_3 with the star graph $K_{1,8}$is a square difference edge cordial graph.

Theorem 2.13 The duplication of any vertex of a bistar$B_{n,n}$is a square difference edgecordial graph.
Proof. Let G be a graph obtained by the duplication of apex vertex v (or u) in a bistar$B_{n,n}$.
LetV(G) ={ u, v, u_i, v_i, v' /1 ≤i ≤n} and E(G) ={$uv, uv', uu_i, vv_i, v'v_i$/1 ≤i ≤n}.
So, |V(G)| = 2n+3 and|E(G)|= 3n+2.
A functionf : V(G) →{1,2,3,...,2n+3} is defined as follows:
f(u) = 1,
f(v) = 2n+2,
f(v') = 2n+3,
f(u_i) = i+1;1 ≤i ≤n,
f(v_i) = n+1+i;1 ≤i ≤n.
Case (i) : n is odd
For 1 ≤i ≤n,

$$f^*(uu_i) = \begin{cases} 1 & if\ i\ is\ odd \\ 0 & if\ i\ is\ even \end{cases}$$
$$f^*(vv_i) = \begin{cases} 1 & if\ i\ is\ odd \\ 0 & if\ i\ is\ even \end{cases}$$
$$f^*(v'v_i) = \begin{cases} 1 & if\ i\ is\ even \\ 0 & if\ \text{i}\ is\ odd \end{cases}$$

$f^*(uv')$ = 0,
$f^*(uv)$ = 1.

Case(ii) : n is even
For 1 ≤i ≤n,

$$f^*(uu_i) = \begin{cases} 1 & if\ i\ is\ odd \\ 0 & if\ i\ is\ even \end{cases}$$
$$f^*(vv_i) = \begin{cases} 1 & if\ i\ is\ even \\ 0 & if\ i\ is\ odd \end{cases}$$
$$f^*(v'v_i) = \begin{cases} 1 & if\ i\ is\ odd \\ 0 & if\ i\ is\ even \end{cases}$$

$f^*(uv')$ = 0,
$f^*(uv)$ = 1.
Thus, $|e_f(0)-e_f(1)| \leq 1$. Hence the graph obtained by the duplication of any vertex of a bistarBn,n is a square difference edge cordial graph.

Example 2.14 Let the duplication of the apex vertex of the bistar $B_{5,5}$be considered. The square difference edge cordial labeling pattern is shown in Figure 2.7.

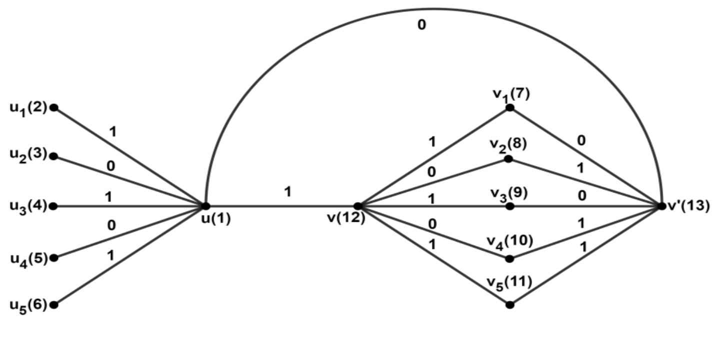

Figure 7: Square difference edge cordial graph of the duplication of the apex vertex of $B_{5,5}$

Clearly, $|e_f(0)|$ = 8 and $|e_f(1)|$= 9. Thus, $|e_f(0)-e_f(1)| \leq 1$. Hence the duplication of the apex vertex of bistar $B_{5,5}$ admits a square difference edge cordial labeling. Therefore the duplication of the apex vertex of bistar $B_{5,5}$ is a square difference edge cordial graph.

Theorem 2.15 The lotus inside a circle LC_n is a square difference edge cordial graph.
Proof. Let G be the lotus inside a circleLC_n.
LetV(G) =$\{v_0, u_i, v_i / 1 \leq i \leq n\}$and
E(G) =$\{u_n u_1, u_n v_1, u_i u_{i+1}, u_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_0 v_i, u_i v_i / 1 \leq i \leq n\}$.
So,|V(G)| = 2n+1 and|E(G)|= 4n.
A function$f : V(G) \rightarrow \{1,2,3,...,2n+1\}$ is defined as follows:
$f(v_0) = 1$,
$f(v_i) = 2i+1; 1 \leq i \leq n$,
$f(u_i) = 2i; 1 \leq i \leq n$.
The induced edge labels are
$f^*(u_n u_1) = 0$,
$f^*(u_n v_1) = 1$,
$f^*(u_i u_{i+1}) = 0; 1 \leq i \leq n-1$,
$f^*(v_0 v_i) = 0; 1 \leq i \leq n$,
$f^*(u_i v_{i+1}) = 1; 1 \leq i \leq n-1$,
$f^*(u_i v_i) = 1; 1 \leq i \leq n$.
Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the lotus inside a circleLC_n is a square difference edge cordial graph.

Example 2.16 Let the lotus inside a circle LC_4 be considered. The square difference edge cordial labeling pattern is shown in Figure 2.8.

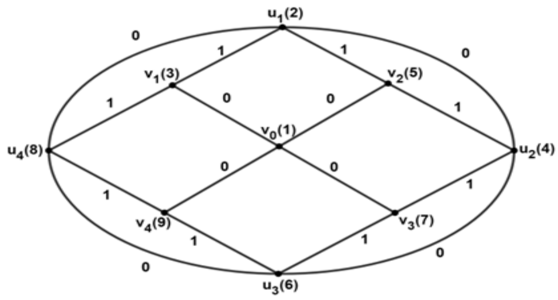

Figure 8: Square difference edge cordial graph of LC_4

Clearly,$|e_f(0)| = 8$ and $|e_f(1)| = 8$. Thus, $|e_f(0) - e_f(1)| \leq 1$. Hence the lotus inside a circle LC_4 admits a square difference edge cordial labeling. Therefore the lotus inside a circle LC_4 is a square difference edge cordial graph.

Theorem 2.17 The bistar$B_{n,n}$is a square difference edge cordial graph.
Proof. Let G be the bistar graph$B_{n,n}$.
LetV(G) =$\{u, v, u_i, v_i / 1 \leq i \leq n\}$and
E(G) =$\{uv, vv_i, uu_i / 1 \leq i \leq n\}$. So,|V(G)|= 2n+2 and|E(G)|= 2n+1.
A function$f : V(G) \rightarrow \{1,2,3,...,2n+2\}$ is defined as follows:
$f(u) = 1$,
$f(v) = n+2$,
$f(u_i) = i+1; 1 \leq i \leq n$,
$f(v_i) = n+2+i; 1 \leq i \leq n$.
The induced edge labels are

$$f^*(uv) = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if n is even} \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(uu_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases}$$
$$f^*(vv_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases}$$

Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the bistar graph$B_{n,n}$ is a square difference edge cordial graph.

Example 2.18 Let the bistar $B_{5,5}$be considered. The square difference edge cordial labeling pattern is shown in Figure 2.9.

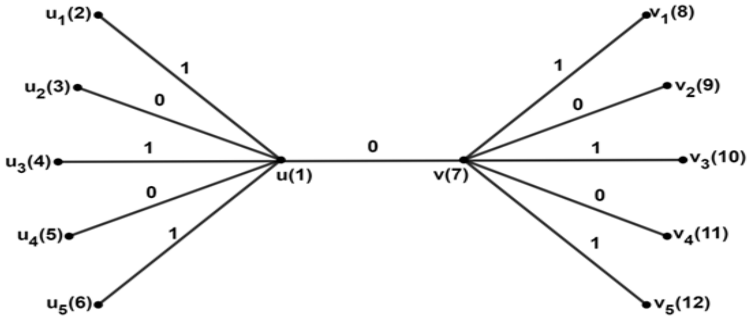

Figure 9: Square difference edge cordial graph of $B_{5,5}$

Clearly,$|e_f(0)| = 5$ and$|e_f(1)| = 6$. Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the bistar $B_{5,5}$ admits a square difference edge cordial labeling. Therefore the bistar $B_{5,5}$ is a square difference edge cordial graph.

Theorem 2.19 The graph $(K_n - e)_n$is a square difference edge cordial graph.
Proof. Let G be the graph $(K_n - e)_n$.
Let$V(G) = \{u_i, v_i \ / \ 1 \leq i \leq n\}$and
$E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} \ / \ 1 \leq i \leq n-1\} \cup \{u_i v_i \ / \ 1 \leq i \leq n\}$.
So, $|V(G)| = 2n$ and $|E(G)| = 4n-3$.
A function$f : V(G) \rightarrow \{1,2,3,...,2n\}$ is defined as follows:
$f(u_i) = 2i-1; 1 \leq i \leq n$,
$f(v_i) = 2i; 1 \leq i \leq n$.
The induced edge labels are
$f^*(u_i u_{i+1}) = 0; 1 \leq i \leq n-1$,
$f^*(v_i v_{i+1}) = 0; 1 \leq i \leq n-1$,
$f^*(u_i v_i) = 1; 1 \leq i \leq n$,
$f^*(u_i v_{i+1}) = 1; 1 \leq i \leq n-1$.
Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the graph $(K_n - e)_n$ is a square difference edge cordial graph.

Example 2.20 Let the graph $(K_4 - e)_7$ be considered. The square difference edge cordial labeling pattern is shown in Figure 2.10.

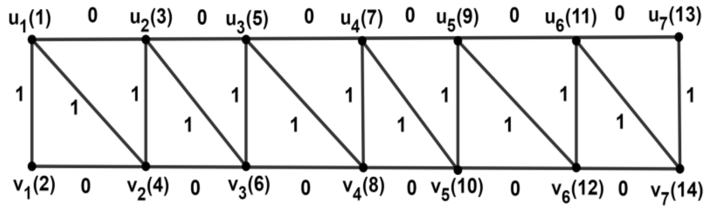

Figure 10: Square difference edge cordial graph of $(K_4 - e)_7$

Clearly,$|e_f(0)| = 5$ and$|e_f(1)| = 6$. Thus,$|e_f(0) - e_f(1)| \leq 1$. Hence the graph $(K_4 - e)_7$admits a square difference edge cordial labeling. Therefore the graph $(K_4 - e)_7$is a square difference edge cordial graph.

Theorem 2.21 The jewel graph J_n is a square difference edge cordial graph.
Proof. Let G be the jewel graph J_n.
Let $V(G) = \{u, v, x, y, v_i \ / \ 1 \leq i \leq n\}$and $E(G) = \{ux, uy, xy, xv, yv, uv_i, vv_i \ / \ 1 \leq i \leq n\}$.
So, $|V(G)| = n+4$ and $|E(G)| = 2n+5$.
A function$f : V(G) \rightarrow \{1,2,3,...,n+4\}$ is defined as follows:
$f(u) = 1$,
$f(v) = 2$,
$f(x) = 3$,

$f(y) = 4$,
$f(v_i) = 4+i;\ 1 \leq i \leq n$.
Further, the induced edge labels are
$f^*(ux) = 0$,
$f^*(uy) = 1$,
$f^*(xy) = 1$,
$f^*(xv) = 1$,
$f^*(yv) = 0$,
For $1 \leq i \leq n$,

$$f^*(uv_i) = \begin{cases} 1 & \text{if i is even} \\ 0 & \text{if i is odd} \end{cases}$$

$$f^*(vv_i) = \begin{cases} 1 & \text{if i is odd} \\ 0 & \text{if i is even} \end{cases}$$

Thus, $|ef(0) - ef(1)| \leq 1$. Hence the jewel graph J_n is a square difference edge cordial graph.

Example 2.22 Let the jewel graph J_4 be considered. The square difference edge cordial labeling pattern is shown in Figure 2.11.

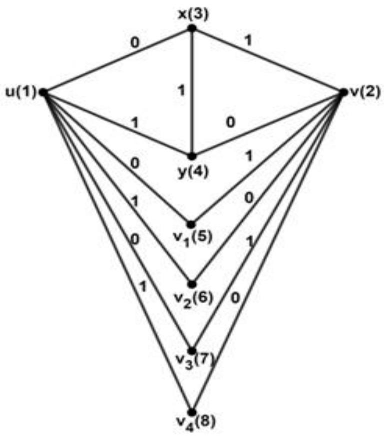

Figure 11: Square difference edge cordial graph of J_4

Obviously, $|e_f(0)| = 6$ and $|e_f(1)| = 7$. Thus, $|e_f(0) - e_f(1)| \leq 1$. Hence the jewel graph J_4 admits a square difference edge cordial labeling. Therefore the jewel graph J_4 is a square difference edge cordial graph.

CONCLUSION

In this chapter, it is found that the square difference edge cordial labeling admits for the squaregraph P_n^2, the duplication of apex vertex of a bistar, the lotus inside the circle LC_n, the bistar $B_{n,n}$, the one point union of the cycle C_3 with the star graph $K_{1,n}$(n is even), the graph $[P_n: S_2]$, the comb graph P_n^+, the jewel graph J_n, the graph $(K_n-e)_n$ and the total graph $T(P_n)$. In future,some more types of labeling for different types of graphs can be proceeded.

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