A novel approach to an integrated inventory system in a fuzzy context

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ABSTRACT
In recent decades, the integration of vendors and buyers in managing inventory has become vital. The perception of inventory costs cannot always be classified as purely static or dynamic. When encountering uncertainty, it is important to consider the fuzzy nature of costs in certain situations. Hence, in this article, we treat the cost components as triangular and trapezoidal fuzzy numbers. Defuzzification is performed using the signed distance method. Employing an algorithmic procedure, the system cost is minimized by determining the optimal values for delivery lot size and total number of deliveries. The illustration of the model includes both a numerical example and a sensitivity analysis.

Keywords: Inventory costs, Deterioration, Fuzzy numbers, Signed distance method.

INTRODUCTION

The inventory model for perishable items in fuzzy environments has not received much attention from researchers. Our study examines these phenomena in the context of the economic order quantity model, with a particular emphasis on the fuzzy cost component. The deterioration of inventory items is an essential consideration for researchers in this field that cannot be ignored. It can be categorized as decay, obsolescence, manufacturing defects, and more. Deteriorating items include vegetables, medicines, milk, oils, and so on. Several researchers have delved into the study of inventory models for deteriorating items in the past few decades.Ghare and Schrader [10] developed a inventory model that incorporates direct spoilage and exponential deterioration. The work of Covert and Philip [7] further developed Ghare and Schrader's [10] findings by employing the Weibull and Gamma distributions. The deterioration rate was modeled by Philip [18] using a three-parameter Weibull distribution. Mishra [17] devised an inventory model that incorporates both variable deterioration rates and a finite production rate. Shah [24] proposed a model for determining lot sizes that considers order levels, exponential and Weibull deterioration distributions, and backordering. Raafat [21] provided a comprehensive survey of the literature on inventory models addressing deteriorating items. In this field, researchers such as Goyal and Giri[12], Skouri and Papachristos [25], Skouri et al. [26], Sarkar [23], Taleizadeh [2], and Geethaand Udayakumar [8] have developed inventory models for deteriorating products by incorporating different deterioration rates.

Managing supply chain management requires the coordination of manufacturing and logistics among various decision-making entities. Numerous coordination mechanisms have been proposed in the supply chain literature. Goyal [11] introduced a joint optimization approach to minimize the total cost functions for both the buyer and the vendor. Banerjee [4] developed a combined economic lot size model suited to scenarios where a vendor supplies exact quantities to meet a purchaser's orders. Viswanathan and Piplani [29] investigated the use of common replenishment epochs for coordinating inventory across the supply chain. The production distribution inventory model developed by Yan et al. [30] considers constant deterioration. Several researchers, including Yang and Wee [31], Ben-Daya and Hariga [5], Abdul-Jalbar et al. [1], Cardenas-Barron et al. [6], Uthayakumar and Priyan [27], and Taleizadeh et al. [3], explored integrated inventory models under different assumptions.

The early literature on inventory models typically used crisp values to represent costs, despite the imprecise nature of real-world inventory costs. Probability-based approaches may not fully account for all uncertainties that can occur in an inventory system. The modeling of inventory systems incorporates fuzzy set theory to handle uncertainties that cannot be adequately accounted for using probability theory.

The origins of fuzzy set theory can be traced back to Zadeh [34]. The application of fuzzy set theory in inventory models has been explored by researchers such as Ishii and Konno [14], Yao and Lee [32], and Vijayan and Kumaran [28]. Sadjadi et al. [22] introduced a model for pricing and marketing that incorporates fuzzy parameters. Liu [15] utilized the extension principle method to propose a solution for the integrated production and marketing planning problem in fuzzy environments. Mahata and Goswami [16] employed trapezoidal and triangular fuzzy numbers in their inventory models, which address imperfect quality and shortage backordering in fuzzy environments. Priyan and Uthayakumar [20] devised an integrated production distribution inventory system for deteriorating products involving fuzzy deterioration with variable set up cost. Hemalatha and Annadurai [13] developed an integrated production-distribution model that incorporates logarithmic investments and fuzzy costs. Priyan et al. [19] developed a two-echelon supply chain model considering fuzzy deterioration and carbon emissions.Geetha and Prabha [9] developed an inventory model for postponement strategy with fuzzy costs.

The primary contribution of this article is to extend the work of Yan et al. [30] by incorporating the fuzzy nature of inventory costs and demonstrating that total inventory costs in a fuzzy environment fluctuates from that of the crisp cost. This article seeks to present a more comprehensive integrated production-distribution inventory model by incorporating fuzzy cost components. The article is organized as follows: Section 1 includes the introduction. The article's preliminary concepts are outlined in Section 2. Section 3 provides notations and assumptions. In Section 4, the model formulation is discussed, while Section 5 explains the fuzzification and defuzzification of the cost components. Numerical and sensitivity analysis is presented in Section 6 to illustrate the model. Section 7 provides the inferences derived from our research work. At last, Section 8 summarizes the conclusions of the study.

2. Preliminaries

Below are all the relevant definitions of fuzzy sets for treating fuzzy total cost in inventory.

Definition 1. A fuzzy set $\widetilde{K_n}$ defined on the set $\mathbb{R}$, is said to be a **fuzzy number** if it satisfies the following conditions:

(i) $\widetilde{K_n}$ is convex.

(ii) There exist $n_0 \in \mathbb{R}$ such that $\mu_{\widetilde{K_n}}(n_0) = 1$, (i.e., $\widetilde{K_n}$ is normal).

(iii) $\mu_{\widetilde{K_n}}$ is piecewise continuous.

(iv) $\mu_{\widetilde{K_n}}(\alpha)$must be bounded closed interval for $\alpha \in [0,1]$.

Definition 2. The α- cut of the fuzzy number $\widetilde{K}$ is represented represented by the crisp set $\widetilde{K}(\alpha) = \{x: \mu_{\widetilde{K}}(x) \geq \alpha\}$, with αbeing a value between 0 and 1.$\widetilde{K}(\alpha)$is a closed interval in the set of real numbers, represented by $[\widetilde{K_L}(\alpha), \widetilde{K_R}(\alpha)]$, and its left and right limits are referred to as the α −cuts of $\widetilde{K}$.

Definition 3. A fuzzy number $\widetilde{K}$ is classified as a **trapezoidal fuzzy number** if it is completely characterized by four crisp values (k_1, k_2, k_3, k_4) such that $k_1 < k_2 < k_3 < k_4$. Its membership function, which represents a trapezoidal shape, is defined as follows:

$$\mu_{\widetilde{K}}(x) = \begin{cases} \frac{x-k_1}{k_2-k_1}, & k_1 \leq x < k_2 \\ 1, & k_2 \leq x \leq k\_3 \\ \frac{k_4-x}{k_4-k_3}, & k_3 < x \leq k_4 \\ 0, & \text{otherwise} \end{cases}$$

Here, the fuzzy number $\widetilde{K}$ is characterized by its lower limit k_1, lower mode k_2, upper mode k_3, and higher limit k_4 . The support of the fuzzy number is the interval$[k_1, k_4]$, which represents the range of all values that are at least fairly possible. The core of the fuzzy number, where the most likely values lie, is represented by the interval $[k_2, k_3]$. The fuzzy number $\widetilde{K}$ is said to have a penumbra consisting of the intervals $[k_1, k_4]$and $[k_1, k_4]$.

Furthermore, a trapezoidal fuzzy number can be shown as $\widetilde{K} = (K - \delta_1, K - \delta_2, K + \delta_3, K + \delta_4)$where δ_i, $(i = 1,2,3,4)$ are positive values, subject to the conditions$K > \delta_1 > \delta_2$ and $\delta_3 < \delta_4$.

Definition 4.A fuzzy number $\widetilde{K}$ is classified as a **triangular fuzzy number** if it is completely characterized by four crisp values (k_1, k_2, k_3) such that $k_1 < k_2 < k_3$. Its membership function is given by:

$$\mu_{\widetilde{K}}(x) = \begin{cases} \dfrac{x-k_1}{k_2-k_1}, & k_1 \leq x \leq k_2 \\ \dfrac{k_3-x}{k_3-k_2}, & k_2 < x \leq k_3 \\ 0, & \text{otherwise} \end{cases}$$

Here, k_1, k_2 and k_3 correspond to the lower limit, mode, and upper limit of the fuzzy number, respectively. The support of the fuzzy number, denoted by the interval $[k_1, k_3]$, signifies the plausible range of values for $\widetilde{K}$.
With the conditions $K > \delta_1 > 0$ and $\delta_2 > 0$, a triangular fuzzy number may alternatively be expressed as $\widetilde{K} = (K - \delta_1, K, K + \delta_2)$, where $\delta_i, (i = 1,2)$ are positive values.

Definition 5. The Signed Distance Method
For any $a_1 \in \mathbb{R}$, the signed distance from a_1 to 0 is defined as $d(a_1, 0) = |a_1|$. If $a_1 > 0$, then the distance from a_1 to 0 is simply $d(a_1, 0) = a_1$. Conversely, if $a_1 < 0$, the distance is given by $-d(a_1, 0) = -a_1$. Let U represent the collection of fuzzy numbers over $\mathbb{R}$.
If $\widetilde{K} \in U$ is a fuzzy number, then the signed distances of the left endpoint $K_L(\alpha)$ and the right endpoint $K_R(\alpha)$ from 0 are given by $d(K_L(\alpha), 0) = K_L(\alpha)$ and $d(K_R(\alpha), 0) = K_R(\alpha)$, respectively. Therefore, the distance from the interval $[K_L(\alpha), K_R(\alpha)]$ to the origin 0 can be computed as: $d([K_L(\alpha), K_R(\alpha)], 0) = \frac{1}{2}[d(K_L(\alpha), 0) + d(K_R(\alpha), 0)] = \frac{1}{2}[K_L(\alpha) + K_R(\alpha)]$.
For each $\alpha \in [0, 1]$, there exists a one-to-one correspondence between the crisp interval $[K_L(\alpha), K_R(\alpha)]$ and the level α fuzzy interval $[K_L(\alpha), K_R(\alpha); \alpha]$. Consequently, the signed distance from $[K_L(\alpha), K_R(\alpha); \alpha]$ to $\tilde{0}$ is given by:

$$d([K_L(\alpha), K_R(\alpha); \alpha], 0) = \frac{1}{2}[K_L(\alpha) + K_R(\alpha)].$$

Since $\widetilde{K} \in U$, the values $K_L(\alpha)$ and $K_R(\alpha)$ exist and are integrable for $\alpha \in [0, 1]$. By applying the decomposition theorem [33], the fuzzy number $\widetilde{K}$ can be expressed as:

$$\widetilde{K} = \bigcup_{0 \leq \alpha \leq 1} [K_L(\alpha), K_R(\alpha); \alpha].$$

For any fuzzy number $\widetilde{K} \in U$, the signed distance from $\widetilde{K}$ to $\tilde{0}$ is defined as:

$$d(\widetilde{K}, \tilde{0}) = \frac{1}{2}\int_0^1 [K_L(\alpha) + K_R(\alpha)]\, d\alpha . \qquad (1)$$

Furthermore, the linearity property of the distance operator d holds (as shown in Vijayan and Kumaran[28]): for n fuzzy numbers $\widetilde{A}_i$ (where $(i = 1, 2, \dots, n)$) and real constants b_i (where $(i = 1, 2, \dots, n)$, it follows that:

$$d\left(\sum_{i=1}^{n} b_i \widetilde{K}_i, \tilde{0}\right) = \sum_{i=1}^{n} b_i d(\widetilde{K}_i, \tilde{0}) \qquad (2)$$

3. Notations and Assumptions

A mathematical model has been developed by employing notations and assumptions that resemble those of Yan et al. [30].

3.1. Notations

n_1	Number of shipments in each production batch,
q_1	Lot size for deliveries (units),
θ_{dr}	Deterioration rate,
C_s	Setup cost for manufacturing a production batch,
P	Production rate (unit/unit time),
A_{oc}	Ordering costs incurred by the buyer,
D_c	Constant demand (units/unit time),
F_c	Constant transportation cost per delivery (\$/delivery),
V	The unit variable cost for order handling and receiving (\$/unit),
C_d	Cost of deterioration per unit (\$/unit),
h_{sc}	Cost of holding inventory for the supplier (\$/unit/unit time),
h_{bc}	Cost of holding inventory for the buyer(\$/unit/unit time),
S_{buy}	Area below the inventory level curve for the buyer,
S_{sup}	Area below the inventory level curve for the supplier,
T	Length of inventory cycle,
T_1	Production time for the supplier,
T_2	Non-production time for the supplier,
T_3	Interval of time between successive deliveries made to the buyer,
TC_1	The average total cost incurred by the buyer and supplier in a crisp system,
$\widetilde{TC}_1$	The average total cost incurred by the buyer and supplier in a fuzzy system,

3.2. Assumptions

i. The supplier's production rate and the buyer's demand rate are constant.
ii. The cost associated with deteriorating items remains constant.

iii. The rate of production exceeds the rate of demand.
iv. Shortages are not permitted.
v. Transportation and order handling costs are to be paid by the buyer.
vi. The deterioration of the item is always proportional to the existing inventory.

4. Model Formulation

In the proposed scenario, the buyer's warehouse receives a fixed quantity of products from the supplier at regular intervals. Each delivery arrives precisely when the previous stock has been depleted, ensuring no delay. Figure 1 and Figure 2 illustrate the inventory levels over time for both the buyer and the supplier. There are two components that make up the total cycle time T: T_1, which is the time for production by the supplier, and T_2, which is the time for non-production. Let T_3 denote the time elapsed between two consecutive deliveries. The decision variables in this model consist of the delivery lot-size q_1 and the number of deliveries n. The average total cost includes fixed setup cost, holding cost for supplier and buyer, deterioration cost for supplier and buyer, ordering cost, and transportation and handling cost. It is expressed as

$$TC_1(q_1, n_1) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)(A_{oc} + C_s + n_1 F_c + V n_1 q_1) + \frac{q_1}{2}\left[(h_{bc} + C_d\theta_{dr}) + (h_{bc} + C_d\theta_{dr})\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \quad (3)$$

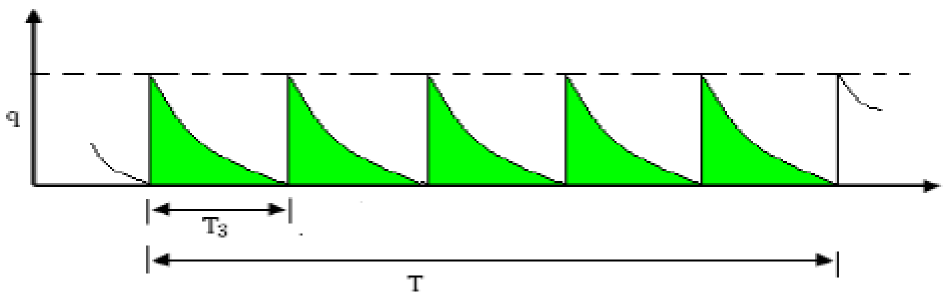

Figure 1: Time vs Inventory level (Buyer)

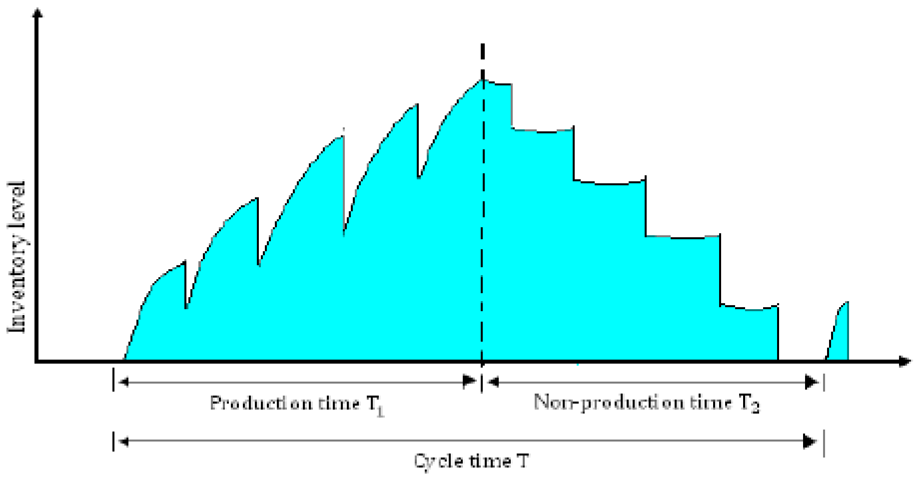

Figure 2: Time vs Inventory level (Supplier)

5. Fuzzification and Defuzzification of the cost components

The cost components outlined below are fuzzified with the use of triangular and trapezoidal fuzzy numbers: Ordering cost A_{oc}, deterioration cost C_d, buyer's holding cost h_{bc}, supplier's holding cost h_{sc}, set up cost C_s. These costs are defuzzified using signed distance method.

5.1.Obtaining Fuzzified cost components using Trapezoidal fuzzy number

The cost components such as ordering cost A_{oc}, deterioration cost C_d, buyer's holding cost h_{bc}, supplier's holding cost h_{sc}, set up cost C_s are fuzzified using trapezoidal fuzzy number.

$$\widetilde{A_{oc}} = (A_{oc} - \gamma_1, A_{oc} - \gamma_2, A_{oc} + \gamma_3, A_{oc} + \gamma_4)$$
$$\widetilde{C_s} = (C_s - \gamma_5, C_s - \gamma_6, C_s + \gamma_7, C_s + \gamma_8)$$
$$\widetilde{C_d} = (C_d - \gamma_9, C_d - \gamma_{10}, C_d + \gamma_{11}, C_d + \gamma_{12}) \quad (4)$$
$$\widetilde{h_{bc}} = (h_{bc} - \gamma_{13}, h_{bc} - \gamma_{14}, h_{bc} + \gamma_{15}, h_{bc} + \gamma_{16})$$
$$\widetilde{h_{sc}} = (h_{sc} - \gamma_{17}, h_{sc} - \gamma_{18}, h_{sc} + \gamma_{19}, h_{sc} + \gamma_{20})$$

The arbitrary positive numbers γ_l, where $l = 1,2,3, \ldots, 20$, must adhere to the following conditions: $A_{oc} > \gamma_1 > \gamma_2, \gamma_3 < \gamma_4$; $C_s > \gamma_5 > \gamma_6, \gamma_7 < \gamma_8$; $C_d > \gamma_9 > \gamma_{10}, \gamma_{11} < \gamma_{12}$; $h_{bc} > \gamma_{13} > \gamma_{14}, \gamma_{15} < \gamma_{14}$; $h_{sc} > \gamma_{17} > \gamma_{18}, \gamma_{19} < \gamma_{20}$.

In equation (5), the costs A_{oc}, C_s, C_d, h_{sc} and h_{bc} from equation (3) are fuzzified as $\widetilde{A_{oc}}, \widetilde{C_s}, \widetilde{C_d}, \widetilde{h_{bc}}$ and $\widetilde{h_{sc}}$. The average total cost for both the buyer and supplier is expressed in a fuzzy sense as

$$\widetilde{TC}_1(q_1, n_1) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)\left(\tilde{A}_{oc} + \tilde{C}_s + n_1 F_c + V n_1 q_1\right) + \frac{q_1}{2}\left[\left(\tilde{h}_{bc} + \tilde{C}_{dr}\theta_{dr}\right) + \left(\tilde{h}_{sc} + \tilde{C}_d \theta_{dr}\right)\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \quad (5)$$

5.2. Defuzzification

Below are the left and right limits of α cuts for $\tilde{A}_{oc}, \tilde{C}_s, \tilde{C}_{dr}, \tilde{h}_{bc}$ and $\tilde{h}_{sc}$.

$$\tilde{A}_{ocL}(\alpha) = A_{oc} - \gamma_1 + (\gamma_1 - \gamma_2)\alpha > 0, \tilde{A}_{ocR}(\alpha) = A_{oc} - \gamma_4 + (\gamma_4 - \gamma_3)\alpha > 0$$
$$\widetilde{C_{sL}}(\alpha) = C_s - \gamma_5 + (\gamma_5 - \gamma_6)\alpha > 0, \quad \widetilde{C_{sR}}(\alpha) = C_s - \gamma_8 + (\gamma_8 - \gamma_7)\alpha > 0$$
$$\widetilde{C_{dL}}(\alpha) = C_d - \gamma_9 + (\gamma_9 - \gamma_{10})\alpha > 0, \quad \widetilde{C_{dR}}(\alpha) = C_d - \gamma_{12} + (\gamma_{12} - \gamma_{11})\alpha > 0$$
$$\tilde{h}_{bcL}(\alpha) = h_{bc} - \gamma_{13} + (\gamma_{13} - \gamma_{14})\alpha > 0, \quad \tilde{h}_{bcR}(\alpha) = h_{bc} - \gamma_{16} + (\gamma_{16} - \gamma_{15})\alpha > 0$$
$$\tilde{h}_{scL}(\alpha) = h_{sc} - \gamma_{17} + (\gamma_{17} - \gamma_{18})\alpha > 0, \quad \tilde{h}_{scR}(\alpha) = h_{sc} - \gamma_{20} + (\gamma_{20} - \gamma_{19})\alpha > 0 \quad (6)$$

The left and right limits of α- cuts ,$(0 \leq \alpha \leq 1)$, for the fuzzified cost function are determined by the following equations

$$\widetilde{TC}_1(q_1, n_1)_L(\alpha) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)\left(\tilde{A}_{ocL}(\alpha) + \tilde{C}_{sL}(\alpha) + n_1 F_c + V n_1 q_1\right) + \frac{q_1}{2}\left[\left(\tilde{h}_{bcL} + \tilde{C}_{dLr}\theta_{dr}\right) + \left(\tilde{h}_{scL} + \tilde{C}_{dL}\theta_{dr}\right)\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \quad (7)$$

and

$$\widetilde{TC}_1(q_1, n_1)_R(\alpha) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)\left(\tilde{A}_{ocR}(\alpha) + \tilde{C}_{sR}(\alpha) + n_1 F_c + V n_1 q_1\right) + \frac{q_1}{2}\left[\left(\tilde{h}_{bcR} + \tilde{C}_{dRr}\theta_{dr}\right) + \left(\tilde{h}_{scR} + \tilde{C}_{dR}\theta_{dr}\right)\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \quad (8)$$

Using eqns. (1), (2), (7) and (8), the defuzzified value of $\widetilde{TC}_1(q_1, n_1)$ is represented as $d\left(\widetilde{TC}_1(q_1, n_1), \tilde{0}\right)$, denoted as $J_1(q_1, n_1)$, and is given by

$$J_1(q_1, n_1) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)(H_1 + H_2 + n_1 F_c + V n_1 q_1) + \frac{q_1}{2}\left[(H_4 + H_3\theta_{dr}) + (H_5 + H_3\theta_{dr})\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \quad (9)$$

where

$$H_1 = A_{oc} + \frac{1}{4}(\gamma_4 + \gamma_3 - \gamma_2 - \gamma_1) > 0$$
$$H_2 = C_S + \frac{1}{4}(\gamma_8 + \gamma_7 - \gamma_6 - \gamma_5) > 0$$
$$H_3 = C_d + \frac{1}{4}(\gamma_{12} + \gamma_{11} - \gamma_{10} - \gamma_9) > 0$$
$$H_4 = h_{bc} + \frac{1}{4}(\gamma_{16} + \gamma_{15} - \gamma_{14} - \gamma_{13}) > 0$$
$$H_5 = h_{sc} + \frac{1}{4}(\gamma_{20} + \gamma_{19} - \gamma_{18} - \gamma_{17}) > 0.$$

5.3. Solution Procedure

This subsection illustrates that $J_1(q_1, n_1)$ is convex with respect to both q_1 and n_1 along with an algorithm to ascertain the optimal values.

Property 1. For fixed $q_1 J_1(q_1, n_1)$ is convex in n_1.

Proof :Calculating the first and second-order partial derivatives of $J_1(q_1, n_1)$ with respect to n_1 yields the following expressions

$$\frac{\partial J_1(q_1, n_1)}{\partial n_1} = \left(-\frac{D_c}{n_1^2 q_1} - \frac{\theta_{dr}}{2n_1^2}\right)(H_1 + H_2) + \frac{q_1}{2}\left(1 - \frac{D_c}{P}\right)(H_5 + H_3\theta_{dr})$$

and $\frac{\partial^2 J_1(q_1,n_1)}{\partial n_1^2} = \left(\frac{2D_c}{n_1^3 q_1} + \frac{\theta_{dr}}{n_1^3}\right)(H_1 + H_2) > 0$

Therefore, for a fixed q_1, it follows that $J_1(q_1, n_1)$ is convex with respect to n_1.

Property 2. For fixed $n_1 J_1(q_1, n_1)$ is convex in q_1.

Proof :Calculating the first and second-order partial derivatives of $J_1(q_1, n_1)$with respect to q_1 yields the following expressions

$$\frac{\partial J_1(q_1,n_1)}{\partial q_1} = -\frac{D_c}{n_1 q_1^2}(H_1 + H_2 + n_1 F_c) + \frac{V\theta_{dr}}{2} + \frac{1}{2}\left\{(H_4 + H_3\theta_{dr}) + (H_5 + H_3\theta_{dr})\left(\frac{(2-n_1)D_c}{P} + n_1 - 1\right)\right\}.$$

and $\frac{\partial^2 J_1(q_1,n_1)}{\partial q_1^2} = \frac{2D_c}{n_1 q_1^3}(H_1 + H_2 + n_1 F_c) > 0.$

Therefore, for a fixed n_1, it follows that $J_1(q_1, n_1)$ is convex with respect to q_1.

From properties 1 and 2, it follows that the expression in equation (9) is a convex function of $q_1$and n_1. As a result, the first order partial derivative of $J_1(q_1, n_1)$ with respect to q_1 is set to zero to find the unique minimum value. This results in the following

$$\frac{\partial J_1(q_1,n_1)}{\partial q_1} = 0 \qquad (10)$$

Solving equation (10), we obtain

$$q_{11}^* = \left\{\frac{2D_c(H_1+H_2+n_1F_c)}{n_1\left(V\theta_{dr} + \left[(H_4+H_3\theta_{dr}) + (H_5+H_3\theta_{dr})\left(\frac{(2-n_1)D_c}{P}+n_1-1\right)\right]\right)}\right\}^{\frac{1}{2}} \qquad (11)$$

To determine the optimal values of $q_{11}^*$and n_1^*, we apply the following algorithm

Algorithm 1.

Step 1: Initialize $n_1 = 1$.

Step 2: Using this value of $n_1$compute q_{11} from equation (11).

Step 3: Calculate$J_1(q_1, n_1)$from equation (9).

Step 4: Increase n_1 by 1 and repeat steps 2 and 3.

Step 5: If $J_2(q_1, n_1) \le J_2(q_1, n_1 - 1)$,go back to Step 4 to continue iterating; otherwise, skip to step 6.

Step 6: If $(q_{11}^*, n_1^*) = (q_{11}^* n_1 - 1^*)$, then $J_2(q_{11}^*, n_1^*)$ represents the minimum estimate of the fuzzy cost function.

5.4.Obtaining Fuzzifiedcost components using Triangular fuzzy number

The cost components A_{oc}, C_s, C_d, h_{sc} and h_{bc} are assumed to be triangular fuzzy numbers as defined below

$$\widetilde{A_{oc}} = (A_{oc} - \beta_1,\ A_{oc}, A_{oc} + \beta_2), \qquad 0 < \beta_1 < A_{oc}, 0 < \beta_2$$
$$\widetilde{C_s} = (C_s - \beta_3, C_s, C_s + \beta_4 \qquad 0 < \beta_3 < C_s, 0 < \beta_4$$
$$\widetilde{C_d} = (C_d - \beta_5, C_d, C_d + \beta_6) \qquad 0 < \beta_5 < C_d, 0 < \beta_6 \qquad (12)$$
$$\widetilde{h_{bc}} = (h_{bc} - \beta_7, h_{bc}, h_{bc} + \beta_8) \qquad 0 < \beta_7 < h_{bc}, 0 < \beta_8$$
$$\widetilde{h_{sc}} = (h_{sc} - \beta_9, h_{sc}, h_{sc} + \beta_{20}) \qquad 0 < \beta_9 < h_{sc}, 0 < \beta_{10}$$

5.5 Defuzzification

Below are the left and right limits of α cuts for $\widetilde{A_{oc}}, \widetilde{C_s}, \widetilde{C_d}, \widetilde{h_{bc}}$ and $\widetilde{h_{sc}}$

$$\tilde{A}_{ocL}(\alpha) = A_{oc} - \beta_1 + \alpha\beta_1 > 0, \tilde{A}_{ocR}(\alpha) = A_{oc} + \beta_2 - \alpha\beta_2 > 0$$
$$\widetilde{C_{sL}}(\alpha) = C_s - \beta_3 + \alpha\beta_3 > 0, \qquad \widetilde{C_{sR}}(\alpha) = C_s + \beta_4 - \alpha\beta_4 > 0$$
$$\widetilde{C_{dL}}(\alpha) = C_d - \beta_5 + \alpha\beta_5 > 0, \qquad \widetilde{C_{dR}}(\alpha) = C_d + \beta_6 - \alpha\beta_6 > 0 \qquad (13)$$
$$\tilde{h}_{bcL}(\alpha) = h_{bc} - \beta_7 + \alpha\beta_7 > 0, \qquad \tilde{h}_{bcR}(\alpha) = h_{bc} + \beta_8 - \alpha\beta_8 > 0$$
$$\tilde{h}_{scL}(\alpha) = h_{sc} - \beta_9 + \alpha\beta_9 > 0, \qquad \tilde{h}_{scR}(\alpha) = h_{sc} + \beta_{10} - \alpha\beta_{10} > 0$$

The left and right limits of the α – cuts of the fuzzified cost function are given by equations (7) and (8). By using eqns. (1), (2), (7) and (8), the defuzzified value of $\widetilde{TC}_1(q_1, n_1)$ is represented as $d(\widetilde{TC}_1(q_1, n_1), \tilde{0})$, denoted as $J_2(q_1, n_1)$, and is expressed as follows

$$J_2(q_1, n_1) = \left(\frac{D_c}{n_1 q_1} + \frac{\theta_{dr}}{2n_1}\right)(G_1 + G_2 + n_1 F_c + V n_1 q_1) + \frac{q_1}{2}\left[(G_4 + G_3\theta_{dr}) + (G_5 + G_3\theta_{dr})\left\{\frac{(2-n_1)D_c}{P} + n_1 - 1\right\}\right] \qquad (14)$$

where $G_1 = A_{oc} + \frac{1}{4}(\beta_2 - \beta_1) > 0$

$$G_2 = C_S + \frac{1}{4}(\beta_4 - \beta_3) > 0$$
$$G_3 = C_d + \frac{1}{4}(\beta_6 - \beta_5) > 0$$
$$G_4 = h_{bc} + \frac{1}{4}(\beta_8 - \beta_7) > 0$$
$G_5 = h_{sc} + \frac{1}{4}(\beta_{10} - \beta_9) > 0.$

5.6. Solution Procedure

This subsection illustrates that $J_2(q_1, n_1)$ is convex with respect to both q_1 and n_1 along with an algorithm to ascertain the optimal values.

Property 3. For fixed q_1,$J_2(q_1, n_1)$ is convex in n_1.

Proof :Calculating the first and second-order partial derivatives of $J_2(q_1, n_1)$with respect to n_1 yields the following expressions

$$\frac{\partial J_2(q_1, n_1)}{\partial n_1} = \left(-\frac{D_c}{n_1^2 q_1} - \frac{\theta_{dr}}{2n_1^2}\right)(G_1 + G_2) + \frac{q_1}{2}\left(1 - \frac{D_c}{P}\right)(G_5 + G_3\theta_{dr})$$

and $\frac{\partial^2 J_2(q_1,n_1)}{\partial n_1^2} = \left(\frac{2D_c}{n_1^3 q_1} + \frac{\theta_{dr}}{n_1^3}\right)(G_1 + G_2) > 0$

Therefore, for a fixed q_1, it follows that $J_2(q_1, n_1)$ is convex with respect to n_1.

Property 4. For fixed n_1,$J_2(q_1, n_1)$ is convex in q_1.

Proof :Calculating the first and second-order partial derivatives of $J_2(q_1, n_1)$with respect to q_1 yields the following expressions

$$\frac{\partial J_2(q_1, n_1)}{\partial q_1} = -\frac{D_c}{n_1 q_1^2}(G_1 + G_2 + n_1 F_c) + \frac{V\theta_{dr}}{2} + \frac{1}{2}\left\{(G_4 + G_3\theta_{dr}) + (G_5 + G_3\theta_{dr})\left(\frac{(2 - n_1)D_c}{P} + n_1 - 1\right)\right\}.$$

and $\frac{\partial^2 J_2(q_1,n_1)}{\partial q_1^2} = \frac{2D_c}{n_1 q_1^3}(G_1 + G_2 + n_1 F_c) > 0.$

Therefore, for a fixed n_1, it follows that $J_2(q_1, n_1)$ is convex with respect to q_1.

From properties 3 and 4, it follows that the expression in equation (14) is a convex function of $q_1$and n_1. As a result, the first order partial derivative of $J_2(q_1, n_1)$ with respect to q_1 is set to zero to find the unique minimum value. This results in the following

$$\frac{\partial J_2(q_1,n_1)}{\partial q_1} = 0 \qquad (15)$$

Solving equation (10), we obtain

$$q_{12}^* = \left\{\frac{2D_c(G_1+G_2+n_1F_c)}{n_1\left(V\theta_{dr} + \left[(G_4+G_3\theta_{dr})+(G_5+G_3\theta_{dr})\left(\frac{(2-n_1)D_c}{P}+n_1-1\right)\right]\right)}\right\}^{\frac{1}{2}} \qquad (16)$$

To determine the optimal values of $q_{12}^*$and n_1^*, we apply the following algorithm

Algorithm 2.

Step 1: Initialize $n_1 = 1$.

Step 2: Using this value of $n_1$compute q_{12} from equation (16).

Step 3: Calculate$J_2(q_1, n_1)$ from equation (14).

Step 4: Increase n_1 by 1 and repeat steps 2 and 3.

Step 5: If $J_2(q_1, n_1) \leq J_2(q_1, n_1 - 1\,)$,go back to Step 4 to continue iterating; otherwise, skip to step 6.

Step 6: If $(q_{12}^*, n_1^*) = (q_{12}^* n_1 - 1^*)$, then $J_2(q_{12}^*\,, n^*)$ represents the minimum estimate of the fuzzy cost function.

6. Numerical example and Sensitivity Analysis

The impact of the level of fuzziness in the cost components on the decision variables is assessed through extensive numerical analysis. The solution to the given example is obtained using MATLAB software. Let us examine the two-echelon supply chain inventory system with the following parameters: D_c=4800units/year, P =19200 units/year, F_c = \$50/delivery and V=\$1/unit, In addition, the crisp costs A_{oc} =\$25 per order, C_s=\$600 per batch, C_d =\$50/unit,
h_{sc}=\$8/unit/year and h_{bc} =\$10/unit/year.

To fuzzify the cost parameters $A_{oc}, C_s, C_d, h_{sc}, h_{bc}$ trapezoidal and triangular fuzzy numbers are utilized. The signed distance method is then applied to determine the defuzzified values. In Tables(1)and(2),$\widehat{A_{oc}}, \hat{C}_s, \hat{C}_d, \hat{h}_{sc}$ *and* $\hat{h}_{bc}$along with $d(\tilde{A}_{oc}, \tilde{0}), d(\tilde{C}_s, \tilde{0}), d(\tilde{C}_d, \tilde{0}), d(\tilde{h}_{sc}\,, \tilde{0})$ and$d(\tilde{h}_{bc}, \tilde{0})$ represent the corresponding percentage differences between the defuzzified value and crisp values and the corresponding defuzzified value respectively. When the degree of fuzziness in all the cost parameter is '0' it corresponds to the crisp case. The third row in Tables (3) – (4) pertains to the crisp case and displays the percentage changes in the defuzzified values of the costs. Section 5 presents the proposed algorithm that computes the optimal order quantity q_1^*, total number of shipments n_1^*, and fuzzy costs for each set of fuzzy numbers. Tables (3) and (4) provide a summary of these results. By using trapezoidal and triangular fuzzy numbers to represent cost components with varying levels of fuzziness, Figures 3 and 4 illustrate the impact of the deterioration rate on total cost.

7. Managerial implications

As shown in Tables (3) and (4), an increase in the deterioration rate results in a decrease in both the optimal order quantity and cycle time, while the total cost increases. Figures 3 and 4further illustrate that as the deterioration rate rises, the total cost consistently increases, while the number of deliveries remains stable. This indicates that suppliers should implement strategies to reduce deterioration rates, as doing so could substantially reduce total costs. Additionally, this finding suggests that buyers may benefit from ordering smaller quantities as deterioration rates increase to minimize losses due to spoilage.
In practical situations, various inventory costs are subject to change. Therefore, it is reasonable to account for these costs in a fuzzy environment when designing an inventory model. The analysis indicates that the optimal solutions in the fuzzy environment show slight variations compared to those in the crisp environment (refer to Tables (3) and (4)). The optimal order quantity q_1, total expected cost TC_1, and cycle time T are notably affected by the degree of fuzziness in cost components. Thus, both vendors and buyers may benefit from incorporating flexibility when managing ordering costs, setup costs, deterioration costs, and holding costs from a managerial perspective.

8. CONCLUSION

An important aspect of a supply chain is the integration between vendor and buyer. Inventory models are extensively used in logistics and supply chain to minimize costs. Controlling inventory and associated costs in a supply chain is a hot topic of research. To be more realistic we have considered the inventory costs in fuzzy environment that is ordering cost, deterioration cost, holding cost for the buyer, holding cost for the supplier, setup cost are treated as fuzzy values. The fuzziness in the cost components is represented by fuzzy numbers, namely trapezoidal and triangular fuzzy numbers. The signed distance method is used to perform defuzzification. Our research results shows that the total cost of the inventory system under fuzzy environment is less than that of the crisp environment. This shows that when there is auncertainity in the parameters of the integrated model it is necessary to consider fuzzy inventory costs. The comparison between the total costs(fuzzy and crisp values) is made and the percentage change in cost components is also given with the aid of numerical analysis. Additionally, potential areas for future research in this field involve investigating extensions of this work, such as multi-echelon supply chains.

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Table 1. Fuzzification of the cost components (Trapezoidal Fuzzy Number)

$\widetilde{A_{oc}}$	$d(\tilde{A}_{oc},\tilde{0})$	$\widehat{A_{oc}}$	$\widetilde{C_s}$	$d(\tilde{C}_s,\tilde{0})$	$\widehat{C_s}$	$\widetilde{C_d}$	$d(\tilde{C}_d,\tilde{0})$	$\hat{C}_d$
(2,4,28,30)	16	-36	(100,150,415,455)	280	-30	(5,20,55,60)	35	-40
(5,7,32,40)	21	-16	(120,200,455,505)	320	-20	(5,30,55,70)	40	-20
(11,20,30,55)	29	+16	(220,290,630,780)	480	+20	(25,45,70,100)	60	+20
(10,17,30,75)	34	+36	(280,320,680,800)	520	+30	(20,40,80,120)	65	+40
(15,20,42,103)	45	+80	(320,360,790,930)	600	+50	(25,50,90,130)	70	+60

$\widetilde{h_{bc}}$	$d(\tilde{h}_{bc},\tilde{0})$	$\widehat{h_{bc}}$	$\widetilde{h_{sc}}$	$d(\tilde{h}_{sc},\tilde{0})$	$\widehat{h_{sc}}$
(1,2,11,12)	6.5	-35	(.3,1,8.1,8.2)	4.4	-45
(3,6,12,15)	9	-10	(.8,5,8.1,8.5)	5.6	-30
(2,8,15,25)	12.5	+25	(6.6,7,10,18)	10.4	+30
(5,8,13,30)	14	+40	(5,7.2,15,24)	12.8	+60
(5,7,22,30)	16	+60	(6,7.5,16,25)	13.4	+80

Table 2. Fuzzification of the cost components (Triangular Fuzzy Number)

$\widetilde{A_{oc}}$	$d(\tilde{A}_{oc},\tilde{0})$	$\widehat{A_{oc}}$	$\widetilde{C_s}$	$d(\tilde{C}_s,\tilde{0})$	$\widehat{C_s}$	$\widetilde{C_d}$	$d(\tilde{C}_d,\tilde{0})$	$\hat{C}_d$
(3,25,27)	20	-20	(70,400,410)	320	-20	(2,25,28)	20	-20
(7,25,31)	22	-12	(140,400,420)	340	-15	(6,25,32)	22	-12
(19,25,43)	28	+12	(390,400,650)	460	+15	(18,25,44)	28	+12
(22,25,48)	30	+20	(380,400,740)	480	+20	(22,25,48)	30	+20
(20,25,70)	35	+40	(380,400,900)	520	+30	(18,25,77)	35	+40

$\widetilde{h_{bc}}$	$d(\tilde{h}_{bc},\tilde{0})$	$\widehat{h_{bc}}$	$\widetilde{h_{sc}}$	$d(\tilde{h}_{sc},\tilde{0})$	$\widehat{h_{sc}}$
(1,10,12)	8.3	-17	(1,8,10)	6.8	-15
(2,10,12)	8.5	-15	(3,8,13)	7.5	-6
(4,10,13)	9.3	-7	(5,8,15)	8.5	+6
(6,10,16)	10.5	+5	(6,8,18)	9.8	+23
(7,10,18)	11.3	+13	(6,8,19)	10.5	+32

Table 3: Impact of the deterioration rate on the integrated system with fuzzy costs using trapezoidal fuzzy numbers

θ_{dr}	$\widehat{A}_{oc}$	$\widehat{C_s}$	$\hat{C}_d$	$\hat{h}_{sc}$	$\hat{h}_{bc}$	q_1^*	n_1^*	T^*	$J_1(q_1,n_1)$	$\widehat{J_1}(q_1,n_1)$
0.1	-36	-30	-40	-35	-45	318.1	2	0.1318	10336	15.5
	-16	-20	-20	-10	-30	224.0	2	0.1457	11329	07.4
	0	0	0	0	0	316.6	2	0.1372	12242	00.0
	+16	+20	+20	+25	+30	206.3	3	0.1342	14406	17.6
	+36	+30	+40	+40	+60	198.9	3	0.1294	15467	26.3
	+80	+50	+60	+60	+80	203.3	3	0.1323	16608	35.6
0.2	-36	-30	-40	-35	-45	247.6	2	0.1071	11975	09.0
	-16	-20	-20	-10	-30	191.3	2	0.0828	12487	05.1
	0	0	0	0	0	282.7	2	0.1222	13166	00.0
	+16	+20	+20	+25	+30	177.5	3	0.1153	16007	21.5
	+36	+30	+40	+40	+60	172.4	3	0.1120	17145	30.2
	+80	+50	+60	+60	+80	176.3	3	0.1145	18455	40.1

0.3	-36	-30	-40	-35	-45	247.6	2	0.1026	12288	12.2
	-16	-20	-20	-10	-30	191.3	2	0.1101	13497	3.6
	0	0	0	0	0	282.7	2	0.1112	14002	00.0
	+16	+20	+20	+25	+30	177.5	3	0.1026	17411	24.3
	+36	+30	+40	+40	+60	172.4	3	0.1001	18626	33.0
	+80	+50	+60	+60	+80	176.3	3	0.1024	20086	43.4
0.4	-36	-30	-40	-35	-45	215.2	2	0.0927	13101	11.3
	-16	-20	-20	-10	-30	154.0	2	0.0998	14405	02.4
	0	0	0	0	0	238.6	2	0.1027	14772	00.0
	+16	+20	+20	+25	+30	144.0	3	0.0933	18678	24.4
	+36	+30	+40	+40	+60	140.9	3	0.0913	19966	35.1
	+80	+50	+60	+60	+80	144.2	3	0.0935	21563	45.9
0.5	-36	-30	-40	-35	-45	198.1	2	0.0852	13843	10.6
	-16	-20	-20	-10	-30	142.1	2	0.0920	15237	01.6
	0	0	0	0	0	223.1	2	0.0958	15488	00.0
	+16	+20	+20	+25	+30	133.0	3	0.0861	19841	28.1
	+36	+30	+40	+40	+60	150.5	3	0.0974	21201	36.8
	+80	+50	+60	+60	+80	133.5	3	0.0864	22922	47.9
0.6	-36	-30	-40	-35	-45	184.5	2	0.0792	14531	10.0
	-16	-20	-20	-10	-30	179.1	2	0.0770	15989	01.0
	0	0	0	0	0	210.3	2	0.0902	16162	00.0
	+16	+20	+20	+25	+30	124.3	3	0.0804	20923	29.4
	+36	+30	+40	+40	+60	122.1	3	0.0790	22351	38.2
	+80	+50	+60	+60	+80	124.9	3	0.0808	24189	49.6
0.7	-36	-30	-40	-35	-45	173.3	2	0.0744	15173	09.6
	-16	-20	-20	-10	-30	168.6	2	0.0724	16704	00.5
	0	0	0	0	0	199.4	2	0.0854	16799	00.0
	+16	+20	+20	+25	+30	117.0	3	0.0756	21939	30.5
	+36	+30	+40	+40	+60	115.1	3	0.0744	23432	39.4
	+80	+50	+60	+60	+80	117.8	3	0.0761	25381	51.0

Cntd.

θ_{dr}	$\hat{A}_{oc}$	$\widehat{C_s}$	$\hat{C}_d$	$\hat{h}_{sc}$	$\hat{h}_{bc}$	q_1^*	n_1^*	T^*	$J_1(q_1, n_1)$	$\widehat{J_1}(q_1, n_1)$
0.8	-36	-30	-40	-35	-45	164.0	2	0.0703	15784	09.3
	-16	-20	-20	-10	-30	159.8	2	0.0685	17380	00.1
	0	0	0	0	0	190.1	2	0.0813	17406	00.0
	+16	+20	+20	+25	+30	110.9	3	0.0716	22899	31.5
	+36	+30	+40	+40	+60	109.2	3	0.0705	24455	40.5
	+80	+50	+60	+60	+80	111.8	3	0.0722	26508	52.2
0.9	-36	-30	-40	-35	-45	156.0	2	0.0668	16362	09.0
	-16	-20	-20	-10	-30	152.2	2	0.0652	18022	00.2
	0	0	0	0	0	182.0	2	0.0777	17987	00.0
	+16	+20	+20	+25	+30	105.7	3	0.0682	23812	32.4
	+36	+30	+40	+40	+60	104.1	3	0.0672	25429	41.4
	+80	+50	+60	+60	+80	106.6	3	0.0688	27582	53.3

Table 4: Impact of the deterioration rate on the integrated system with fuzzy costs using triangular fuzzy numbers

θ_{dr}	$\widehat{A}_{oc}$	$\widehat{C_s}$	$\widehat{C}_d$	$\widehat{h}_{sc}$	$\widehat{h}_{bc}$	q_1^*	n_1^*	T^*	$J_2(q_1, n_1)$	$\widehat{J_2}(q_1, n_1)$
0.1	-20	-20	-20	-15	-17	324.6	2	0.1406	10845	11.4
	-12	-15	-12	-6	-15	321.9	2	0.1395	11212	08.4
	0	0	0	0	0	316.6	2	0.1372	12242	00.0
	+12	+15	+12	+6	+7	339.2	2	0.1469	12587	02.8
	+20	+20	+20	+23	+5	326.0	2	0.1412	13222	08.0
	+40	+30	+40	+32	+13	232.0	3	0.1509	13929	13.7
0.2	-20	-20	-20	-15	-17	294.7	2	0.1273	11489	12.7
	-12	-15	-12	-6	-15	291.5	2	0.1259	11912	09.5
	0	0	0	0	0	282.7	2	0.1222	13166	00.0
	+12	+15	+12	+6	+7	304.3	2	0.1314	13516	02.6
	+20	+20	+20	+23	+5	211.9	3	0.1376	14171	07.6
	+40	+30	+40	+32	+13	208.1	3	0.1351	15012	14.0
0.3	-20	-20	-20	-15	-17	271.7	2	0.1171	12080	13.7
	-12	-15	-12	-6	-15	268.4	2	0.1157	12553	10.3
	0	0	0	0	0	257.9	2	0.1112	14002	00.0
	+12	+15	+12	+6	+7	201.1	3	0.1303	14359	02.5
	+20	+20	+20	+23	+5	194.6	3	0.1261	15032	07.3
	+40	+30	+40	+32	+13	190.3	3	0.1233	15994	14.2
0.4	-20	-20	-20	-15	-17	253.4	2	0.1090	12629	14.0
	-12	-15	-12	-6	-15	250.0	2	0.1075	13147	11.0
	0	0	0	0	0	238.6	2	0.1027	14772	00.0
	+12	+15	+12	+6	+7	186.6	3	0.1207	15128	02.4
	+20	+20	+20	+23	+5	180.9	3	0.1171	15829	07.1
	+40	+30	+40	+32	+13	176.4	3	0.1142	16900	14.4

Cntd.

	$\widehat{A}_{oc}$	$\widehat{C_s}$	$\widehat{C}_d$	$\widehat{h}_{sc}$	$\widehat{h}_{bc}$	q_1^*	n_1^*	T^*	$J_2(q_1, n_1)$	$\widehat{J_2}(q_1, n_1)$
0.5	-20	-20	-20	-15	-17	238.4	2	0.1023	13143	15.2
	-12	-15	-12	-6	-15	234.9	2	0.1008	13703	11.5
	0	0	0	0	0	223.1	2	0.0958	15488	00.0
	+12	+15	+12	+6	+7	174.8	3	0.1129	15845	02.3
	+20	+20	+20	+23	+5	169.8	3	0.1097	16574	07.0
	+40	+30	+40	+32	+13	165.2	3	0.1068	17744	14.5
0.6	-20	-20	-20	-15	-17	225.7	2	0.0967	13630	15.6
	-12	-15	-12	-6	-15	222.3	2	0.0953	14229	11.9
	0	0	0	0	0	210.3	2	0.0902	16162	00.0
	+12	+15	+12	+6	+7	165.0	3	0.1065	16520	02.2
	+20	+20	+20	+23	+5	160.4	3	0.1035	17277	06.8
	+40	+30	+40	+32	+13	155.8	3	0.1006	18539	14.7
0.7	-20	-20	-20	-15	-17	214.9	2	0.9019	14092	16.1
	-12	-15	-12	-6	-15	211.5	2	0.0905	14728	12.3
	0	0	0	0	0	199.4	2	0.0854	16799	00.0
	+12	+15	+12	+6	+7	156.7	3	0.1016	17160	02.1
	+20	+20	+20	+23	+5	152.5	3	0.0983	17943	06.8
	+40	+30	+40	+32	+13	147.9	3	0.0954	19292	14.8
0.8	-20	-20	-20	-15	-17	205.5	2	0.0878	14534	16.5
	-12	-15	-12	-6	-15	202.1	2	0.0864	15205	12.6
	0	0	0	0	0	190.1	2	0.0813	17406	00.0
	+12	+15	+12	+6	+7	149.5	3	0.0962	17770	02.0
	+20	+20	+20	+23	+5	145.7	3	0.0938	18579	06.7
	+40	+30	+40	+32	+13	141.1	3	0.0909	20009	14.9
0.9	-20	-20	-20	-15	-17	197.2	2	0.0841	14958	16.8
	-12	-15	-12	-6	-15	193.9	2	0.0827	15662	12.9
	0	0	0	0	0	182.0	2	0.0777	17987	00.0
	+12	+15	+12	+6	+7	143.2	3	0.0921	18353	02.0
	+20	+20	+20	+23	+5	139.6	3	0.0898	19188	06.6
	+40	+30	+40	+32	+13	135.2	3	0.0870	20695	15.0

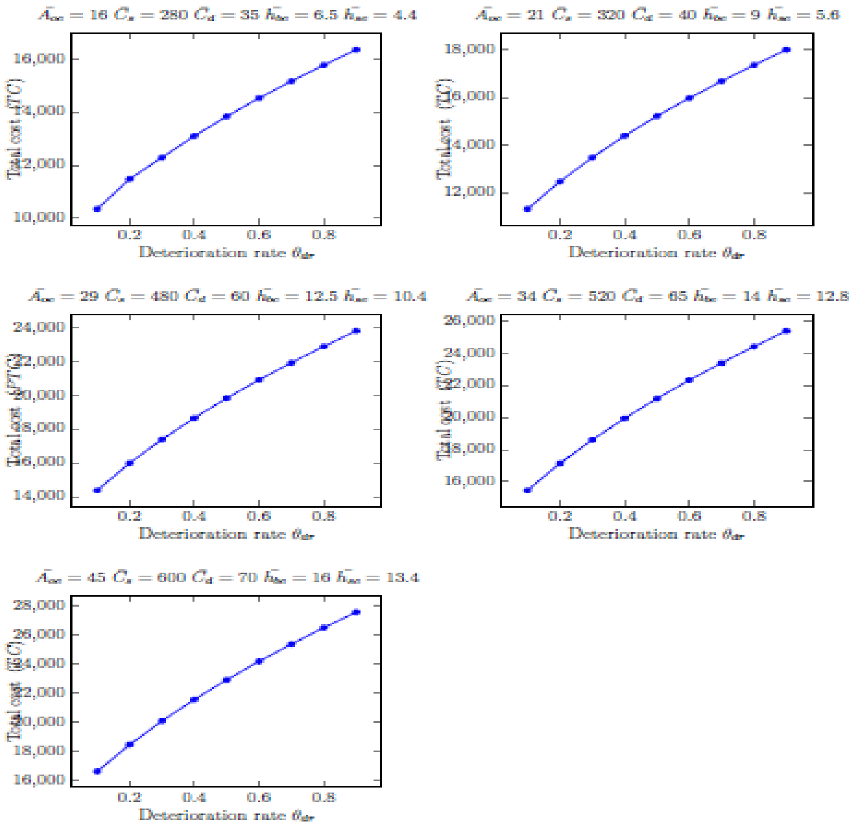

Figure 3: Deterioration rate Vs Total cost (Trapezoidal fuzzy number)

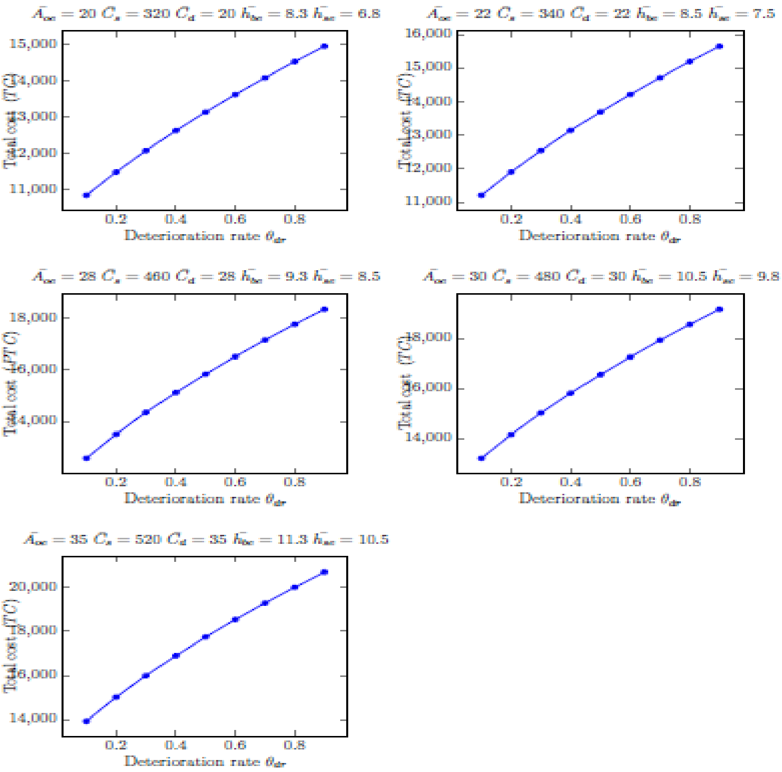

Figure 4: Deterioration rate Vs Total cost (Triangular fuzzy number)