Fixed Point Theorems for Admissible Mappings in Fuzzy Metric Spaces

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ABSTRACT
The aim of authors in this manuscript is to establish the sufficient condition to determine the fixed points for continuous mappings under (α, β)-weakly contraction mapping of type A and B in fuzzy metric spaces. To demonstrate the established result an example is also given. Our result is generalization of [5, Theorem 2.1] from metric spaces to fuzzy metric spaces.

Keywords: Fixed point theorem; Weak contraction.

1. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh [6] in 1965 to obtain a more accurate and natural method for mathematical modeling of situations which involve vagueness and uncertainty because of the existence of non-probabilistic elements. This concept was thoroughly investigated by several authors in the form of fuzzy metric space, which was originally introduced by Kramosil and Michalek [2]. Kramosil and Michalek further developed this theory since then and have shown quite a number of interesting applications for it, mostly in topology and analysis. The formal definition of the concept is as follows:

Definition 1.1: [2] A fuzzy metric space is a triple $(X, M, *)$, where $X \neq \phi$, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ such that following properties hold:

1. $M(x, y, 0) = 0, \quad \forall\, x, y \in X$;
2. $M(x, y, t) = 1, \forall\, t > 0$ iff $x = y$;
3. $M(x, y, t) = M(y, x, t), \ \forall\, x, y \in X, t > 0$;
4. $M(x, y, \cdot): [0, \infty) \to [0,1]$ is left continuous for all $x, y \in X$;
5. $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$for all $x, y, z \in X \& s, t > 0$.

This space is referred as KM - Fuzzy metric space and has been generalized by George and Veeramani [1] in the following manner:

Definition 1.2: A fuzzy metric space is a triple $(X, M, *)$, where $X \neq \phi$, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ such that following properties hold:

1. $M(x, y, t) > 0, \quad \forall\, x, y \in X, t > 0$;
2. $M(x, y, t) = 1, \forall\, t > 0$ iff $x = y$;
3. $M(x, y, t) = M(y, x, t), \forall\, x, y \in X, t > 0$;
4. $M(x, y, \cdot): (0, \infty) \to (0,1]$ is continuous for all $x, y \in X$;
5. $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$ for all $x, y, z \in X$ and $s, t > 0$.

Value of $M(x, y, t)$ is known as degree of nearness between x and y with respect to't' and from axiom (2) we can relate the value 0 & 1 of a fuzzy metric to the notions of ∞ and 0 of classical metric, respectively. The condition (5) is a fuzzy version of triangular inequality.

Example 1.3: Consider the metric space $(\mathbb{R}, d)$, where $d(x, y) = |x - y|$ is the usual Euclidean distance on the real line. Now, let us define the fuzzy set $M(x, y, t)$ as $M(x, y, t) = \frac{t}{t + |x - y|}$for $t > 0$. Now, let the maximum norm $*$ be defined as $a * b = \max\{a, b\}$. Then the triplet $(\mathbb{R}, M, *)$ forms a fuzzy metric space.
Recently, in the reeling notion of weak contraction mapping of type $A$and B, Tiwari and Som [5] have established a fixed-point result for (ϕ, ψ)-weak contraction in fuzzy metric space.

Definition 1.4: [4] Let X be a nonempty set and $\alpha: X \times X \to [0,\infty)$. We say a self mapping f on X is α-admissible if $\alpha(x,y) \geq 1$, then $\alpha(fx,fy) \geq 1$, for all $x,y \in X$.To understand the aforementioned concept, we have the following example:

Example 1.5: [4] Let $X = [0,\infty)$. Then define $f: X \to X$ such that $f(x) = \frac{x}{2}$ for all $x \in X$ and $\alpha: X \times X \to [0,\infty)$ such that

$$\alpha(x,y) = \begin{cases} 0, & x < y \\ \dfrac{1}{1+|x-y|}, & x \geq y \end{cases}$$

Clearly for $\alpha(x,y) \geq 1$, we have $\alpha(fx,fy) \geq 1$. Therefore, f is α-admissible.
The concept of generalized (α,β)-weakly contraction mapping of type A is as follow:

Definition 1.6: Let $(X,M,*)$ be a fuzzy metric space and let $\alpha,\beta: X \times X \to [0,\infty)$ be two given mappings. Assume f is a self mapping on $X$and $\psi,\phi: [0,\infty) \to [0,\infty)$, where ψ is altering distance function and ϕ is a continuous function such that $\phi(t) = 0$ iff $t = 0$. We say f is generalized (α,β)-weakly contraction mapping of type A, if $\forall x,y \in X$,

$$\psi\left(\frac{1}{M(fx,fy,t)} - 1\right) \leq \beta(x,y)\psi(m(x,y)) - \alpha(x,y)\phi\left(\max\left\{\begin{matrix}\left(\frac{1}{M(x,y,t)} - 1\right), \\ \left(\frac{1}{M(y,fy,t)} - 1\right)\end{matrix}\right\}\right) \tag{1}$$

where

$$m(x,y) = \max\left\{\left(\frac{1}{M(x,y,t)} - 1\right), \left(\frac{1}{M(x,fx,t)} - 1\right), \left(\frac{1}{M(y,fy,t)} - 1\right), \frac{1}{2}\left[\left(\frac{1}{M(x,fy,t)} - 1\right) + \left(\frac{1}{M(y,fx,t)} - 1\right)\right]\right\}$$

The concept of generalized (α,β)-weakly contraction mapping of type B is as follow:

Definition 1.7: Let $(X,M,*)$ be a fuzzy metric space $\alpha,\beta: X \times X \to [0,\infty)$ be two given mapping. Assume that f is a self-mapping on $X$and $\phi: [0,\infty) \to [0,\infty)$, where ψ is altering distance function and $\phi(t) = 0 \Leftrightarrow t = 0$. We say f is generalized (α,β)-weakly contraction mapping of type B, if $\forall x,y \in X$,

$$\psi\left(\frac{1}{M(fx,fy,t)} - 1\right) \leq \beta(x,y)\psi\big(m(x,y)\big) - \phi\left(\max\left\{\left(\frac{1}{M(x,y,t)} - 1\right), \left(\frac{1}{M(y,fy,t)} - 1\right)\right\}\right), \tag{2}$$

where

$$m(x,y) = \max\left\{\left(\frac{1}{M(x,y,t)} - 1\right), \left(\frac{1}{M(x,fx,t)} - 1\right), \left(\frac{1}{M(y,fy,t)} - 1\right), \frac{1}{2}\left(\left(\frac{1}{M(x,fy,t)} - 1\right) + \frac{1}{M(y,fx,t)} - 1\right)\right)\right\}.$$

Definition 1.8: Let X be a nonempty set and $\beta: X \times X \to [0,\infty)$. We say a mapping $f: X \to X$ is β_0 sub-admissible if for all $x,y \in X$ it satisfies the following inequalities

$$0 < \beta(x,y) \leq 1 \Longrightarrow 0 < \beta(fx,fy) \leq 1 \tag{3}$$

Definition 1.9: [3] Let X be a nonempty set. The mapping $\alpha: X \times X \to [0,\infty)$ is called forwarded transitive if $\alpha(x,y) \geq 1$and $\alpha(y,z) \geq 1$, then $\alpha(x,z) \geq 1$, for all $x,y,z \in X$.

Definition 1.10: [3] Let X be a nonempty set. The mapping $\alpha: X \times X \to [0,\infty)$ is called 0 $-$backward transitive if $0 < \alpha(x,y) \leq 1$and $0 < \alpha(y,z) \leq 1$, then $0 < \alpha(x,z) \leq 1$, for all $x,y,z \in X$.

2. Main Results

Theorem 2.1: Suppose $(X,M,*)$ is fuzzy metric space and $\alpha,\beta: X \times X \to [0,\infty)$ be two given mappings such that β is 0 $-$backward transitive and α is forward transitive. Assumethat f is generalized (α,β)-weakly contraction mapping of type A. If f is continuous, α-admissible, β_0 sub-admissible, and there exist $x_0 \in X$ such that $(x_0, fx_0) \geq 1 \geq \beta(x_0, fx_0) > 0$, then f has fixed point in X.

Proof: If we consider an arbitrary element$x_0 \in X$, then in view of the forward transitiveness of α along with the sequence $x_{n+1} = fx_n, \forall n \in ℕ \cup \{0\}$, one can find an element $n_0 \in ℕ \cup \{0\}$ such that $x_{n_0} = x_{n_0+1}$. Thus x_{n_0} is fixed point of f.

Now suppose that $x_n \neq x_{n+1} \forall n \in ℕ \cup \{0\}$. Using the condition that there exist $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1 \geq \beta(x_0, fx_0) > 0$ one have

$$\alpha(x_0, x_1) \geq 1 \geq \beta(x_0, x_1) > 0 \tag{4}$$

Since f is α-admissible,

$$\alpha(fx_0, fx_1) \geq 1 \geq \beta(fx_0, fx_1) > 0 \tag{5}$$

and hence

$$\alpha(x_1, x_2) \geq 1 \geq \beta(x_1, x_2) > 0 \quad (6)$$

Proceeding as above, we get a sequence $\{x_n\}$ in X such that $x_{n+1} = fx_n$ and

$$\alpha(x_n, x_{n+1}) \geq 1 \geq \beta(x_n, x_{n+1}) > 0 \forall n \in ▯ \cup \{0\} \quad (7)$$

Clearly, for all $n \in ▯ \cup \{0\}$, one has

$$\psi\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) = \psi\left(\frac{1}{M(fx_n, fx_{n+1}, t)}\right) \leq \beta(x_n, x_{n+1})\psi(m(x_n, x_{n+1}))$$
$$-\alpha(x_n, x_{n+1})\phi\left(\max\left\{\left(\frac{1}{M(x_n, x_n, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, fx_{n+1}, t)} - 1\right)\right\}\right)$$
$$\leq \psi(m(x_n, x_{n+1})) - \phi\left(\max\left\{\left(\frac{1}{Mx_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

where

$$m(x_n, x_{n+1}) = \max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_n, fx_n, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, fx_{n+1}, t)} - 1\right),\right.$$
$$\left.\frac{1}{2}\left(\left(\frac{1}{M(x_n, fx_{n+1}, t)} - 1\right) + \frac{1}{M(x_{n+1}, fx_n, t)} - 1\right)\right)\right\}$$
$$= \max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right),\right.$$
$$\left.\frac{1}{2}\left(\left(\frac{1}{M(x_n, x_{n+2}, t)} - 1\right) + \frac{1}{M(x_{n+1}, x_{n+1}, t)} - 1\right)\right)\right\}$$
$$= \max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right),\right.$$
$$\left.\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right), \frac{1}{2}\left(\frac{1}{M(x_n, x_{n+2}, t)} - 1\right)\right\} \quad (8)$$

Equations (7) and (8) can be utilized to obtain the following

$$\psi\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right) \leq \psi\left(\max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$
$$-\phi\left(\max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right), \forall n \in ▯ \cup \{0\} \quad (9)$$

We now aim to demonstrate that the sequence $\left\{\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right\}$ is monotonic decreasing. On contrary, for some n

$$\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right) < \left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) \quad (10)$$

In light of equation (9), one obtains

$$\psi\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right) \leq\leq \psi\left(\max\left\{\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right), \left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right)\right\}\right) \quad (11)$$

By employing equations (10) and (11), one has

$$\psi\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right) \leq \psi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) - \phi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) \quad (12)$$
$$\Rightarrow \quad \phi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) \leq 0$$

this provides that $\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) = 0$, and hence $x_{n+1} = x_{n+2}$, a contradiction. Therefore, the sequence $\left\{\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right\}$ is monotonic decreasing.

We will now prove that $\{x_n\}$ is a Cauchy sequence. To do this suppose that

$$\lim_{n\to\infty}\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) = \Lambda(t)$$

Now we shall prove that $\Lambda(t) = 0$, $\forall t > 0$. On contrary there corresponds some $t > 0$ such that $\Lambda(t) < 0$. Assuming that limit $n \to \infty$ in (12), we get

$$\psi(\Lambda(t)) \leq \psi(\Lambda(t)) - \phi(\Lambda(t))$$

this implies that $\phi(\Lambda(t)) \leq 0$, a contradiction. Therefore, $\{x_n\}$ is a Cauchy sequence. Since X is complete metric space, there exist x^* such that $x_n \to x^*$ as $n \to \infty$. By continuity of f,

$$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} fx_n = fx^*$$

Therefore, one conclude that $fx^* = x^*$.

Theorem 2.2: Suppose $(X, M, *)$ is fuzzy metric space and $\alpha, \beta: X \times X \to [0, \infty)$ be two given mappings such that β is 0 −backward transitive and α is forward transitive. Assume that f is generalized (α, β)-weakly contraction mapping of type B. If f is continuous, α-admissible, β_0 sub-admissible, and there exist $x_0 \in X$ such that $(x_0, fx_0) \geq 1 \geq \beta(x_0, fx_0) > 0$, then f has fixed point in X.

Proof: Suppose that for any element $x_0 \in X$, owing to the forward transitiveness of α and sequence $x_{n+1} = fx_n \forall n \in$ ▯ $\cup \{0\}$, we get an element $n_0 \in$ ▯ $\cup \{0\}$ with the property that $x_{n_0} = x_{n_0+1}$. Therefore, x_{n_0} is fixed point of f.

Let $x_n \neq x_{n+1}, \forall n \in$ ▯ $\cup \{0\}$. Using the condition that $x_0 \in X$ exists such that $\alpha(x_0, fx_0) \geq 1 \geq \beta(x_0, fx_0) > 0$ is satisfied, one have

$$\alpha(x_0, x_1) \geq 1 \geq \beta(x_0, x_1) > 0 \tag{13}$$

Since f is α-admissible,

$$\alpha(fx_0, fx_1) \geq 1 \geq \beta(fx_0, fx_1) > 0 \tag{14}$$

and hence

$$\alpha(x_1, x_2) \geq 1 \geq \beta(x_1, x_2) > 0 \tag{15}$$

In the similar vein, we get $\{x_n\}$ is sequence in X such that $x_{n+1} = fx_n$ and

$$\alpha(x_n, x_{n+1}) \geq 1 \geq \beta(x_n, x_{n+1}) \text{ for all } n \in ▯ \cup \{0\} \tag{16}$$

Moreover, for each $n \in$ ▯ $\cup \{0\}$,

$$m(x_n, x_{n+1}, y) = \max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t} - 1\right)\right\}$$

Since f is generalized (α, β)-weakly contraction mapping of type B, for all $n \in$ ▯ $\cup \{0\}$, we get

$$\psi\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) = \psi\left(\frac{1}{Mfx_n, fx_{n+1}, t)} - 1\right) \leq \alpha(x_n, x_{n+1})\psi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right)$$

$$\leq \beta(x_n, x_{n+1})\psi\big(m(x_n, x_{n+1})\big) - \phi\left(\max\left\{\left(\frac{1}{M(x_n, x_n, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

$$\leq \psi\big(m(x_n, x_{n+1})\big) - \phi\left(\max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

$$\leq \psi\left(\max\left\{\left(\frac{1}{M(x_n, x_n, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

$$- \phi\left(\max\left\{\left(\frac{1}{M(x_n, x_n, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right).$$

This indicates that

$$\psi\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) \leq \psi\left(\max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

$$-\phi\left(\max\left\{\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right), \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right)\right\}\right)$$

We have now demonstrated that the sequence $\left\{\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right\}$ is monotonically decreasing sequence. On contrary, for some n,

$$\left(\frac{1}{M(x_n, x_{n+1}, t)} - 1\right) < \left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) \tag{17}$$

By inequality (16), we have

$$\psi\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right) \leq \psi\left(\max\left\{\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right), \left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right)\right\}\right) \tag{18}$$

Using (17) and (18),

$$\psi\left(\frac{1}{M(fx_{n-1}, fx_n, t)} - 1\right) \leq \psi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) - \phi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) \tag{19}$$

$$\Rightarrow \phi\left(\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right) \leq 0$$

$$\Rightarrow M(x_{n+1}, x_{n+2}, t) - 1) = 1 \text{ that implies } x_{n+1} = x_{n+2}$$

a contradiction. Therefore $\left\{\frac{1}{M(fx_n, fx_{n+1}, t)} - 1\right\}$ is monotonically decreasing sequence. Now, we shall prove that $\{x_n\}$ is a Cauchy sequence. To do this first suppose that

$$\lim_{n\to\infty}\left(\frac{1}{M(x_{n+1}, x_{n+2}, t)} - 1\right) = \Lambda(t) \tag{20}$$

Now our aim is to show that $\Lambda(t) = 0, \forall t > 0$. On the contrary there corresponds some $t > 0$ such that $\Lambda(t) < 0$. Taking limit $n \to \infty$ in (17), we get

$$\psi(\Lambda(t)) \leq \psi(\Lambda(t)) - \phi(\Lambda(t)) \tag{21}$$

This implies $\phi(\Lambda(t)) \leq 0$, a contradiction, and hence $\{x_n\}$ is Cauchy sequence. Since X is complete metric space, $\exists x^*$ such that $x_n \to x^*$when $n \to \infty$. By the continuity of f, we have

$\lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} f x_n = f x^*$.

Therefore, we conclude that x^* is a fixed point.

Example 2.3: Let $X = [0,1]$ and $(X, M, *)$ be a complete fuzzy metric space. Consider $M(x,y,t) = e^{\frac{-|x-y|}{t}}, \phi(t) = \frac{t}{2}, \psi(t) = t$, and $f(x) = \frac{x}{2}$. Clearly, these functions satisfy all conditions of Theorem 2.1. Without loss of generality assume that $x > y$. Since f is contraction mapping of type A, equation (1) holds. Clearly,

$$\max = \left\{|x-y|, \frac{|x|}{2}, \frac{|y|}{2}, \frac{1}{2}\left(\left|x - \frac{y}{2}\right| + \left|y - \frac{x}{2}\right|\right)\right\} = \begin{Bmatrix} x-y, & 0 \leq y \leq \frac{x}{2} \\ \frac{x}{2}, & \frac{x}{2} \leq y \leq x \end{Bmatrix}.$$

We shall consider the cases separately;

Case1: Let $0 \leq y \leq \frac{x}{2}$. Then

$$\psi\left(e^{\frac{|x-y|}{2t}} - 1\right) = \left(e^{\frac{|x-y|}{2t}} - 1\right) \tag{22}$$

and

$$m(x,y) = \left(e^{\frac{|x-y|}{t}} - 1\right) \tag{23}$$

employing (23) in (22), we get

$$\psi(m(x,y)) = \left(e^{\frac{|x-y|}{t}} - 1\right) \tag{24}$$

Since $\phi(t) = \frac{t}{2}$,

$$\phi\left(\max\left\{\left(\frac{1}{M(x,y,t)} - 1\right), \left(\frac{1}{M(y,fy,t)} - 1\right)\right\}\right) = \left(e^{\frac{|x-y|}{t}} - 1\right) \tag{25}$$

$$\phi\left(e^{\frac{|x-y|}{t}} - 1\right) = \frac{\left(e^{\frac{|x-y|}{t}} - 1\right)}{2} \tag{26}$$

Using (1), (22) and (23), we get

$$\left(e^{\frac{|x-y|}{2t}} - 1\right) \leq \left(e^{\frac{|x-y|}{t}} - 1\right) - \frac{1}{2}\left(e^{\frac{|x-y|}{t}} - 1\right) = \frac{1}{2}\left(e^{\frac{|x-y|}{t}} - 1\right) \tag{27}$$

Case2: Let $\frac{x}{2} \leq y \leq x$. Then

$$m(x,y) = \left(e^{\frac{|x|}{2t}} - 1\right) \tag{28}$$

$$\psi(m(x,y)) = \left(e^{\frac{|x|}{2t}} - 1\right) \tag{29}$$

$$\max\left\{\left(\frac{1}{M(x,y,t)} - 1\right), \left(\frac{1}{M(y,fy,t)} - 1\right)\right\}\Big) = \left(e^{\frac{|x|}{2t}} - 1\right) \tag{30}$$

$$\phi\left(e^{\frac{|y|}{2t}} - 1\right) = \frac{\left(e^{\frac{|y|}{2t}} - 1\right)}{2} \tag{31}$$

Using (28), (29), (30), (31), we get

$$\left(e^{\frac{|x-y|}{2t}} - 1\right) \leq \left(e^{\frac{|x|}{2t}} - 1\right) - \frac{1}{2}\left(e^{\frac{|y|}{2t}} - 1\right) = e^{\frac{|x|}{2t}} - 1 - \frac{1}{2}e^{\frac{|y|}{2t}} - \frac{1}{2} = e^{\frac{|x|}{2t}} - \frac{1}{2} - \frac{1}{2}e^{\frac{|y|}{2t}} \tag{32}$$

Hence, in both cases (1) inequality holds. Therefore f has a fixed point.

CONFLICTS OF INTEREST

Authors do not have conflicts of interest.

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