# Some Results on Line-Triangle Graph of a Graph

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Received: 18.01.2022 Revised: 20.02.2022 Accepted: 24.03.2022

**ABSTRACT**
In this paper, we obtain the results determining the number of points and lines in the line-triangle graph of a graph.We characterize the graphs whose line-triangle graphs are isomorphic. Also,we present the characterization of planar line-triangle graph.

**Keywords:** k-simplex, isomorphic, line graph, line-triangle graph, maximal planar graph.

## 1. INTRODUCTION

All graphs considered here are finite, undirected and without loops or multiple lines. We use the terminology of [1].

The line graph of G, denoted L(G), is the intersection graph Ω(E), where E is the set of all lines of G. Thus, the points of L(G) are the lines of G, with two points of L(G) are adjacent whenever the corresponding lines of G are adjacent.

Manygraphs which are encountered in the study of graph theory are characterized by a type of configuration or subgraphs they possess. A simplex of a graph is any complete subgraph. A k-simplex is any simplex with exactly k-points. The middle graph M(G) of a graph G is the intersection graph of the set of all 1 or 2 – simplices of G. The simplex graph Simp(G) of a graph G is the intersection graph of the set of all complete subgraphs of G. These concepts are introduced in [2].

The line-triangle graph LT(G) of a graph Gis the intersection graph of the set of all 2 or 3 simplices of G.This concept was introduced by Biradar [4].

A graph is said to be embedded in a surface S when it is drawn on S so that no two lines intersect. A graph is planar if it can be embedded in the plane. A maximal planar graph is one to which no line can be added without losing planarity.Many other graph valued functions in graph theory were studied, for example, in [5-16]

The following will be useful in the proof of our results.

Theorem A. [1, p.72] If G is a (p, q) graph whose points have degree $d_i$ then L(G) has q points and $-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ lines.

Theorem B. [1, p.104] If G is a (p,q) maximal plane graph with r faces, then every face is a triangle and i) 2q=3r ii) q=3p–6.

Theorem C.[4]The graphs L(G) and LT(G) are isomorphic if and only if G is triangle free.

Theorem D. [1, p.109] A graph is planar if and only if it has no subgraph homeomorphic to $K_5$ or $K_{3,3}$.

Theorem E.[3] The line graph L(G) of a graph G is planar if and only if G is planar, $\Delta(G)\leq 4$ and if degv=4 for a point v of G, then v is a cutpoint.

## 2. Results determining the number of points and lines in LT(G)

When defining any class of graphs, it is desirable to know the number of points and lines in each; the next theorem determines the same.

**Theorem 1.** If G is a non-trivial (p, q) graph whose points have degree $d_i$and $t_i$be the number of triangles in Gthen LT(G) has (q+ $t_i$) points and $[-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2 + 3t_i]$ lines.

**Proof.** By definition, LT(G) is an intersection of 2 or 3-simplices of G. Since G has q number of 2-simplices and $t_i$ number of 3-simplices, LT(G) has (q+ $t_i$) points.

Now every triangle of G is a 3-simplex which corresponds to a point of LT(G) and is adjacent to three other points of LT(G) corresponding to the three lines of a triangle of G.

Also, L(G) is a subgraph of LT(G). By Theorem [A], L(G) has ( $-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ ) lines. Therefore,

The number of lines in LT(G)=[Number of lines in L(G)+3t$_i$] =[ $-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2 + 3t_i$ ]

In the next theorem, we determine the number of points and lines in LT(G) when G is a maximal planar graph.

**Theorem 2.** If G is a maximal planar graph with p≥ 3 points. Then LT(G) has (5p–11) points and [ $(3p-9)+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ ] lines.

**Proof.** Let G be a maximal plane graph with p≥ 3 points,q lines and r faces. Thenby Theorem [B], every face of G is a triangle, which is a 3-simplex.Also, we have
i) 2q=3r and ii) q=3p–6.
That is, r = (2/3)q →(1)
and q=3p–6→(2).
Now consider, the line-triangle graph LT(G) of a graph G. The number of points in LT(G) is the sum of all 2 or 3-simplices of G. Let $p_r$ and $q_r$ be the number of points and lines of LT(G) respectively.
Then $p_r$ = q+(r–1), since exterior face of G is not considered in LT(G).
$p_r$ = q+[(2/3)q –1], using (1)
$p_r$ =(5p–11), using (2).
Thus LT(G) has (5p–11) points.
Now, by Theorem 1,

$q_r$=[ $-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2 + 3t_i$ ] lines, where t$_i$be the number of triangles in G.

$q_r$=[ $-q+\frac{1}{2}\sum_{i=1}^{p} d_i^2 + 3(r-1)$ ], excluding the exterior face of G.

$q_r$=[ $(q-3)+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ ], using (1)

$q_r$ = [ $(3p-9)+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ ],using (2).

Thus, LT(G) has [ $(3p-9)+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ ] lines.

## 3. Isomorphic graphsto the line-triangle graphs

We now characterize those graphs whose line-triangle graphs are isomorphic to the graph G.

**Theorem 3.** A connected graph G and LT(G) are isomorphic if and only if G=$C_n$ ,n ≥ 4.
**Proof.** Suppose G = LT(G). We consider three cases.
**Case 1.** Assume G is acyclic. Since G is connected, it is a tree. Then G has p points and (p–1) lines. It implies L(G) has (p–1) points.Since G is acyclic, it is a triangle free graph. By Theorem [C], L(G) = LT(G). It implies LT(G) also has (p–1) points.Hence G≠LT(G), as G has p points,which is a contradiction. Hence G is a cyclic graph.
**Case 2.**Assume G is a cycle graph other than cycle. Then G≠$C_n$ , n≥ 3 but contains a cycle as its subgraph. If it is triangle free, then L(G) = LT(G) and G≠L(G), which implies G≠LT(G), which is again a contradiction. Hence G=$C_n$ ,n ≥ 3.
**Case 3.**Assume G=$C_3$. By definition LT(G) has 4 points corresponding to 3 lines (2-simplices) and a triangle (3-simplex). This implies G≠L(G), which is again a contradiction. Hence G=$C_n$ ,n ≥ 4.
Conversely, suppose G=$C_n$ ,n ≥ 4.Then G = L(G) and since G is a triangle free graph, by Theorem [C], L(G)=LT(G) and which implies G = LT(G). Hence the proof.

**4. Planarity of the line-triangle graph**

We now present characterization ofgraphs whose line-triangle graphs are planar.

**Theorem 4.**For any graph G, LT(G) is planar if and only if L(G) is planar.

**Proof.** Suppose LT(G) is planar. Since L(G) is a subgraph of LT(G), obviously L(G) is also planar.

Conversely, suppose L(G) is planar. Assume LT(G) is nonplanar. Then LT(G) has a subgraph homeomorphic from $K_5$ or $K_{3,3}$.

Since L(G) is planar,by Theorem [E], G is planar, $\Delta(G)\leq 4$ and if degv=4 for a point v of G, then v is a cutpoint.

Suppose $\Delta(G)=4$. Then G has a cutpoint v. The 4 lines incident with v form $K_4$ as a subgraph in LT(G). Even if v lies on some triangle of G, two of its incident lines will be on that triangle. Then the points corresponding to these lines and the point corresponding to the triangle form $K_4$ as a subgraph in LT(G), which is a contradiction. Thus LT(G) has no subgraph homeomorphic from $K_5$ or $K_{3,3}$. Hence LT(G) is planar.

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