Enhancing Time Series Forecasting and Uncertainty Estimation with Bayesian Neural Networks

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ABSTRACT

Forecasts of time series are quite important in economics and finance as well as in weather predictions. Traditional models like ARIMA and SARIMA face a problem in handling complex patterns; therefore, there is a need to design more complex models such as LSTM networks and Transformers. While these models have been able to overcome quite a few limitations, they still fall short when it comes to measuring uncertainty in prediction. This is required in high stake applications like stock market forecasting. Bayesian Neural Networks assist by bringing uncertainty directly into the model to offer probabilistic predictions that enhance decision making. It is focused on the integration of BNNs into time series forecasting in the stock market and compares their performance against a great combination of traditional and modern machine learning models. Results of the study show that BNNs boost accuracy in predictions and add richness and robustness to the ability to explain its prediction. This is a particularly useful property of BNNs when the tasks involve dealing with uncertainty. BNNs enable one to make informed decisions in uncertain environments, such as the stock market. It thereby provides a detailed perspective on how much reliance may be placed on the prediction. The focus of the study is to create a novel hybrid methodology combining BNN and custom Transformer-based Deep Learning model, taking advantage of the strongest capabilities each technique has to offer. The practical outcome of using BNNs in financial prediction will also be discussed, which include and are not limited to improving risk management tactics and methods of portfolio optimization.

Keywords: Time Series Forecast, Stock Market Prediction, Bayesian Neural Networks, Transformer Model, Uncertainty Estimation, Machine Learning, Deep Learning, Financial Analytics

1. INTRODUCTION

The predetermination of the future has always been pertinent to investors, analysts, and researchers involved in financial markets. The trends of the stock market are mainly predicted correctly: not in a theoretically abstracted sense but practically with economic connotations. Thus, time series forecasting can be considered crucially important in this environment to understand and predict market movements. Traditional models, that is, the reliability and simplicity of ARIMA and Seasonal ARIMA, have been in place for a long time. Traditional approaches have been at the core of predictive analytics in stock market analysis as well as other financial applications, but these are often neutralized by the more complex designs and relationships within the data. These disadvantages have been mitigated by using a hybrid approach, which combines ARIMA with neural networks [19], which represents that such models can significantly enhance the accuracy of forecasting by exploiting merits of both statistical and machine learning approaches. During the last few decades, machine and deep learning have experienced great strides, which led to the emergence of models like Long Short-Term Memory networks, Support Vector Regression, and Transformer models. These models have shown increased capacity in terms of capturing complex temporal dynamics and nonlinear interactions. However, a big gap still exists in this regard and that is the determination of the uncertainty of these predictions. Time series forecasting does involve uncertainty estimation. In high impact applications particularly, a very significant estimation of uncertainty is required, such as in the case of forecasting stock market prices. Machine learning models provide single estimates without considering the uncertainty of the predictions, which may not easily cut through risk-based challenges in certain situations. This issue addressed the development of novel approach in which dropout could be regarded as Bayesian approximation such that model uncertainty in deep learning could be evaluated using the developed method.

BNNs add uncertainty directly to the model and hence provide a useful answer. They combine the benefits of neural networks with Bayesian inference in order to produce probabilistic predictions that will be qualified by certain amounts of uncertainty. This probabilistic paradigm generates more informative forecasts and thus results in better decision-making under uncertainty. This work will examine how BNNs can be tailored to the forecasting of data from the stock market, specifically regarding the precision of predictions and, importantly, assessments of uncertainty. The performance of BNNs is compared to that of traditional statistical models as well as to current best machine learning algorithms for specific advantages and potential applications of BNNs in uncertainty-sensitive forecasting tasks. This work aims to bring trustworthiness and clarity to time series predictions by the probabilistic aspects of BNNs; such possibility will strengthen and fortify decision-making processes to be more informed and robust. The contribution lays within the application of BNNs towards time series forecasting and provides a deeper look into their value in reducing uncertainty while enhancing prediction accuracy.

The paper focuses on recent developments by investigating the real-world application of BNNs in financial time series forecasting, with an emphasis on the share market in particular, where volatility and uncertainty pose major challenges. It aims to address the limitations of current deep learning approaches and traditional statistical methods by combining BNNs with advanced machine learning algorithms like Transformers net- works and Long Short-Term Memory (LSTM), aiming to not only enhance prediction accuracy but also to offer a measurable indicator of confidence when making forecasts. It shows that BNNs enhance the capability to predict uncertainty, such as precision and interpretability in output, an especially crucial attribute in financial markets where knowledge of risk, as well as control, is crucial in informed decisions. The results show the transformative power of BNNs in time series forecasting, providing both accurate and reliable predictions-thus strengthening more robust strategies in unpredictable environments. Machine learning models currently give single estimates but do not concentrate on the uncertainty of the prediction, which presents a problem in some risk-related scenarios. [18] approached this problem by introducing a novel approach where dropout-the regularization technique-is treated as a Bayesian approximation and may thus be used to compute the model uncertainty in deep learning.

2. LITERATURE REVIEW

Within the field of financial analytics, forecasting and time series analysis have become essential tools for comprehending and identifying patterns in the stock market. Merging statistical techniques with advanced machine learning methods such as transformers and

Bayesian neural networks provides a comprehensive approach to increase the accuracy and reliability of stock market predictions.

With the widespread adoption of machine learning in finance, it is essential to look at different machine learning and deep learning models used in time series analysis and forecasting. Sonkavde et al. [1] examined machine learning and deep learning models used in finance, provides a framework for stock price prediction and classification, as well as an ensemble model for forecasting stock prices, comparing it with other popular models. Zhao et al. [2] proposed a model called the time series relational model (TSRM) which combined time series data from LSTM networks with relationship data from a graph convolutional network to predict stock prices. Chhajer et al. [3] talked about different pros and cons of utilizing machine learning and artificial intelligence for predictive analytics in the stock market.

In time series forecasting, deep learning methods such as long short-term memory (LSTM) networks and convolutional neural networks (CNNs) have become more effective instruments. These models offer improved performance in detecting long-term dependencies in sequential data. Mehtab et al. [4] constructed deep learning models based on LSTMs and models utilising CNNs to achieve a high accuracy in forecasting of stock prices. Zaheer and colleagues [5] introduced a hybrid deep-learning model that forecasts two stock indicators: the closing price and the high price for the next day. Aldhyani et al. [6] proposed a combination of CNN with LSTM and a framework based on LSTM for predicting the closing prices of Apple Inc. and Tesla Inc. When it came to stock market price prediction, the CNN-LSTM model outperformed both the single LSTM and the systems that already exist. Zhang et al. [7] used a CNN and bi-directional LSTM network to extract temporal features, an attention mechanism to automatically assign weights to the features, and a dense layer to produce the final prediction results. Abbasimehr et al. [8] proposed a hybrid model based on LSTM and multi-head attention and the results indicate the predictive efficacy of the attention mechanism in time series forecasting.

The shift from LSTM networks to transformers in time series forecasting was driven by the advantages of transformers' parallel processing and self-attention mechanisms. Sanjay et al. [9] studied the ability of transformer-based models to reliably predict future stock closing prices. This study optimized the hyperparameters of the original transformer, which was the basis for a revolutionary design, for each of the four major stock market sectors: technology, banking, pharmaceuticals, and fast-moving consumer goods. The model's performance was compared to well-known time series models like the LSTM network using test data that had not before been seen. Xu et al. [10] described a financial time series prediction model that incorporates multiplexed attention techniques with linear transformers. The linear transformer model improved training efficiency and expands forecasting capability and it minimised the complexity of the original transformer while preserving the decoder's multiplexed attention mechanism.

Muhammad et al. [11] presented a case study on Dhaka Stock Exchange (DSE) and suggested the use of the transformer model to anticipate future stock prices. It used two effective methodologies to investigate forecasting skills in DSE's volatile stock market. The experiments showed good results for the majority of the stocks. Additionally, the performance of the model is contrasted with that of the popular benchmark stock forecasting model, ARIMA, which yields acceptable outcomes. Using pricing data from the Saudi Stock Exchange, Malibari et al. [12] introduced a machine learning technique that uses Transformer neural networks to estimate stock prices. Self-attention processes were utilized in time-series data characterized by significant volatility and nonlinearity to identify nonlinear patterns and dynamics. The system predicted the closing values for the next trading day by utilizing various stock price inputs.

BNNs give probabilistic forecasts, providing important information on model uncertainty and confidence. Dezhkam and colleagues [13] outlined a method of labelling price data patterns into three categories: up, down, and no-action. The performance of our labelling system has been tested using machine learning and deep learning models and it is improved by using the Bayesian optimisation technique to identify the best hyperparameter tuning values. Chandra et al. [14] employed unique Bayesian neural networks to estimate market prices in many steps before and during COVID- 19, and investigates the relevance of pre-COVID-19 datasets for pandemic modelling. The results demonstrate that Bayesian neural networks can produce accurate forecasts with uncertainty quantification during the first peak of COVID-19, despite the increased difficulty in forecasting caused by excessive volatility. Blasco et al. [15] analyzed the existing studies on uncertainty quantification methods in deep learning for predicting the performance of financial assets such as stocks.

Pinapatruni et al. [16] investigated the effectiveness of the Bayesian LSTM Neural Network Model in stock price predicting. The performance was evaluated using ML models such as Vanilla LSTM, XGBoost, and Random Forest and the data about the public shares of five Indian companies—Reliance, Dr. Reddy, Dmart, TCS, and Hindunilvr—is gathered from YahooFinance. Ray et al. [17] suggested a hybrid method that comprises of two primary processes for forecasting multiple correlated time-series data. A multivariate Bayesian structural time series method was initially employed. Second, the technique incorporated a diagnostic post-model fitting step wherein a multi-input/output temporal convolutional network (M-TCN) with multiple time scale feature learning processes the residuals obtained from the MBSTS stage. The results showed that the hybrid model performed better than several benchmark models for forecasting accuracy.

3. RESEARCH GAPS

Advances in the field of stock market forecasting involve major changes in deep learning and machine learning models. Such progressions notwithstanding, there still exist a whole host of significant challenges yet to overcome. The most principal is building models that properly handle extremely nonlinear patterns and high volatility of financial data. Although CNNs and LSTM networks have been proved to be efficient for time-series prediction, the need for more advanced methods remains exigent to obtain a better grasp of sophisticated market behavior. Transformer models, having advanced features of self-attention mechanisms and the possibility of parallel processing, have great promises but are often too complex and inefficient if used for real-time financial tasks.

Of the major deficiencies in existing forecasting models, it is probably that these systems lean toward offering single estimates rather than managing prediction uncertainty as a prominent aspect of important financial decision-making. BNNs provide useful information regarding model uncertainty, yet such networks are often overlooked in financial prediction. There seems to exist a lack of studies into the successful implementation of BNNs with sophisticated deep learning architectures such as transformers. Further, comparative effectiveness analyses for hybrid models, which are the integration of traditional, deep learning, and Bayesian methods, are lacking empirical studies in various financial situations and market environments. Without a proper performance evaluation, our understanding is barred as to how the models tend to behave under different market conditions.

It attempts to fill the above gaps by suggesting a novel hybrid approach based on the direct combination of a custom Transformer model with Bayesian Neural Networks. The approach incorporated both the encoding ability of the Transformer for complex sequential patterns and the predictive probabilities and quantification of uncertainty of the BNN. It made use of advanced feature engineering techniques, such as various moving averages, and temporal dynamics of multiple time scales. It solves the problem of handling multiscaling in the financial data by resampling them into sequences of constant length. A comprehensive assessment system based on multiple metrics and visual comparisons underpinned high-level performance evaluation. It is trying to bring out more accurate prediction, transparency, and reliability into the model for the stock market, which has stronger solutions for fulfilling the complexities in financial analysis and decision-making.

4. METHODOLOGY

4.1 Dataset Description

This study relies on two major sets of data: the data set on stock prices and the data set on stock trading volume. The stock prices dataset is an arrangement of information about stock prices chronologically, having every day's closing prices of various stocks. The stock volume dataset gives daily trading volumes for listed companies. A detailed perspective of the performance of the stocks can be obtained by putting these datasets together by date. Some of the key columns to consider are Date, Close (closing price), and Volume.

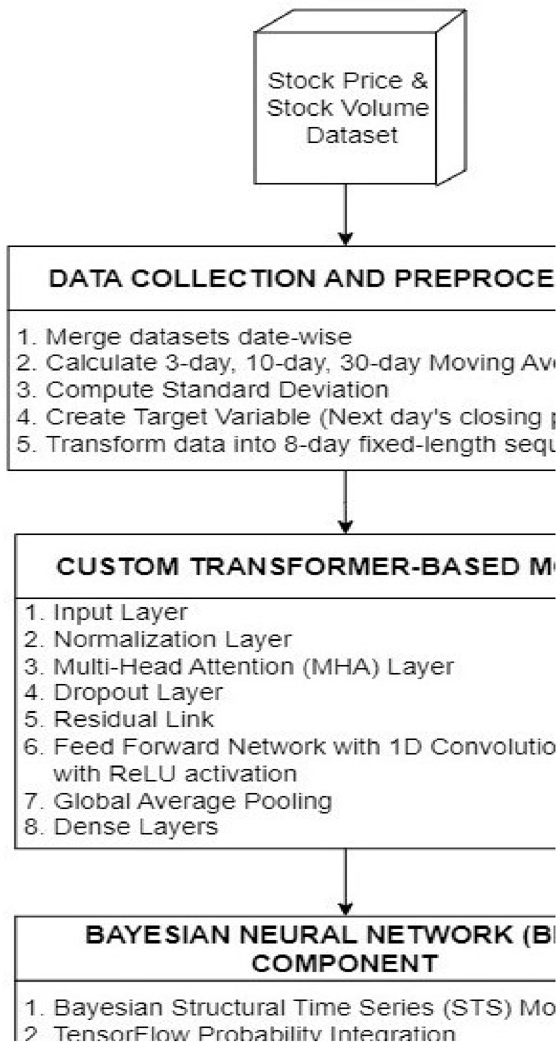
A critical aspect of data preparation is the creation of the target variable using a trading window approach. The 'Close' price column shifts by one day when the target variable is set to the next day's closing price. This links present characteristics to the upcoming final cost, indicating the forecast of stock value as a supervised learning problem. Enhancing predictive abilities significantly depends on feature engineering, which involves creating the 3-day, 10-day, and 30-day moving averages as key metrics. These measurements assist in capturing price trends of different durations - short, medium, and long-term. Standard deviation measurement also accounts for price volatility. The data is transformed into fixed-length sequences of eight days, allowing the model to capture both recent and older information impacting future stock values. The dataset is split into training (80%) and test (20%) sets to ensure thorough evaluation of the model's performance and generalization capabilities.

4.2 Proposed System

This research builds on prior work in stock market forecasting by using a hybrid strategy that combines advanced deep learning techniques with BNNs to estimate stock values. The major objective is to boost the accuracy and reliability of stock market forecasts by utilizing the benefits of both machine learning and deep learning models. Measuring prediction uncertainty and capturing intricate market dynamics can be challenging in financial forecasting. The goal of this work is to address these problems.

Sequential models convert the data into fixed-length sequences of eight days, which are then used for time series forecasting. This transformation captures both recent and slightly older information that may impact future stock values by utilizing a sliding window strategy to help the model find patterns and relationships in the time series data. To test the model's performance on never-before-seen data and ensure a reliable evaluation of the model's generalization skills, the dataset is divided into training (80%) and test (20%) sets after this transformation. The model uses the Adam optimizer with a batch size of 32 and a learning rate of 0.001. Mean Squared Error (MSE), the loss function used, which considers the average of the squares of the errors between the predicted and actual stock prices.

The core is a unique Transformer architecture created especially for stock price forecasting with makes use of a hybrid model that seamlessly combines the Transformer-based deep learning with BNNs. This architecture, which combines global average pooling, thick layers, and several Transformer encoder blocks, provides a solid foundation for handling complex time-series data.



With several critical layers, the Transformer model is designed to maximize performance. The normalization layer, being the first layer in the model, is considered critical to stabilizing and speeding up the training since it ensures uniform input distributions at all levels. Thereafter, the MHA layer receives the normalized tensor. In this, many attention heads pay attention to different parts of the input sequence simultaneously, which makes it easier for the model to find interesting patterns or relationships between data. The output of MHA layer is augmented with a dropout layer for preventing overfitting. The residual link, such as adding results back to the input tensor, is an important part of this structure. This link is crucial in

making it possible to transfer data throughout the network and reduces the vanishing gradient problem, which is a major problem in deep learning models.
Then the processing pipeline for the model continues with another normalization step before this tensor passes through the feed-forward network. Its structure has enhanced the non-linear relations in the data and comprises a 1D convolutional layer with a kernel size of 1, and it applies a ReLU activation function to its output. To complement this Transformer design, Bayesian Structural Time Series (STS) modelling with TensorFlow Probability components is added into the approach. This integration does allow Bayesian probabilistic techniques and deterministic deep learning to have a strong synergy. BNN is a mix of a Probabilistic Model and a Neural Network. In a way, this new design pursues the benefits of the model characterised in stochastic modeling and neural networks that can give an even more profound and complex method of predicting stock prices. With this hybrid approach, the intricacy of research into finance-based time series is covered through the ability of the Transformer to capture long-range relationships and that of the BNN to describe uncertainty. This resulted in a model not only predicting stock prices highly accurately but also giving its users immense insight into the underlying uncertainties and probabilities associated with these predictions.

5. RESULTS

This is an evaluation of the Transformer model's performance on a task for the prediction of a stock price. With a mean absolute error of 8.8802, the transformer model thus gives a proof that the very same shows to be correct when it comes to the projection of the stock price with the help of data. The RMSE of 11.8345 of the transformer also demonstrated its ability to decrease the average of the squared differences between the predicted values and the observed ones. Another attractive feature is the significant value of R-squared, which in the case of this model is at 0.9701. This demonstrates that there is a deep correlation between real and expected stock values, ensuring that the model may explain about 97% of the stock price fluctuation. All of the above statistics illustrate how effective the transformer model is concerning capturing and predicting the fundamental patterns in the fluctuations of stock prices.

Fig. 2. Predicted vs actual values

Besides this numerical information, the effectiveness of the transformer model can be further depicted through a chart in Fig. [2] that contrasts real and predicted stock prices in the test dataset. This provides intuition on how well the model's predictions match real stock price movements over time. The graph shows how well the model, in fact, represents both the long-term market movements and short-term fluctuations by nearly matching the actual stock prices with the expected values. This graph is useful to pinpoint periods where the model may over or under-project in addition to the easier-to-read method of checking its performance.

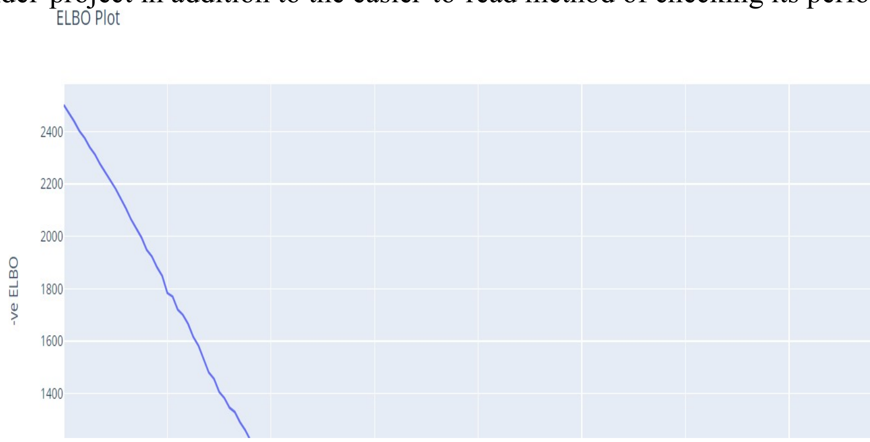


Fig. 3. ELBO convergence over iterations

For the Bayesian Neural Network (BNN), the Evidence Lower Bound (ELBO) plot as shown in Fig. [3] is prima- rily what is required to assess the performance of the model at training. ELBO succeeds in providing an estimate of how closely the model approximates the actual posterior distribution. The closer ELBO values are to the true posterior; otherwise, meaning the better the model is capturing the underlying data distribution. In the initial iteration, the graph had a very negative value close to -2400 that translates to an essentially poor approximation of the posterior distribution. This, then, translates to the drastic fall of the graph as a sign of fast adjustments to parameters in order to improve prediction. Since the flatness of the curve during these iterations means that it's a positive sign, and the model has converged, and there is not much chance of major variation, it happened after almost 80 iterations whereby the model had come to a position of negligible returns, and therefore, the train performed well enough. The ELBO plot indicates that it has converged in the process of variational inference, and as such, insights may be sought regarding its stability and effectiveness during the training procedure.

To assess the performance of the Transformer model versus the BNN, a graph, Fig. [4], showing the prediction of BNN (red) versus the Transformer as the target value (green) is utilized, providing a visual depiction of the models' capability to accurately identify patterns. The comparative analysis helps in understanding thoroughly the advantages and disadvantages of each model. Although the Transformer model is skilled at capturing intricate sequential patterns, the BNN is distinguished by its capability to measure uncertainty in predictions. In challenging financial forecasting situations, the level of agreement between the forecasts of the two models can indicate the reliability of the predictions and potentially aid in developing ensemble strategies for even more accurate predictions.

Fig. 4. Amazon forecast

Both the plots follow a similar upward trend , particularly from March 2020, demonstrating that both can capture the general direction of market's movement. However, it can be noticed that BNN's predictions are smoother as well as less volatile than Transformer's, suggesting that BNN has rather more conservative predictions. During the high volatility period of March 2020, Transformer is able to capture sharp drops and spikes, indication of ability to catch rapid changes, where the BNN lags a bit and exhibits less fluctuations. Towards the end around July 2020, predictions and target values diverge with each other. BNN's prediction increase steadily, whereas Transformer exhibits variability, also showing a sharp peak and decline in August 2020.

As mentioned, Transformer outperforms in capturing complex sequential patterns, evident from its ability to track sharp changes in target values, making it particularly useful for tasks that require emphasis on pattern recognition, and sensitivity to short-term market movements. BNN's on the other hand, less responsive to sudden changes, offer quantification of uncertainty. This is valuable in financial forecasting, where incorporating uncertainty in prediction can lead to robust and improved decision making.

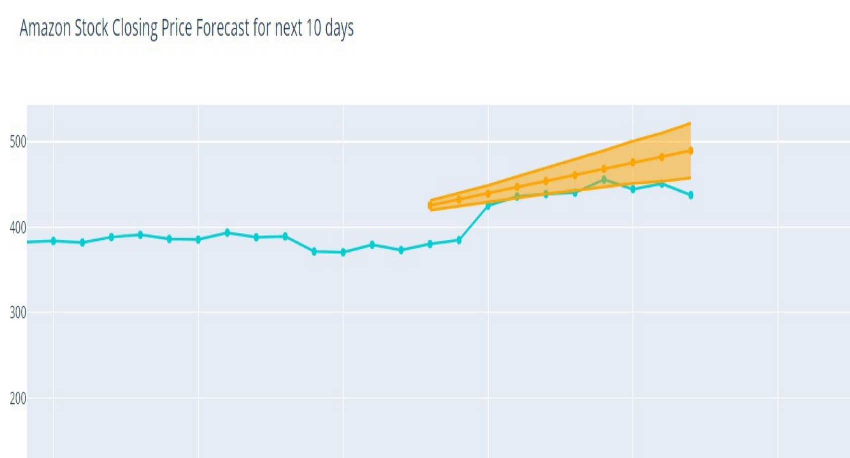


Fig. 5. Amazon comparison

Fig [5] depicts the forecast of the stock closing prices for 10 days, comparing model's forecast with ground truth. [5] shoes two lines, forecast spectrum (in orange) and ground truth (in cyan) indicating actual observed stock prices. In the start, forecast follows ground truth, but as the prediction extends, it begins to diverge, towards the latter part of 10 day period. It starts overestimating the stock prices, highlighting potential bias or limitation when predicting in the future.

This observation highlights the critical role of uncertainty quantification in financial forecasting, enabling better risk management and decision-making for investors. Future work can aim on exploring the integration of ensemble learning techniques or more advanced models to enhance the accuracy and reliability of stock price forecasts, particularly over longer time horizons.

6. DISCUSSION AND CONCLUSION

This research in breadth showed that if combined with a Transformer-based deep learning model, BNNs yield promising results regarding the prediction of stock prices. Taking into account the combination of both approaches, in the new technique, is most effective in dealing with challenging problems concerning financial time series data analysis. Using the custom-designed Transformer model, great predicting ability is seen, with a Mean Absolute Error at 8.8802 and a Root Mean Squared Error of 11.8345. However, the model's R-squared value is dramatically high at 0.9701, being an indicator of its performance in assuming the intricacy of the market dynamics, which explains 97% of the variance in stock prices.

The probabilistic component added on to the forecasting approach integrates Bayesian Neural Networks. This enhancement significantly boosts the capability of the model when the situation involves risk, like in the stock market, known to be highly volatile. The plot of ELBO for the BNN component indicates effective convergence of the variational inference, suggesting that the model could potentially estimate the accurate posterior distribution. It improves the accuracy of forecasts and offers important information on potential risks in predictions, making it a powerful analytical tool for financial decision-making. Further evidence for the precision and reliability of these methods is evident from visual comparisons of the forecasts produced by the Transformer and BNN models and corresponding real stock prices.

As such, there are many future research and improvement directions in this area. This includes improving the architecture of the hybrid model further expansion of the analysis towards more financial instruments and conditions, and implementation of a real-time system for prediction. Adding more information from other sources, including perhaps even sentiment analysis on social media or macroeconomic data, may even make it stronger in its predictions.

Further, this technique may enhance the predictive capability of the model. There are financial applications where clarity and interpretability are of great importance. Improvement of transparency in decision-making processes shall, therefore be highly beneficial. Performance of the model during longer term forecasting can prove to be useful input for strategic financial planning.

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