# Dominator color class dominating sets in Grid graphs

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**ABSTRACT**
Let $G = (V, E)$ be a graph. Let $\mathcal{C} = \{\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3 \ldots\ldots\ldots \mathcal{C}_x\}$ be a proper coloring of G. $\mathcal{C}$ is called dominator color class dominating set if each vertex v in G is dominated by a color class $\mathcal{C}_i \in \mathcal{C}$and each $\mathcal{C}_i \in \mathcal{C}$ is dominated by a vertex v in G. The dominator color class domination number is the minimum cardinality taken over all dominator color class dominating sets in G is denoted by $\gamma_\chi^d(G)$. In this paper, we obtain $\gamma_\chi^d(G)$ for Grid graph.

**Keywords:** Chromatic number, Domination number, Color Class Dominating set, Domination Color Class Dominating set, Color Class Domination number, Dominator ColorClass Domination number

## 1. INTRODUCTION

All graphs considered in this paper are finite, undirected graphs and we follow standard definitions of graph theory as found in [3]. Let $G = (V, E)$ be a graph of order P. The open neighborhood $N(v)$of vertex $v \in V(G)$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $N[v] = N(v) \cup \{v\}$. For a set$S \subseteq V$, the open neighborhood $N(S)$is defined to be $\cup_{v \in S} N(v)$, and the closed neighborhood of S is$N[S] = N(S) \cup S$. A subset S of V is called a dominating set if every vertex in $V - S$ is adjacent to some vertex in S. A dominating set is a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(G)$is the minimum cardinality taken over all minimal dominating sets of G. A $\gamma$-set is any minimal dominating set with cardinality $\gamma$ . A proper coloring of G is an assignment of colors to the vertices of G, such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G is denoted by $\chi(G)$. A total dominator coloring (td-coloring) of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(G)$ and is defined by the minimum number of colors needed in a total dominator coloring of G.This concept was denoted by A.Vijayalekshmi [1].

The dominator coloring of G is a proper coloring of G dominates atleast one color class. The dominator chromatic number is denoted by $\gamma_d(G)$ and it is defined by the minimum number of colors needed in a dominator coloring of G.

A dominator color class dominating set of G is a proper coloring of G with the extra property that each vertex v in G is dominated by a color class $\mathcal{C}_i \in \mathcal{C}$ and every color class $\mathcal{C}_i \in \mathcal{C}$ is dominated by a vertex in G. A dominator color class dominating set is said to be a minimal dominator color class dominating set if no proper subset of$\mathcal{C}$ is a dominator color class dominating set of G. The dominator color class domination number of G is the minimum cardinality taken over all minimal dominator color class dominating set of G is denoted by $\gamma_\chi^d(G)$. This notion was introduced by Vijayalekshmi et. al in 2021 [4].

A Cartesian product of two subgraphs $G_1$ and $G_2$ is the graph $G_1 \times G_2$ such that its vertex set is $V(G_1 \times G_2) = \{(x, y) \,/\, x \in V(G_1), y \in V(G_2)\}$ and the edge set is$E(G_1 \times G_2) = \{(x_1, x_2), (y_1, y_2) / x_1 = y_1$ and$(x_2 \times y_2) \in E(G_2)$ or $x_2 = y_2$ and $(x_1, y_1) \in E\,(G_1)\}$, where $P_n$is the path graph with n vertices. A two dimensional grid graph is the Cartesian product of path graphs $P_m$ and $P_n$.

## 2. Main Results

**Theorem 2.1**

The grid graph $G_m^n = P_m \times P_n$ has

$$\gamma_\chi^d(G_m^n) = \begin{cases} \left(\frac{mn}{2}\right) \text{ifmandnareeven} \\ \frac{(m-1)n}{2} + \left\lceil \frac{2n}{3} \right\rceil \text{ifmodd} \\ \frac{m(n-1)}{2} + \left\lceil \frac{2m}{3} \right\rceil \text{ifnodd} \\ \frac{(m-1)(n-1)}{2} + \left\lceil \frac{2(m+n-1)}{3} \right\rceil \text{ifmandnareodd} \end{cases}$$

**Proof**

Let $G_m^n = P_m \times P_n$ and$V(G_m^n) = v_j^i (i = 1 \text{ tomandj} = 1 \text{ ton})$

We consider 4 cases

**Case (i)** Let $m, n \equiv 0 \pmod 2$ Decompose $G_m^n$ in to $\frac{mn}{4}$copies of $G_2^2$ for $1 \le i \le \frac{m}{2}$and$1 \le j \le \frac{n}{2}$. Let$G_2^2 = P_2 \times P_2$. Assign two distinct colors, say (2r-1)and 2r$(1 \le r \le \frac{mn}{4})$ to the vertices say, $\left(v_{2i-1}^{2j-1}, v_{2i}^{2j}\right)$ and $\left(v_{2i-1}^{2j}, v_{2j-1}^{2j}\right)$ respectively, we have obtain $\gamma_\chi^d$ of $G_{m.}^n$So$\gamma_\chi^d(G_m^n) = 2\left(\frac{mn}{4}\right) = \frac{mn}{2}$

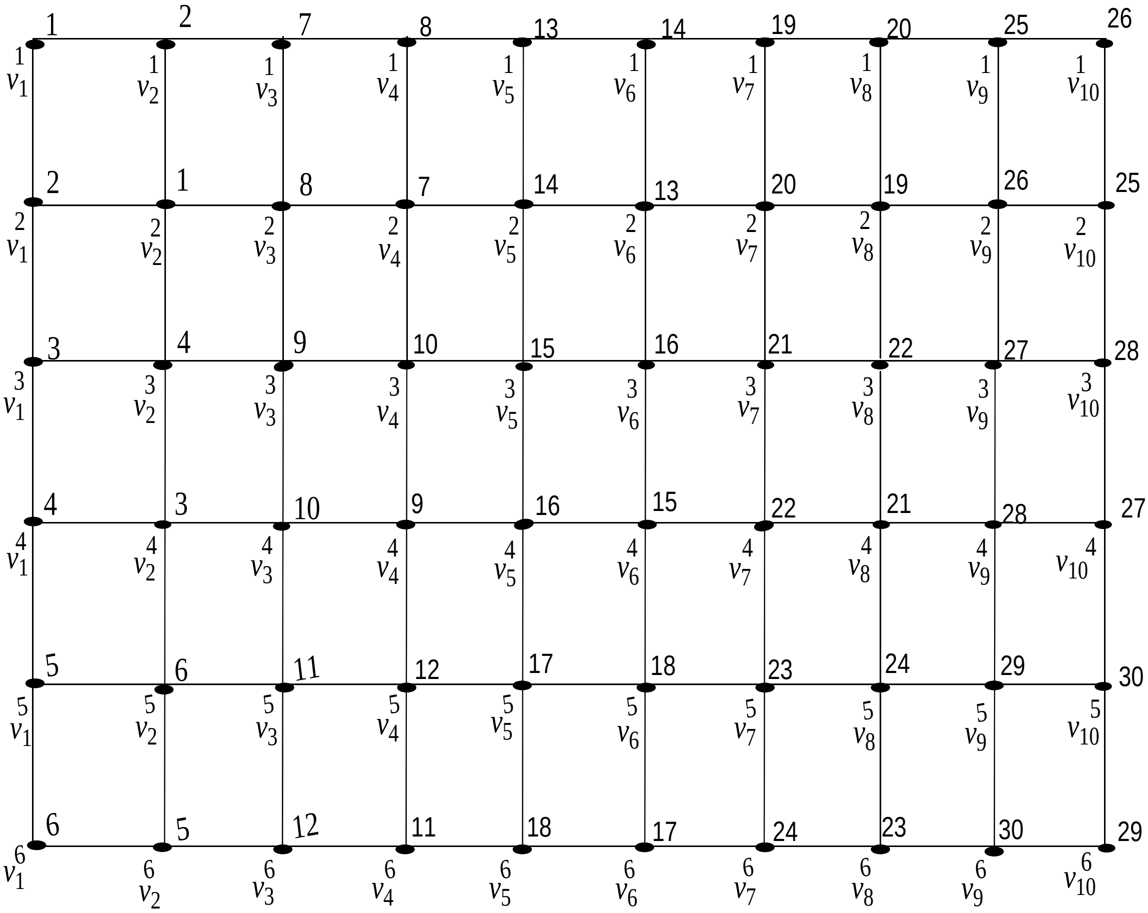


**Figure 1**

$\gamma_\chi^d(G_{10}^6) = 30$

**Case (ii)**$m \equiv 1 \pmod 2$and$n \equiv 0 \pmod 2$

Since $m - 1 \equiv 0 \pmod 2$and$n \equiv 0 \pmod 2$$G_m^n$is obtained by $G_{m-1}^n$followedby$G_1^n$

As in case (i) $\gamma_\chi^d(G_m^n) = \gamma_\chi^d(G_{m-1}^n) + \gamma_\chi^d(G_1^n) = \frac{(m-1)n}{2} + \left\lceil \frac{2n}{3} \right\rceil$

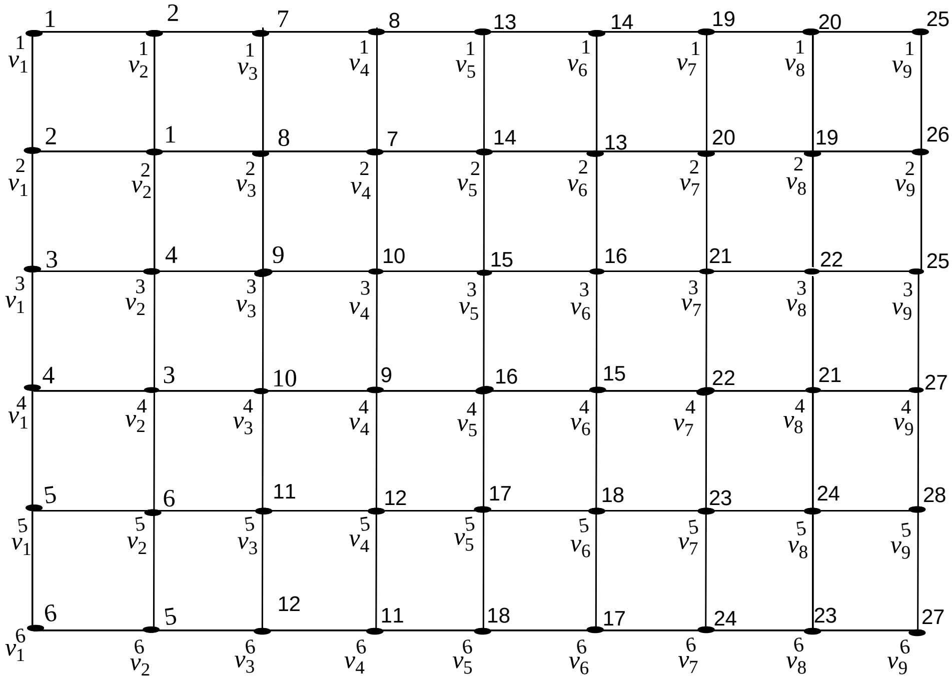


**Figure 2**

$\gamma_\chi^d(G_9^6) = 28$

**Case (iii)**m ≡ 0 (mod2)and n ≡ 1(mod2)

By case (ii), interchanging m and n, we obtain the required results.

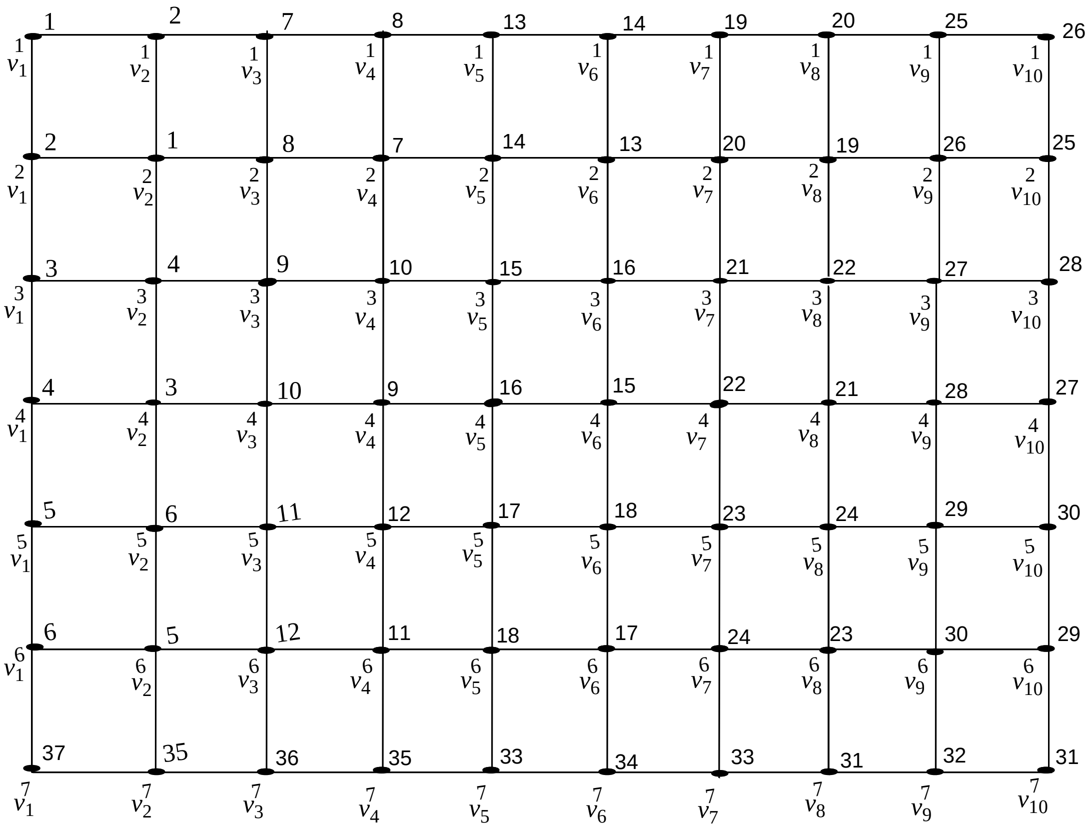


**Figure 3**

$\gamma_\chi^d(G_{10}^7) = 37$

**Case (iv)** $m \equiv 1(\mathrm{mod}2)$ and $n \equiv 1(\mathrm{mod}2)$

Since $m-1 \equiv 1(\mathrm{mod}2)$ and $n-1 \equiv 1(\mathrm{mod}2)$ $(G_m^n)$ is obtained by $(G_{m-1}^{n-1})$ followed by $(G_1^n)$ and $(G_m^1)$. So by previous cases

$$\gamma_\chi^d(G_m^n) = \gamma_\chi^d(G_{m-1}^{n-1}) + \gamma_\chi^d(G_m^1) + \gamma_\chi^d(G_1^n) = \frac{(m-1)(n-1)}{2} + \left\lceil \frac{2m}{3} \right\rceil + \left\lceil \frac{2n}{3} \right\rceil$$

$$= \frac{(m-1)(n-1)}{2} + \left\lceil \frac{2(m+n)}{3} \right\rceil$$

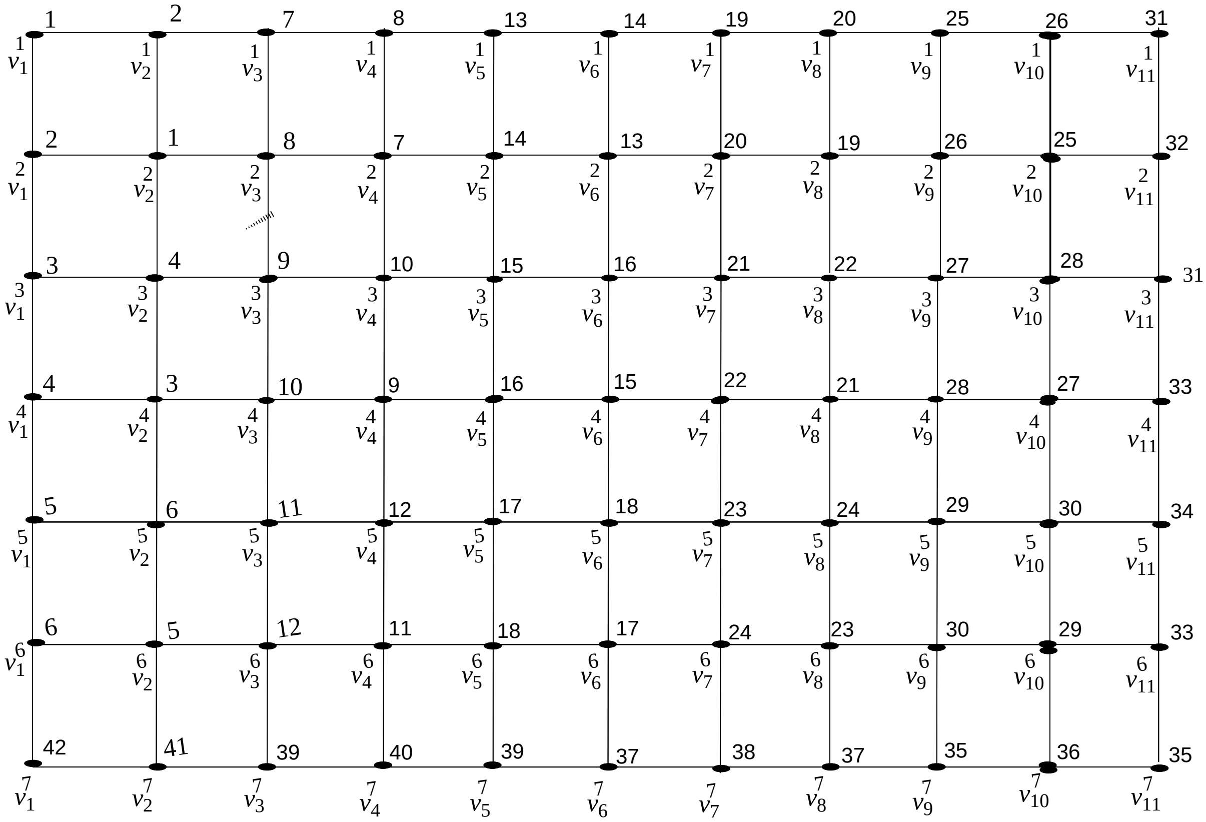


**Figure 4**

$\gamma_\chi^d(G_{11}^7) = 42$

## References

[1] A. Vijayalekshmi, Total dominator colorings in graphs, International Journal of Advancements in Research & Technology, Vol. 1, No. 4, pp. 01-06, 2012.

[2] A. Vijayalekshmi, A.E. Prabha, Introduction of color class dominating set in Graphs, Malaya Journal of Mathematik, Vol. 8, No. 4, 2186-2189, 2020.

[3] Harrary, Graph theory, Addition – Wesley Reading Mass, 1969.

[4] A. Vijayalekshi, P.Niju, An Introduction of Dominator color class dominating set in Graphs, Malaya Journal of Mathematik,Vol. 9, No.1, pp.1015-1018, 2021.

[5] P. Niju, A. Vijayalekshmi, Color Class Dominating sets in Open Ladder and Slanting Ladder Graphs, International Journal of New Innovations in Engineering and Technology, Vol. 17, No. 1, pp. 57-61, 2021.

[6] A. Vijayalekshmi & P. Niju, Color Class Dominating sets in Triangular ladder and Mobius ladder Graphs, International Journal of New innovations in Engineering and Technology, Vol. 18, No. 1, pp. 30-37, 2021.

[7] A. Vijayalekshmi, & P. Niju, Color Class Dominating sets in Umbrella Graph and Dutch Windmill Graphs, Infokara research, Vol.10, No. 12, pp. 38-48, 2021.

[8] A.Vijayalekshmi, Total dominator colorings in Paths, International Journal of Mathematical Combinatorics, Vol. 2, pp. 89-95, 2012.

[9] P. Niju, P and A. Vijayalekshmi, Dominator color class dominating sets on firecracker, gear and flower graphs, Malaya Journal of Matematik, Vol.9, No.1, pp. 1043-1046, 2021.

[10] A. Vijayalekshmi, and P. Niju,Dominator Color class Dominating sets on Ladder, Open Ladder and Slanting Ladder Graphs, Journal of Computational Mathematica, Sacred Heart Research, Vol. 5, No. 02, pp.132-137, 2021.
[11] Terasa W. Haynes, Stephen T. Hedetniemi, Peter J Slater, Determination in Graphs, Maral Dekker, New York, 1998.