Hilbert Mean Labeling Of Some Special Graphs

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ABSTRACT
Let $G = (V(G), E(G))$be a graph with p vertices and q edges. A hilbert mean labeling is an injective function $\mu: V(G) \to \{0, 1, 2, \ldots, H_q\}$where H_q is the q^{th}hilbert number and$H_q = 4(q-1)+1$,$q \geq 1$that induces a bijection$\mu^*: E(G) \to \{H_1, H_2, \ldots, H_q\}$defined by $\mu^*(uv) = \begin{cases} \frac{\mu(u)+\mu(v)+1}{2} & \text{if } \mu(u)+\mu(v) \text{ is odd} \\ \frac{\mu(u)+\mu(v)}{2} & \text{if } \mu(u)+\mu(v) \text{ is even} \end{cases}$
for all $uv \in E(G)$. A graph which admits such labeling is called a hilbert mean graph. In this paper, the hilbert mean labeling of some special graphs are studied.

Keywords: Hilbert numbers, Hilbert mean labeling, Hilbert mean graph.

1.INTRODUCTION

The paper deals with finite, undirected graphs that lack loops or multiple edges. Let $G = (V, E)$represent a graph with p vertices and q edges, where terms follow Harary's [5] definitions. Unspecified terms adhere to [1] for number theoretic terminology. A graph labeling involves assigning integers to the vertices, edges or both under specific conditions. Vertex, edge or total labeling is determined by the domain of the mapping. Gallian [2] maintains a dynamic survey of graph labeling regularly updated and published by the Electronic Journal of Combinatorics. The notation of mean labeling for graphs was introduced by S. Somasundaram and R. Ponraj [7]. The concept of hilbert mean labeling was introduced in [8]. Also, we have referred [3],[4],[6] and [9] for my results.

2.Preliminaries

Definition 2.1:The n^{th} hilbert number H_n is given by the formula $4(n-1)+1$ for $n \geq 1$. The first few hilbert numbers are1,5,9,13,17, 21, 25, 29, 33, 37,41,45,49,53,57 etc.
Definition 2.2:Let G be a graph with p vertices and q edges. A hilbert mean labeling is an injective function $\mu: V(G) \to \{0, 1, 2, \ldots, H_q\}$where H_q is the q^{th}hilbert number and$H_q = 4(q-1)+1, q \geq 1$that induces a bijection $\mu^*: E(G) \to \{H_1, H_2, \ldots, H_q\}$ defined by $\mu^*(uv) = \begin{cases} \frac{\mu(u)+\mu(v)+1}{2} & \text{if } \mu(u)+\mu(v) \text{ is odd} \\ \frac{\mu(u)+\mu(v)}{2} & \text{if } \mu(u)+\mu(v) \text{ is even} \end{cases}$
for all $uv \in E(G)$. A graph which admits such labeling is called a hilbert mean graph.

3. Main Results

Theorem 3.1:$M(P_m)$ is a hilbert mean graph where m is odd and $m \geq 3$.
Proof: Let $G = M(P_m)$,$V(G) = \{w_{i_1}, v_{i_1}: 1 \leq i_1 \leq m\}$and
$E(G) = \{w_{i_1}v_{i_1}: 1 \leq i_1 \leq m, w_{i_1}w_{i_{1+1}}, v_{i_1}v_{i_{1+1}}: 1 \leq i_1 \leq m-1\}$.
We observe that G has $2m$ vertices and $3m-2$ edges.
Define $\mu : V(G) \to \{0, 1, 2, \ldots, H_{3m-2}\}$ as follows.
For $1 \leq i_1 \leq m$,$\mu(w_{i_1}) = \begin{cases} 4\,(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 4(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$

$$\mu(v_{i_1}) = \begin{cases} 4\,(2m + i_1 - 3) + 1 & \text{if } i_1 \text{ is odd} \\ 4(2m + i_1 - 2) & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly,μ is an injective functionand the induced bijectiveedge labeling$\mu^* : E(G) \to \{H_1, H_2, \dots, H_{3m-2}\}$is defined as follows.

$\mu^*(w_{i_1} w_{i_1+1}) = H_{i_1}$ $\quad 1 \le i_1 \le m - 1$

$\mu^*(w_{i_1} v_{i_1}) = H_{(m+i_1-1)}$ $\quad 1 \le i_1 \le m$

$\mu^*(v_{i_1} v_{i_1+1}) = H_{(2m+i_1-1)}$ $\quad 1 \le i_1 \le m - 1$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3m-2}$.

Hence $M(P_m)$ is a hilbert mean graph where m is odd and $m \ge 3$.

Example 3.2: The hilbert mean labeling of$M(P_3)$ is given in figure 1.

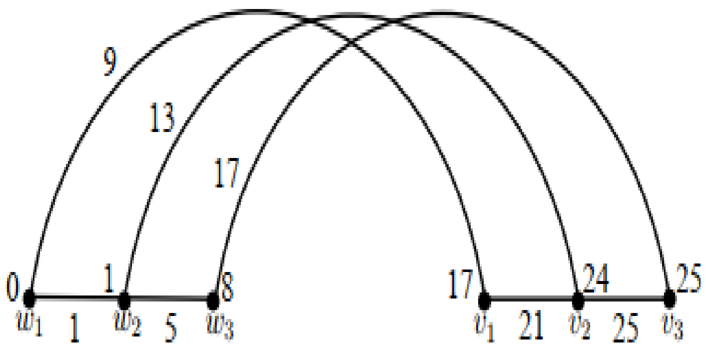

Figure 1: Hilbert mean labeling of$M(P_3)$

Theorem 3.3: $VD(P_m)$is a hilbert mean graph where m is even and $m \ge 4$.

Proof: Let $G = VD(P_m)$, $V(G) = \{w_{i_1} : 1 \le i_1 \le m - 1, w'_{m-1}\}$ and $E(G) = \{w_{i_1} w_{i_1+1} : 1 \le i_1 \le m - 1, w_{m-2} w'_{m-1}, w_m w'_{m-1}\}$.

We observe that G has $m + 1$ vertices and $m + 1$ edges.Also,$N(w_{m-1}) = N(w'_{m-1})$.

Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{m+1}\}$ as follows.

For $1 \le i_1 \le m - 2$,$\mu(w_{i_1}) = \begin{cases} 4\,(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 4(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$

$\mu(w_{m-1}) = H_{m+1} - 1$, $\mu(w_m) = H_{m+1}$, $\mu(w'_{m-1}) = H_{m-1}$

Clearly,$\mu$is an injective function and the induced bijective edge labeling$\mu^* : E(G) \to \{H_1, H_2, \dots, H_{m+1}\}$is defined as follows.

$\mu^*(w_{i_1} w_{i_1+1}) = H_{i_1}$ $\quad 1 \le i_1 \le m - 3$

$\mu^*(w_{m-2} w_{m-1}) = H_{m-1}$,$\mu^*(w_{m-1} w_m) = H_{m+1}$

$\mu^*(w_m w'_{m-1}) = H_m$, $\mu^*(w_{m-2} w'_{m-1}) = H_{m-2}$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{m+1}$.

Hence $VD(P_m)$ is a hilbert mean graph where m is even and $m \ge 4$.

Example 3.4: The hilbert mean labeling of$VD(P_4)$ is given in figure 2.

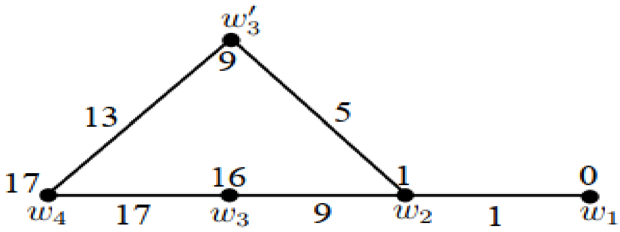

Figure 2: Hilbert mean labeling of$VD(P_4)$

Theorem 3.5: $G = P_m(Qs_1)$where $m$is even is a hilbert mean graph.
Proof: Let $V(G) = \{w_{i_1}: 1 \le i_1 \le m, x_{i_1}: 1 \le i_1 \le 2m, y_{i_1}: 1 \le i_1 \le m\}$ and
$E(G) = \{w_{i_1}w_{i_1+1}: 1 \le i_1 \le m-1, y_{i_1}x_{2i_1-1}, y_{i_1}x_{2i_1}, w_{i_1}x_{2i_1-1}, w_{i_1}x_{2i_1}: 1 \le i_1 \le m\}$.
We observe that G has $4m$ vertices and $5m-1$ edges.
Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{5m-1}\}$ as follows.
For$1 \le i_1 \le m$,$\mu(w_{i_1}) = \begin{cases} 20(i_1-1)+16 & \text{if } i_1 is\ odd \\ 20(i_1-2)+17 & \text{if } i_1\ is\ even \end{cases}$
$\mu(x_{2i_1-1}) = \begin{cases} 20(i_1-1)+1 & \text{if } i_1\ is\ odd \\ 20(i_1-2)+25 & \text{if } i_1\ is\ even \end{cases}$
$\mu(x_{2i_1}) = \begin{cases} 20(i_1-1)+9 & \text{if } i_1\ is\ odd \\ 20(i_1-2)+33 & \text{if } i_1\ is\ even \end{cases}$
$\mu(y_{i_1}) = \begin{cases} 20(i_1-1) & \text{if } i_1\ is\ odd \\ 20(i_1-2)+32 & \text{if } i_1\ is\ even \end{cases}$
Clearly,μ is an injective function and the induced bijectiveedge labeling is defined as follows.
$\mu^*(w_{i_1}w_{i_1+1}) = H_{5(i_1-1)+5}$ $1 \le i_1 \le m-1$
For $1 \le i_1 \le m$,
$\mu^*(w_{i_1}x_{2i_1-1}) = \begin{cases} H_{5(i_1-1)+3} & \text{if } i_1\ is\ odd \\ H_{5(i_1-2)+6} & \text{if } i_1\ is\ even \end{cases}$
$\mu^*(w_{2i_1}x_{2i_1}) = \begin{cases} H_{5(i_1-1)+4} & \text{if } i_1\ is\ odd \\ H_{5(i_1-2)+7} & \text{if } i_1\ is\ even \end{cases}$
$\mu^*(y_{i_1}x_{2i_1-1}) = \begin{cases} H_{5(i_1-1)+1} & \text{if } i_1\ is\ odd \\ H_{5(i_1-2)+8} & \text{if } i_1\ is\ even \end{cases}$
$\mu^*(y_{2i_1}x_{2i_1}) = \begin{cases} H_{5(i_1-1)+2} & \text{if } i_1\ is\ odd \\ H_{5(i_1-2)+9} & \text{if } i_1\ is\ even \end{cases}$
Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{5m-1}$.
Hence $G = P_m(Qs_1)$where m is even is a hilbert mean graph.

Theorem 3.6: $G = P_2(Qs_m)$ where $m \ge 1$is a hilbert mean graph.
Proof: Let $G = P_2(Qs_m), V(G) = \{y_{i_1}: 1 \le i_1 \le 2m+2, x_{i_1}: 1 \le i_1 \le 4m\}$ and
$E(G) = \{x_{2i_1-1}y_{i_1} : 1 \le i_1 \le m, x_{2i_1}y_{i_1}: 1 \le i_1 \le m, x_{2i_1-1}y_{i_1+1} : 1 \le i_1 \le m, x_{2i_1}y_{i_1+1}: 1 \le i_1 \le m, x_{2m+i_1+1}y_{m+i_1+1}: 1 \le i_1 \le m, x_{2m+i_1}y_{m+i_1+2}: 1 \le i_1 \le m, x_{2m+i_1+1}y_{m+i_1+2}: 1 \le i_1 \le m, y_{m+1}y_{m+2}\}$
We observe that G has $6m+2$ vertices and $8m+1$ edges.
Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{8m+1}\}$ as follows.
$\mu(y_{i_1}) = 16(i_1-1)$, $1 \le i_1 \le m+1$
$\mu(y_{m+i_1+1}) = 16(m+i_1-1)+1$, $1 \le i_1 \le m+1$
For $1 \le i_1 \le 2m$,$\mu(x_{i_1}) = \begin{cases} 8(i_1-1)+1 & \text{if } i_1\ is\ odd \\ 8(i_1-2)+9 & \text{if } i_1\ is\ even \end{cases}$
$\mu(x_{2m+i_1}) = \begin{cases} 8(2m+i_1)+1 & \text{if } i_1\ is\ odd \\ 8(2m+i_1) & \text{if } i_1 is\ even \end{cases}$
Clearly,μ is an injective function and the inducedbijective edge labelingis defined as follows.
For $1 \le i_1 \le m$,
$\mu^*(x_{2i_1-1}y_{i_1}) = H_{4(i_1-1)+1}$, $\mu^*(x_{2i_1}y_{i_1}) = H_{4(i_1-1)+2}$
$\mu^*(x_{2i_1-1}y_{i_1+1}) = H_{4(i_1-1)+3}$, $\mu^*(x_{2i_1}y_{i_1+1}) = H_{4(i_1-1)+4}$
$\mu^*(x_{2m+i_1}y_{m+i_1+1}) = H_{4(m+i_1-1)+2}$, $\mu^*(x_{2m+i_1+1}y_{m+i_1+1}) = H_{4(m+i_1-1)+3}$
$\mu^*(x_{2m+i_1}y_{m+i_1+2}) = H_{4(m+i_1-1)+4}$, $\mu^*(x_{2m+i_1+1}y_{m+i_1+2}) = H_{4(m+i_1)+1}$
$\mu^*(y_{m+1}y_{m+2}) = H_{4m+1}$
Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{8m+1}$.
Hence $G = P_2(Qs_m)$where $m \ge 1$ is a hilbert mean graph.

Example 3.7: The hilbert mean labeling of$P_2(Qs_2)$ is given in figure 3.

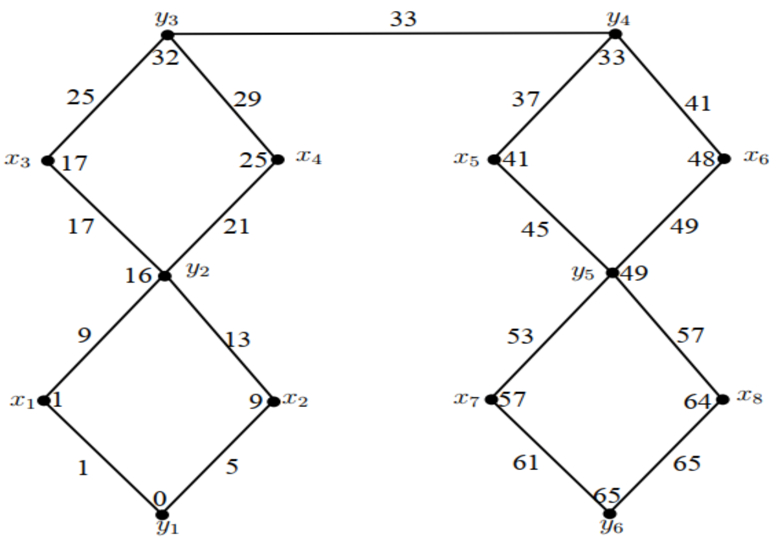

Figure 3: Hilbert mean labeling of$P_2(Qs_2)$

Theorem 3.8: Alternate Quadrilateral Snake graph $G = A(C_4^n)$is a hilbert mean graph where n is even.

Proof: Let $G = A(C_4^n)$,$V(G) = \{w_{i_1}, x_{i_1}: 1 \leq i_1 \leq n\}$ and

$$E(G) = \{x_{i_1}w_{i_1}, x_{2i_1}w_{2i_1-1}, x_{i_1}\, w_{i_1+1}, x_{2i_1}w_{2i_1}: 1 \leq i_1 \leq n\}$$

We observe that G has $2n$ vertices and $2n+1$edges.Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{2n+1}\}$ as follows.

For $1 \leq i_1 \leq n$,$\mu(w_{2i_1-1}) = \begin{cases} 20(i_1-1)+1 & \text{if } i_1 \textit{ is odd} \\ 20(i_1-2)+25 & \text{if } i_1 \textit{ is even} \end{cases}$

$\mu(w_{2i_1}) = \begin{cases} 20(i_1-1)+9 & \text{if } i_1 \textit{ is odd} \\ 20(i_1-2)+32 & \text{if } i_1 \textit{ is even} \end{cases}$

$\mu(x_{2i_1}) = \begin{cases} 20(i_1-1)+16 & \text{if } i_1 \textit{ is odd} \\ 20(i_1-2)+33 & \text{if } i_1 \textit{ is even} \end{cases}$

$\mu(x_{2i_1-1}) = \begin{cases} 20(i_1-1) & \text{if } i_1 \textit{ is odd} \\ 20(i_1-2)+17 & \text{if } i_1 \textit{ is even} \end{cases}$

Clearly,μ is an injective function and the inducedbijective edge labeling $\mu^* : E(G) \to \{H_1, H_2, \dots, H_{2n+1}\}$is defined as follows.

$\mu^*(x_{i_1}w_{i_1}) = H_{5(i_1-1)+1}$ $\quad 1 \leq i_1 \leq n$

$\mu^*(x_{2i_1}w_{2i_1-1}) = H_{5(i_1-1)+3}$ $\quad 1 \leq i_1 \leq n$

$\mu^*(x_{2i_1-1}\, w_{2i_1}) = H_{5(i_1-1)+2}$ $\quad 1 \leq i_1 \leq n$

$\mu^*(x_{2i_1}w_{2i_1}) = H_{5(i_1-1)+4}$ $\quad 1 \leq i_1 \leq n$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{2n+1}$.

Hence Alternate Quadrilateral Snake graph $G = A(C_4^n)$is a hilbert mean graph where n is even.

Example 3.9: The hilbert mean labelling of$A(C_4^2)$is given in figure 4.

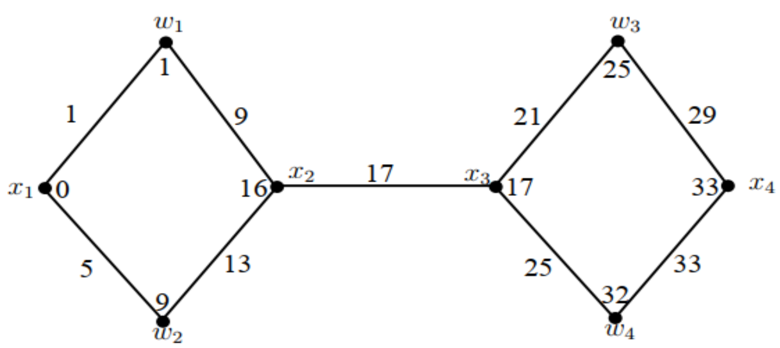

Figure 4: Hilbert mean labelling of $A(C_4^2)$

Theorem 3.10: The Bull graph is a Hilbert mean graph.
Proof: Let us denote Bull graph by BG.Let $V(BG) = \{w_1, w_2, \cdots, w_5\}$ and
$E(BG) = \{w_{i_1}w_{i_1+1} : 1 \le i_1 \le 4, w_2w_{n-1}\}$. We observe that BG has 5 vertices and 5 edges.
Define $\mu : V(BG) \to \{0, 1, 2, \dots, H_5\}$ as follows.
$\mu(w_1) = H_1 - 1, \qquad \mu(w_2) = H_1$
For $3 \le i_1 \le 5, \mu(w_{i_1}) = \begin{cases} H_{i_1} & \text{if } i_1 \text{ is odd} \\ H_{i_1+1} - 1 & \text{if } i_1 \text{ is even} \end{cases}$
Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(BG) \to \{H_1, H_2, \dots, H_5\}$ is defined as follows.

$$\mu^*(w_{i_1}w_{i_1+1}) = H_{i_1}, i_1 = 1,2 \quad \mu^*(w_{i_1+2}w_{i_1+3}) = H_{i_1+3}, i_1 = 1,2 \quad \mu^*(w_2w_4) = H_3$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_5$.
Hence the Bull graph is a hilbert mean graph.

Example 3.11: The hilbert mean labeling of BG is given in figure 5.

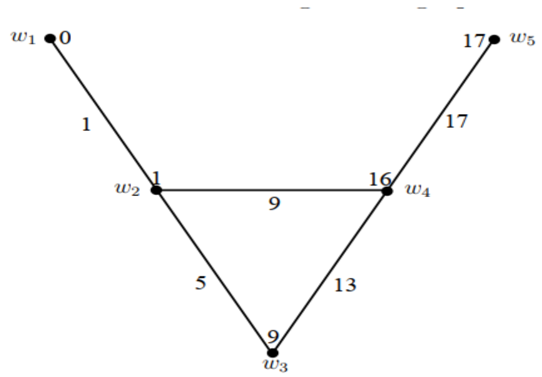

Figure 5: Hilbert mean labeling of BG

Theorem 3.12: The splitting graph $Spl(P_m)$ is a hilbert mean graph, where $m \ge 4$ *and* m is even.
Proof: Let $G = Spl(P_m)$, $V(G) = \{w_{i_1}, x_{i_1} : 1 \le i_1 \le m\}$ and

$$E(G) = \{x_{i_1}\, x_{i_1+1}, w_{i_1}\, x_{i_1+1}, x_{i_1}\, w_{i_1+1} : 1 \le i_1 \le m-1\}$$

We observe that G has $2m$ vertices and $3m - 3$ edges.
Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{3m-3}\}$ as follows.
For $1 \le i_1 \le m, \mu(w_{i_1}) = \begin{cases} 12(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 12(i_1 - 2) + 9 & \text{if } i_1 \text{ is even} \end{cases}$
$\mu(x_{i_1}) = \begin{cases} 12(i_1 - 1) + 8 & \text{if } i_1 \text{ is odd} \\ 12(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$
Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(G) \to \{H_1, H_2, \dots, H_{3m-3}\}$ is defined as follows.
For $1 \le i_1 \le m-1, \mu^*(x_{i_1}\, w_{i_1+1}) = \begin{cases} H_{3(i_1-1)+3} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+4} & \text{if } i_1 \text{ is even} \end{cases}$
$\mu^*(w_{i_1}\, x_{i_1+1}) = \begin{cases} H_{3(i_1-1)+1} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+6} & \text{if } i_1 \text{ is even} \end{cases}$
$\mu^*(x_{i_1}\, x_{i_1+1}) = \begin{cases} H_{3(i_1-1)+2} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+5} & \text{if } i_1 \text{ is even} \end{cases}$
Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3m-3}$.
Hence the splitting graph $Spl(P_m)$ is a hilbert mean graph, where $m \ge 4$ and m is even.

Theorem 3.13: An alternate trianglular snake graph $ATS(P_n)$ is a hilbert mean graph, where $n \ge 4$ and n is even.
Proof: Let $G = ATS(P_n)$, $V(G) = \left\{x_{i_1} : 1 \le i_1 \le n, y_{i_1} : 1 \le i_1 \le \frac{n-2}{2}\right\}$ and

$$E(G) = \left\{x_{i_1}x_{i_1+1} : 1 \le i_1 \le n-1, x_{2i_1}y_{i_1} \cup x_{2i_1+1}y_{i_1} : 1 \le i_1 \le \frac{n-2}{2}\right\}$$

We observe that G has $\frac{3n-2}{2}$ vertices and $2n - 3$ edges.
Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{2n-3}\}$ as follows.

For $1 \le i_1 \le n$, $\mu(x_{i_1}) = \begin{cases} 8\,(i_1-1) & \text{if } i_1 \text{ is odd} \\ 8\,(i_1-2)+1 & \text{if } i_1 \text{ is even} \end{cases}$

$\mu(y_{i_1}) = 16(i_1-1)+9 \quad 1 \le i_1 \le \frac{n-2}{2}$

Clearly,μ is an injective function and the induced bijective edge labelingμ^* is defined as follows.

$\mu^*(x_{i_1}x_{i_1+1}) = H_{2(i_1-1)+1} \quad 1 \le i_1 \le n-1$

$\mu^*(x_{2i_1}\,y_{i_1}) = H_{4(i_1-1)+2} \quad 1 \le i_1 \le \frac{n-2}{2}$

$\mu^*(x_{2i_1+1}y_{i_1}) = H_{4(i_1-1)+4} \quad 1 \le i_1 \le \frac{n-2}{2}$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{2n-3}$.

Hence the alternate trianglular snake graph $ATS(P_n)$is a hilbert mean graph, where $n \ge 4$and n is even.

Example 3.14: The hilbert mean labeling of$ATS(P_6)$ is given in figure 6.

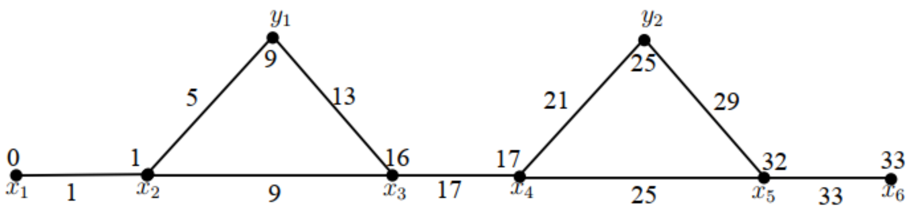

Figure 6: Hilbert mean labeling of$ATS(P_6)$

Theorem 3.15: A double alternate triangular snake graph $DATS(P_n)$ is a hilbert mean graph, where $n \ge$ 4and $n$is even.

Proof: Let $G = DATS(P_n)$,$V(G) = \left\{x_{i_1}: 1 \le i_1 \le n, y_{i_1}: 1 \le i_1 \le \frac{n-2}{2}, z_{i_1}: 1 \le i_1 \le \frac{n-2}{2}\right\}andE(G) = \left\{x_{i_1}x_{i_1+1}: 1 \le i_1 \le n-1 \cup x_{2i_1}y_{i_1} \cup x_{2i_1+1}y_{i_1} \cup x_{2i_1}z_{i_1} \cup x_{2i_1+1}\,z_{i_1}: 1 \le i_1 \le \frac{n-2}{2}\right\}$. We observe that G has$2n-2$vertices and $3n-5$ edges.

Define $\mu : V(G) \to \{0, 1, 2, \dots, H_{3n-5}\}$as follows.

For $1 \le i_1 \le n$,$\mu(x_{i_1}) = \begin{cases} 12\,(i_1-1) & \text{if } i_1 \text{ is odd} \\ 12(i_1-2)+1 & \text{if } i_1 \text{ is even} \end{cases}$

For $1 \le i_1 \le \frac{n-2}{2}$,$\mu(y_{i_1}) = 24(i_1-1)+9$, $\mu(z_{i_1}) = 24(i_1-1)+17$

Clearly,μ is an injective function and the induced bijective edge labelingμ^* is defined as follows.

$\mu^*(x_{i_1}x_{i_1+1}) = H_{3(i_1-1)+1} \quad 1 \le i_1 \le n-1$

For $1 \le i_1 \le \frac{n-2}{2}$,$\mu^*(x_{2i_1}y_{i_1}) = H_{6(i_1-1)+2}$, $\mu^*(x_{2i_1+1}y_{i_1}) = H_{6(i_1-1)+5}$

$\mu^*(x_{2i_1}z_{i_1}) = H_{6(i_1-1)+3}$, $\mu^*(x_{2i_1+1}\,z_{i_1}) = H_{6(i_1-1)+6}$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3n-5}$.

Hence the double alternate triangular snake graph $DATS(P_n)$ is ahilbert mean graph,where $n \ge 4$and n is even.

Example 3.16: The hilbert mean labeling of $DATS(P_6)$ is given in figure 7.

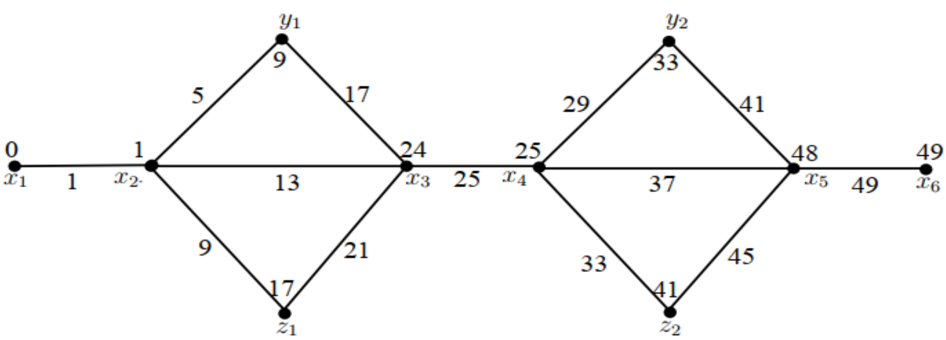

Figure 7: Hilbert mean labeling of $DATS(P_6)$

4. CONCLUSION

In this paper, we have investigated the hilbert mean labelling of certain special graphs. This work contributes several new results to the theory of graph labeling.

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