Fuzzy Set-Based Inventory Model for TPD Demand under effect of Inflation and Carbon Emissions with Partial Backordering

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Received: 15.07.2024 Revised: 14.08.2024 Accepted: 15.09.2024

ABSTRACT:
Climate change is a global challenge that is gaining attention. Reducing carbon emissions is challenging for achieving both economic growth and sustainable development. This study focuses on optimizing profits in an inventory system, accounting for additional costs due to carbon emissions at various process stages. Deterioration and inflation also affect prices and demand over time. To address these real-world complexities, the authors consider demand to be sensitive to both selling price and time, under inflationary conditions. The model incorporates partial backlogging during shortages to maintain a certain level of customer service even when stock is unavailable. A fuzzy approach is employed to handle the unpredictability associated with cost components. Trapezoidal fuzzy numbers are assigned to the cost variables and are defuzzified using the graded mean integration representation method. The fuzzy model shows a 3.82% increase in profit while prices were reduced by 0.27% as compared to the crisp model. The key objective of this model is to maximize profits. Numerical validation and sensitivity analysis are also conducted to evaluate the impact of various changes on the proposed inventory model.

Keywords*:* Inflation, carbon cost, shortages, partial backlogging, trapezoidal fuzzy number, graded mean representation method.

1. INTRODUCTION

Various countries and organizations set de-carbonization or net zero carbon emission targets. Businesses and the economic sector also commit to net zero, accelerating this trend. Significant reductions in greenhouse gas emissions across the economy must reach the zero-carbon emission target. It is a challenge for enterprises to meet the aims of sustainable development in traditional production. Inventory control has arisen as a critical component of maintaining profitability and reducing risks for firms coping with deteriorating goods in a dynamic and always-changing business environment. The conventional inventory models fall short in handling these complicated scenarios as industries struggle with the various issues such as shifting demand, product deterioration, supply chain disruptions, and environmental concerns. The effect of carbon emissions has a severe impact on human health as well as nature. Carbon emissions cost can occur in different production stages while placing the order, holding the produced and raw stock, and due to deterioration.

As environmental decay became increasingly serious, researchers and policymakers paid attention to the reduction of carbon emissions to achieve sustainable goals. Bonny and Jaber (2011) modeled an inventory system that was environmentally responsible in classical as well as non-classical eras. Bouchery et al. (2012) studied the criteria for how to include sustainability in inventory models. Datta (2017) observed the effect on the production-inventory system after implementing the carbon tax policy. An EOQ model was presented by Kazemi et al. (2018) for defective objects taking carbon emission into account. A sustainable production inventory model consisting of shortages with managed carbon emission was provided by Mishra et al. in 2020. Singh et al. (2021) use a supply chain management system with a two-level trade-credit policy to analyze the effects of energy and carbon emissions. Yadav et al. (2021) aimed to lower waste and carbon emissions and develop a sustainable supply chain with preservation technologies by selecting goods with cross-price elasticity of demand. Based on government subsidy and carbon tax policies, Ran and Xu (2023) proposed a concept for coordinating low-carbon supply chains.

The impact of inflation cannot be ignored in some inventory systems when the demand for specific commodities, such as trendy items, depends on price and time. Whether or not to accept backlog depends on when the next refill will be available. Hariga (1995) used an inventory model that took time-reliant demand rates and shortages into account to examine the inflationary effect and the time value of money. Hou, (2006) formulated an inventory model that accounts for time discounting, decaying goods, and inflation. They also took into account how the rate of consumption is affected by both surpluses and shortages. When demand is dependent on both price and time, Maihami and Kamalabadi (2012) proposed a model in which they jointly optimized pricing for non-instantaneous deterioration and total profit with partial backlogging. An EOQ model for defective products under the influence of inflation, including partial back ordering, consumer price-dependent demand, and learning, was proposed by Yadav and Singh (2015). Teksan and Geunes (2016) presented an EOQ model where supply is dependent on price and demand. An inventory model with non-instantaneous deterioration due to inflation, fully backlogged shortages, and demand dependent on price and stock was developed by Shaikh et al. (2017). Saha and Sen (2019) provided an inventory model that took into account the effects of inflation, shortages, and time- and price-dependent demand. For an EOQ model with bivariate demand, Sundararajan et al. (2021) observed the influence of payment delays, inflation, and shortages. Again, Sundararajan et al. (2022) calculate the cost of the EOQ model with payment delays and shortages due to inflation for a non-immediate decline. An EOQ model under inflation for stock- and lifetime-dependent demand on expiring items was optimized by Singh and Ambedkar (2023). An inventory model with sustainability and cap-and-trade pricing was first presented by Sharma and Sharma (2023).

Zadeh (1965) introduced the concept of fuzzy set theory in the inventory model to handle imprecision and uncertainty in inventory management to address the complexities of the real business environment. Maragatham and Lakshmidevi (2014) created an inventory model for degrading goods that took into account price-dependent demand. Pal et al. (2015) provided a production model with fuzziness that allowed for inflation, shortages, and ramp-type demand for degrading goods. Yadav et al. (2015) provided an ideal course of action for retailers with hazy trade credit under inflation. A fuzzy inventory model under inflation, where demand is dependent on price and time, was developed by Hossen et al. (2016). For items that are deteriorating, Saha and Chakrabarti (2017) provided a fuzzy inventory model with demand being price-dependent and back ordering being prohibited in a supply chain system. A multi-object inventory model was provided by Garai et al. (2019), in which the demand rate was based on both the stock and the holding cost rate in a fuzzy environment. Singh et al. (2020) offered a comparative analysis for the best reordering strategy for perishable goods in crisp and fuzzy styles. Bhavani et al. (2022) introduced a sustainable green inventory system under fuzziness with eco-friendly demand and partial backlogging. In a fuzzy context, Das (2022) discussed a multi-objective inventory model with population-dependent production and setup costs. Research of an integrated fuzzy inventory model with an unpredictable rate of deterioration and inflation was completed by Padiyar et al. in 2022. In their intuitionistic fuzzy inventory model (2022), Chaudhary and Kumar take time-dependent holding costs, shortages, and quadratic demand rates into account. Shaikh and Gite (2022) proposed a fuzzy inventory model that took into account demand based on selling price and variable production for degrading items under inflation. Poswal et al. (2022) investigated and examined an EOQ model for stock and price-sensitive demand in a shortage-driven environment. For items that are deteriorating, Kumar et al. (2023) proposed a fuzzy inventory model where demand follows a recurrent seasonal pattern with ramp-type growth seasonally. In their EOQ inventory model with time-dependent demand, Karmakar and De (2022, 2023) assigned triangular fuzzy numbers to the fluctuating number of tourists, although in their other EOQ inventory model, they considered the Pythagorean fuzzy number. Kumar et al. (2023) provided sustainability for deteriorating items under the effect of learning along with social and environmental responsibility and partial backlogging.

This study presents a fuzzy inventory model for deteriorating items that offers proper inventory management and pricing along with consideration of environmental impact. Due to the usual inverse relationship between demand and price, demand is dependent on both time and price. Partial backlogging of shortages is considered. The backlog rate determines how long it will take to restock. A trapezoidal fuzzy integer is allocated to the cost parameter to address imprecision and uncertainty. For defuzzification, the graded mean representation method was used. The main goal of the suggested model is to offer the best possible selling price and replenishment cycle time to identify inventory control techniques for items that are deteriorating. A numerical case has been solved to assess the performance of the suggested fuzzy inventory model. The study illustrates how the model can assist organizations in making knowledgeable decisions regarding pricing, inventory control, shortages, and partially backlogged orders while accounting for the impact of inflation and the naturally deteriorating nature of the items. This is done by looking at various scenarios and parameters. The model seeks to maximize total profit while meeting customer demand and minimizing the consequences of item deterioration through the optimization of pricing and inventory management.

Table 1 Previous research contribution related to this paper

Authors	Demand depends on Time & price dependent	Shortage	Inflation	Fuzziness	Carbon Concern
Hariga (1995)	✗	✓	✓	✗	✗
Hou (2006)	✗	✓	✓	✗	✗
Maihami&Kamalabadi (2012)	✓	✓	✗	✗	✗
Maragatham & Lakshmidevi (2014)	✗	✓	✗	✓	✗
Yadav et al. (2015)	✗	✗	✓	✓	✗
Hossen et al. (2016)	✓	✓	✓	✓	✗
Datta (2017)	✗	✗	✗	✗	✓
Saha and Chakrabarti (2017)	✗	✗	✗	✓	✗
Garai, et al. (2019)	✗	✓	✗	✓	✗
Saha and Sen (2019)	✓	✓	✓	✗	✗
Mishra et al. (2020)	✗	✓	✗	✗	✓
Sundararajan et. al. (2021)	✓	✓	✓	✗	✗
Bhavani et al. (2022)	✗	✓	✗	✓	✓
Chaudhary and Kumar, (2022)	✗	✓	✗	✓	✗
Das (2022)	✗	✗	✗	✓	✗
Padiyar et al. (2022)	✗	✗	✗	✓	✗
Poswal, et al. (2022)	✗	✓	✗	✓	✗
Shaikh and Gite (2022)	✓	✗	✓	✓	✗
Kumar et al. (2023)	✗	✓	✗	✓	✓
Sharma and Sharma (2023)	✗	✗	✓	✗	✗
Singh and Ambedkar (2023)	✗	✗	✓	✗	✗
This paper	✓	✓	✓	✓	✓

2. Problem description

The study's objective is to offer answers for issues that organizations face when handling perishable goods including food, vegetables, pharmaceuticals, and other items whose demand relies on time and price. Moreover, the model considers inflationary effects, partial backlogs, and scarcity of accounts. In addition, the cost of carbon emissions associated with inventory transit, storage, and decay raises environmental concerns. The objective is to develop an economical, environmentally friendly inventory plan that guarantees sustainable practices. To account for uncertainty regarding different cost parameters, authors apply fuzzy logic to tackle the complexities of the real world. This model's ultimate objective is to handle deteriorating inventory items in a way that strikes a balance between economic efficiency and environmental impact.

2.1 Assumptions:

- A sustainable inventory model is developed for a single item that is instantaneously deteriorating.
- Carbon emission cost added to counter environmental impact in different operational activities.
- The inflationary environment is considered throughout the study.
- Constant inflation rate is assumed.

- The rate of replenishment is not bounded by zero lead time.
- The time horizon is finite.
- Trapezoidal fuzzy number is assigned to the cost component.
- The defuzzification method employs the graded mean representation integration method.
- There may be shortages. In this scenario, the backlog of unmet demand equals the amount of shortage that is back-ordered by $S(y) = K_0 e^{\kappa y}, (0 < K_0 \le 1, \kappa > 0)$, where *y* is the duration for which customers have to wait for the next refill and is a constant $0 \le S(y) \le 1,\ S(0) = 1$. To ensure that an optimal solution will exist, we have assumed that $S(y) + H(S^{'}(y)) > 0$, where is the derivative of $S(y)$. Also note, if for all *t*, that means shortage occurred is fully backlogged.
- The fundamental rate of demand, $D(p,t) = \Delta e^{\ell t},\ where\ \Delta = (\zeta - \xi p),$, where *ζ>0, and* ξ *>0* are linearly dependent prices and exponentially increase or decrease over time when ℓ < 0 (ℓ > 0). We employ the exponential time effect to represent the underlying demand rate, which is appropriate to elaborate on the time-varying demand. This form can describe the majority of situations where the demand rate changes over time, depending on a variable, which can be either positive or negative. It is useful to consider the time and price-reliant demand for deteriorating goods including high-tech equipment, fashion accessories, veggies, and fruits.

2.2 Notations

A	Per order cost (Dollar) of ordering.
$A^{'}$	Carbon emission cost (Dollar) due to placement of new order.
c	Purchase cost (Dollar) for one unit.
$c^{'}$	Additional carbon emission charges (Dollar) in the purchase of one unit.
C_h	Unit holding Cost (Dollar) for one unit of time.
$C_h^{'}$	Per unit carbon emission cost (Dollar) due to holding the item.
C_s	Unit back ordering cost (Dollar) for one unit time.
$C_s^{'}$	Per unit carbon emission cost (Dollar) due to backorder.
C_o	The cost (Dollar) of lost sales per unit.
C_d	The cost (Dollar) of deterioration per unit.
$C_d^{'}$	Per unit carbon emission cost (Dollar) due to deterioration per unit.
p	Per unit consumer price (Dollar).
d	Deterioration rate.
t_1	Time from shortage occurs.
T	Cycle length.
Q	Ordered Quantity.
$I_1(t)$	Inventory level when $t \in [0, t_1]$.
$I_2(t)$	Inventory level when $t \in [t_1, T]$.
q_1	The maximum inventory level.
q_2	Backlogged amount.
i	Inflation rate
$Z(p,t_1,T)$	The total profit.
A	Fuzzy Per order cost (Dollar) of ordering.
$A^{'}$	Fuzzy Carbon emission cost (Dollar) due to placement of new order
$\tilde{c}$	Fuzzy purchase cost (Dollar) per unit.
$\tilde{c}^{'}$	Fuzzy per unit carbon emission cost (Dollar) due to purchase for one unit.
C_h	Fuzzy Cost (Dollar) of holding per unit per unit time.

$C_h^{'}$	Fuzzy per unit carbon emission cost due to holding the item.
C_s	Fuzzy back ordering cost (Dollar) per unit per unit time.
$C_s^{'}$	Fuzzy per unit carbon emission cost (Dollar) due to backordering.
C_o	Fuzzy cost (Dollar) of lost sale per unit.
C_d	Fuzzy cost (Dollar) of deterioration/unit.
$C_d^{'}$	Fuzzy per unit carbon emission cost (Dollar) due to deterioration.
$Z(p,t_1,T)$	Fuzzy total profit (Dollar).
$Z^*(p^*,t_1^*,T^*)$	Fuzzy optimal total profit (Dollar).

3. Mathematical Formulation

3.1 Crisp Model

We assume that the period of shortage is less than or equal to the period of availability. The inventory level drops within the time interval [0, t_1] due to demand and deterioration.

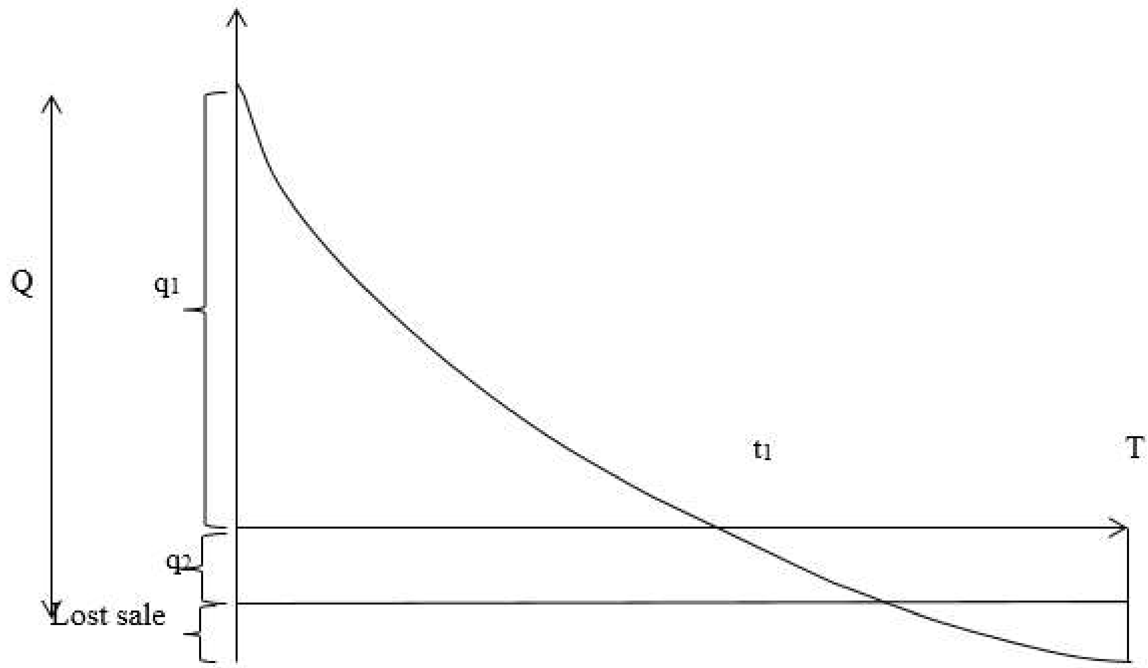

Fig. 1 Inventory system in a graphical way

At any time t, the positive inventory level is controlled by

$$\frac{dI_1(t)}{dt} + dI_1(t) = -D(p,t), \qquad 0 \le t \le t_1 . \tag{1}$$

with initial condition $I_1(0) = q_1$ and boundary conditions $I(t_1) = 0$,
from equation (1),

$$I_1(t)e^{dt} = -\frac{\Delta}{(d+\ell)} e^{(d+\ell)t} + K_1 \tag{2}$$

Using, $I(0) = q_1$, we obtain $K_1 = q_1 + \frac{\Delta}{(d+\ell)}$,

The equation (2), becomes

$$I_1(t) = q_1 e^{-dt} + \frac{\Delta}{(d+\ell)} e^{-dt} - \frac{\Delta}{(d+\ell)} e^{\ell t} \tag{3}$$

By applying the boundary condition $I(t_1)=0$, we get

$$q_1 = \frac{\Delta}{(d+\ell)}(e^{(d+\ell)t_1} - 1)$$

Equation (3), becomes

$$I_1(t) = \frac{\Delta}{(d+\ell)}(e^{d(t_1-t)}e^{\ell t_1} - e^{\ell t}) \qquad 0 \le t \le t_1 \tag{4}$$

The demand during the shortage backlogged at the rate $e^{-\kappa(T-t)}, \kappa > 0$ and customers have to wait till the next replenishment *(T-t)* and $t \in [t_1, T]$

$$\frac{dI_2(t)}{dt} = -\Delta e^{\ell t - \kappa(T-t)}, \qquad t_1 \le t \le T \tag{5}$$

with initial conditions $I_2(t_1)=0$, and boundary conditions: $-I_2(T) = q_2$
solution of (6), given by

$$I_2(t) = \frac{\Delta}{(\ell+\kappa)} e^{-\kappa T}\left(e^{(\ell+\kappa)t_1} - e^{(\ell+\kappa)t}\right) \tag{6}$$

Maximum back-ordered quantity

$$q_2 = -I_2(T)$$

$$q_2 = \frac{e^{-\kappa T}\Delta\left(e^{(\ell+\kappa)T} - e^{(\ell+\kappa)t_1}\right)}{(\ell+\kappa)} \tag{7}$$

Total quantity ordered per cycle

$$Q = q_1 + q_2$$

$$Q = \frac{\Delta}{(d+\ell)}(e^{(d+\ell)t_1} - 1) + \frac{e^{-\kappa T}\Delta\left(e^{(\ell+\kappa)T} - e^{(\ell+\kappa)t_1}\right)}{(\ell+\kappa)} \tag{8}$$

With the following cost components, we can now calculate inventory expenses and sales income each cycle:

1. Ordering cost per order and corresponding carbon emission cost is calculated as $A + A^{'}$.
2. For maintaining emission norms, there must be some environmental cost. Consequently, the holding cost of the inventory system has two components. One is related withholding of inventory as C_h and another $C_h^{'}$ is due to carbon emissions while inventory is being held

$$HC = \left(C_h + C_h^{'}\right)\int_0^{t_1} e^{-it} I_1(t)dt$$

$$= \frac{\left(C_h + C_h^{'}\right)\Delta}{(d+\ell)}\left\{+\frac{1-e^{(\ell-i)t_1}}{(\ell-i)} + \frac{e^{(d+\ell)t_1} - e^{(\ell-i)t_1}}{(d+i)}\right\} \tag{9}$$

3. Since this model is developed keeping in mind deteriorating products. So, due to the decaying of products, some costs must be placed in an inventory system. Deterioration of any product leads to environmental consequences, so two components are being introduced one C_d for deterioration of products and second $C_d^{'}$ for carbon emission cost. Cumulative cost due to deterioration and carbon emission is given by

$$DC = \left(C_d + C_d^{'}\right)\left\{Q - \int_{t_1}^{T} D(p,t)e^{-it}e^{-\kappa(T-t)}dt - \int_{0}^{t_1} D(p,t)e^{-it}dt\right\}$$

$$= \left(C_d + C_d^{'}\right)\Delta\left\{\begin{array}{l}\dfrac{e^{-\kappa T}(e^{(\kappa+\ell)T} - e^{(\kappa+\ell)t_1})}{(\kappa+\ell)} + \dfrac{e^{(d+\ell)t_1} - 1}{(d+\ell)} \\ -\dfrac{e^{-\kappa T}(e^{(\kappa+\ell-i)T} - e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)} - \dfrac{e^{(\ell-i)t_1} - 1}{(\ell-i)}\end{array}\right\} \quad (10)$$

4. Non-availability of stock caused shortages and the cost due to shortages and the corresponding environmental costs are C_s and $C_s^{'}$ respectively. The cost arises due to stock-out is given by

$$SC = -\left(C_s^{'} + C_s\right)\int_{t_1}^{T} e^{-it} I_2(t)dt$$

$$SC = \frac{\Delta e^{-\kappa T}\left(C_s + C_s^{'}\right)}{(\ell+\kappa)}\left\{\frac{(e^{-iT} - e^{-it_1})e^{(\kappa+\ell)t_1}}{i} + \frac{(e^{(\kappa+\ell-i)T} - e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)}\right\} \quad (11)$$

5. Due to shortages, the required demand could not be fulfilled. So, there is a loss of opportunity that could be accomplished in case of availability of stock. The cost due to this opportunity is calculated as

$$OC = C_o\int_{t_1}^{T} D(p,t)e^{-it}(1 - e^{-\kappa(T-t)})dt$$

$$OC = C_o\Delta\left\{\frac{(e^{(\kappa+\ell-i)t_1} - e^{(\kappa+\ell-i)T})e^{-\kappa T}}{(\kappa+\ell-i)} + \frac{(e^{(\ell-i)T} - e^{(\ell-i)t_1})}{(\ell-i)}\right\} \quad (12)$$

6. While Q quantity being purchased at the product price of c and c′ is the additional environmental cost. Therefore, the purchase cost is

$$PC = \left(c + c^{'}\right)(q_1 + q_2)$$

$$PC = \left(c + c^{'}\right)\left\{\frac{\Delta(e^{(d+\ell)t_1} - 1)}{(d+\ell)} + \frac{e^{-\kappa T}\Delta(e^{(\ell+\kappa)T} - e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}\right\} \quad (13)$$

7. Total sales Revenue generated by selling items at selling price *p*

$$SR = p\left\{\int_{0}^{t_1} e^{-it}D(p,t)dt + q_2\right\}$$

$$= p\Delta\left\{\frac{(e^{(\ell-i)t_1} - 1)}{(\ell-i)} + \frac{e^{-\kappa T}(e^{(\ell+\kappa)T} - e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}\right\} \quad (14)$$

Therefore, the overall profit per unit of time is obtained below

$$Z(p,t_1,T)=\frac{SR-A-HC-DC-SC-OC-PC}{T}$$

$$Z(p,t_1,T)=\frac{1}{T}\left[\frac{e^{(\ell-i)t_1}-1}{(\ell-i)}\left(p\Delta+\left(C_d+C_d^{'}\right)\right)+\left(p-\left(c+c^{'}\right)-\left(C_d+C_d^{'}\right)\right)\frac{\Delta e^{-\kappa T}(e^{(\ell+\kappa)T}-e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}\right.$$
$$-\left(\left(c+c^{'}\right)+\left(C_d+C_d^{'}\right)+\frac{\left(C_h+C_h^{'}\right)}{(d+\ell)}\right)\frac{\Delta\left(e^{(\ell-i)t_1}-1\right)}{(\ell-i)}-\left(\left(C_d+C_d^{'}\right)+\left(C_s+C_s^{'}\right)+C_o\right)\times$$
$$\frac{e^{-\kappa T}\Delta(e^{(\kappa+\ell-i)T}-e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)}-\frac{\left(C_h+C_h^{'}\right)\Delta\left(e^{(d+\ell)t_1}-e^{(\ell-i)t_1}\right)}{(d+\ell)(d+i)}$$
$$\left.-\frac{\Delta e^{-\kappa T}e^{(\kappa+\ell)t_1}\left(C_s+C_s^{'}\right)(e^{-iT}-e^{-it_1})}{i.(\ell+\kappa)}-\frac{C_o\Delta(e^{(\ell-i)T}-e^{(\ell-i)t_1})}{(\ell-i)}\right] \tag{15}$$

3.2 Fuzzy Model

Cost is uncertain in the business world and is vaguely defined. So, to handle this vagueness, cost components viz. *A c,* C_h, C_d, C_s, and C_o, are assumed as fuzzy numbers. Trapezoidal fuzzy number introduced for the parameters as below

$$A=(a_1,a_2,a_3,a_4),\ A^{'}=(a_1^{'},a_2^{'},a_3^{'},a_4^{'}),\quad \tilde{c}^{'}=(c_1^{'},c_2^{'},c_3^{'},c_4^{'}),\quad \tilde{c}=(c_1,c_2,c_3,c_4),$$
$$C_h=(C_{h1},C_{h2},C_{h3},C_{h4}),\ C_h^{'}=(C_{h1}^{'},C_{h2}^{'},C_{h3}^{'},C_{h4}^{'}),C_d=(C_{d1},C_{d2},C_{d3},C_{d4}),$$
$$C_d^{'}=(C_{d1}^{'},C_{d2}^{'},C_{d3}^{'},C_{d4}^{'}),C_s=(C_{s1},C_{s2},C_{s3},C_{s4}),C_s^{'}=(C_{s1}^{'},C_{s2}^{'},C_{s3}^{'},C_{s4}^{'})$$
$$and\qquad C_o=(C_{o1},C_{o2},C_{o3},C_{o4}),$$

Now, the total profit obtained in equation (15) in a fuzzy sense is expressed as

$$Z(p,t_1,T)=\frac{1}{T}\left[\frac{e^{(\ell-i)t_1}-1}{(\ell-i)}\left(p\Delta+\left(C_d+C_d^{'}\right)\right)+\left(p-\left(\tilde{c}+\tilde{c}^{'}\right)-\left(C_d+C_d^{'}\right)\right)\frac{\Delta e^{-\kappa T}(e^{(\ell+\kappa)T}-e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}\right.$$
$$-\left(\left(\tilde{c}+\tilde{c}^{'}\right)+\left(C_d+C_d^{'}\right)+\frac{\left(C_h+C_h^{'}\right)}{(d+\ell)}\right)\frac{\Delta\left(e^{(\ell-i)t_1}-1\right)}{(\ell-i)}-\left(\left(C_d+C_d^{'}\right)+\left(C_s+C_s^{'}\right)+C_o\right)\times$$
$$\frac{e^{-\kappa T}\Delta(e^{(\kappa+\ell-i)T}-e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)}-\frac{\left(C_h+C_h^{'}\right)\Delta\left(e^{(d+\ell)t_1}-e^{(\ell-i)t_1}\right)}{(d+\ell)(d+i)}-\left(A+A^{'}\right)$$
$$\left.-\frac{\Delta e^{-\kappa T}e^{(\kappa+\ell)t_1}\left(C_s+C_s^{'}\right)(e^{-iT}-e^{-it_1})}{i.(\ell+\kappa)}-\frac{C_o\Delta(e^{(\ell-i)T}-e^{(\ell-i)t_1})}{(\ell-i)}\right] \tag{16}$$

Since costs fluctuate in a non-symmetrical way, we have applied a graded mean integration representation method for defuzzification that gives varying contributions of different levels of costs in a symmetric and graded manner. The output is a smooth and continuous crisp value that effectively captures the central tendency of the fuzzy output.

$$d(Z(p,t_1,T))=\frac{1}{T}\Bigg[\frac{e^{(\ell-i)t_1}-1}{(\ell-i)}\left(p\Delta+d\left(\left(C_d+C_d^{'}\right),0\right)\right)+\left(p-d\left(\left(\tilde{c}+\tilde{c}^{'}\right),0\right)-d\left(\left(C_d+C_d^{'}\right),0\right)\right)\times$$

$$\frac{\Delta e^{-\kappa T}(e^{(\ell+\kappa)T}-e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}-\left(d\left(\left(\tilde{c}+\tilde{c}^{'}\right),0\right)+d\left(\left(C_d+C_d^{'}\right),0\right)+\frac{d\left(\left(C_h+C_h^{'}\right),0\right)}{(d+\ell)}\right)\frac{\Delta\left(e^{(\ell-i)t_1}-1\right)}{(\ell-i)}$$

$$-\left(d\left(\left(C_d+C_d^{'}\right),0\right)+d\left(\left(C_s+C_s^{'}\right)\right)+d\left(C_o,0\right)\right)\times\frac{e^{-\kappa T}\Delta(e^{(\kappa+\ell-i)T}-e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)}$$

$$-\frac{d\left(\left(C_h+C_h^{'}\right),0\right)\Delta\left(e^{(d+\ell)t_1}-e^{(\ell-i)t_1}\right)}{(d+\ell)(d+i)}-d\left(\left(A+A^{'}\right),0\right)$$

$$-\frac{\Delta e^{-\kappa T}e^{(\kappa+\ell)t_1}d\left(\left(C_s+C_s^{'}\right),0\right)(e^{-iT}-e^{-it_1})}{i.(\ell+\kappa)}-\frac{d\left(C_o,0\right)\Delta(e^{(\ell-i)T}-e^{(\ell-i)t_1})}{(\ell-i)}\Bigg] \qquad (17)$$

In equation (17), the values of expressions $d(A,0)$, $d(C_h,0)$, $d(p,0)$, $d(C_d,0)$, $d(C_s,0)$, $d(C_o,0)$ and $d(\tilde{c},0)$ are obtained by using the arithmetic operations on the trapezoidal fuzzy number and defuzzify using graded mean integration representation function as below

$$X+Y=(x_1+y_1,x_2+y_2,x_3+y_3,x_4+y_4)$$

$$d(X+Y^{'},0)=\frac{x_1+y_1+2(x_2+y_2+x_3+y_3)+x_4+y_4}{6},$$

Using this method of defuzzification in equation (17), we get

$$d(Z(p,t_1,T))=\frac{1}{T}\Bigg[\frac{e^{(\ell-i)t_1}-1}{(\ell-i)}\left(p\Delta+\left(\frac{C_{d1}+C_{d1}^{'}+2(C_{d2}+C_{d2}^{'}+C_{d3}+C_{d3}^{'})+C_{d4}+C_{d4}^{'}}{6}\right)\right)$$

$$\left(p-\left(\frac{c_1+c_1^{'}+2(c_2+c_2^{'}+c_3+c_3^{'})+c_4+c_4^{'}}{6}\right)\right.$$

$$\left.-\left(\frac{C_{d1}+C_{d1}^{'}+2(C_{d2}+C_{d2}^{'}+C_{d3}+C_{d3}^{'})+C_{d4}+C_{d4}^{'}}{6}\right)\right)\frac{\Delta e^{-\kappa T}(e^{(\ell+\kappa)T}-e^{(\ell+\kappa)t_1})}{(\ell+\kappa)}$$

$$-\left(\left(\frac{c_1+c_1^{'}+2(c_2+c_2^{'}+c_3+c_3^{'})+c_4+c_4^{'}}{6}\right)\right.$$

$$+\left(\frac{C_{d1}+C_{d1}^{'}+2(C_{d2}+C_{d2}^{'}+C_{d3}+C_{d3}^{'})+C_{d4}+C_{d4}^{'}}{6}\right)$$

$$\left.+\frac{\left(\frac{C_{h1}+C_{h1}^{'}+2(C_{h2}+C_{h2}^{'}+C_{h3}+C_{h3}^{'})+C_{h4}+C_{h4}^{'}}{6}\right)}{(d+\ell)}\right)\frac{\Delta\left(e^{(\ell-i)t_1}-1\right)}{(\ell-i)}$$

$$-\left(\left(\frac{C_{d1}+C_{d1}^{'}+2(C_{d2}+C_{d2}^{'}+C_{d3}+C_{d3}^{'})+C_{d4}+C_{d4}^{'}}{6}\right)\right.$$

$$-\left(\frac{C_{s1}+C_{s1}^{'}+2(C_{s2}+C_{s2}^{'}+C_{s3}+C_{s3}^{'})+C_{s4}+C_{s4}^{'}}{6}\right.$$

$$\left.+\left(\frac{C_{o1}+2C_{o2}+2C_{o3}+C_{o4}}{6}\right)\right)\frac{e^{-\kappa T}\Delta(e^{(\kappa+\ell-i)T}-e^{(\kappa+\ell-i)t_1})}{(\kappa+\ell-i)}$$

$$-\frac{\left(\frac{C_{h1}+C_{h1}^{'}+2(C_{h2}+C_{h2}^{'}+C_{h3}+C_{h3}^{'})+C_{h4}+C_{h4}^{'}}{6}\right)\Delta\left(e^{(d+\ell)t_1}-e^{(\ell-i)t_1}\right)}{(d+\ell)(d+i)}$$

$$-\left(\frac{a_1+a_1^{'}+2(a_2+a_2^{'}+a_3+a_3^{'})+a_4+a_4^{'}}{6}\right)$$

$$-\frac{\Delta e^{-\kappa T}e^{(\kappa+\ell)t_1}\left(\frac{C_{s1}+C_{s1}^{'}+2(C_{s2}+C_{s2}^{'}+C_{s3}+C_{s3}^{'})+C_{s4}+C_{s4}^{'}}{6}\right)(e^{-iT}-e^{-it_1})}{i.(\ell+\kappa)}$$

$$\left.-\frac{\left(\frac{C_{o1}+2C_{o2}+2C_{o3}+C_{o4}}{6}\right)\Delta(e^{(\ell-i)T}-e^{(\ell-i)t_1})}{(\ell-i)}\right] \quad (18)$$

4. Solution Methodology

1. Firstly, we have to find the solution for *(t_1, T)* at a given price with the help of Wolfram Mathematica 9.0.
2. Obtain optimal selling *p* price at the value of *(t_1, T)* obtained in the first step.
3. Assign a fuzzified value to the cost components.
4. Defuzzified by using the graded mean integration method.
5. Apply steps 1 and 2.

4.1 Example 1. (Crisp model)

The proposed inventory model is illustrated and validated by taking the following data (from Maihami and Kamalabadi (2012) and rate of inflation from Saha and Sen (2019)) and we have used Wolfram Mathematica 9.0 to illustrations and validate

$$A=\$250/order,\ A^{'}=\$2.5/order,\ c=\$20/\text{unit}, c^{'}=\$0.2/unit, C_h=\$1/unit\,time,$$
$$C_h^{'}=\$0.1/unit\,time,\ C_s=\$5/unit\,time,\ C_s^{'}=\$0.05/unit\,time, C_o=\$25/unit\,time,$$
$$C_d=\$5/unit\,time,\ C_d^{'}=\$0.05/unit\,carbon\,emission,$$
$$d=0.08,\ \zeta=200,\ \xi=4, \ell=-0.98,\ \kappa=0.7,\ i=0.03, p=\$35$$

First we assume the value of p = 35 and optimized value of t_1 and T, we get Z = \$249.397, t^* = 0.856249 unit and T^* = 0.964649unit

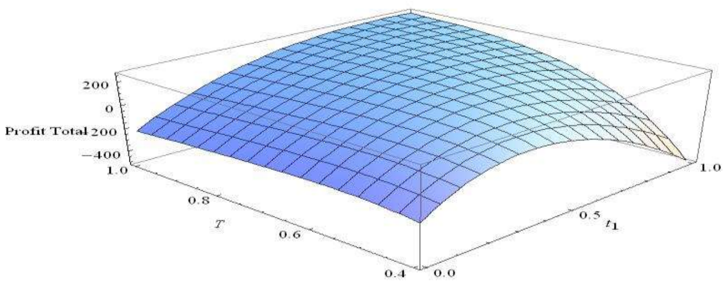

Figure 1 Concavity of profit function

Now we used t^* = 0.856249 and T* = 0.964649 to optimize the value of p and we get
p^* = 35.8332 and Z^*=251.162

4.2 Example 2. (Fuzzy model)

Here we assigned a trapezoidal fuzzy number to the cost parameter as below

$$d = 0.08,\ \zeta = 200,\ \xi = 4,\ \ell = -0.98,\ \kappa = 0.7,\ i = 0.3,\ p = 35$$

$$A = (a_1, a_2, a_3, a_4) = (230, 240, 255, 260),\ A' = (a_1', a_2', a_3', a_4') = (2.3, 2.4, 2.55, 2.6)$$

$$\tilde{c} = (c_1, c_2, c_3, c_4) = (17, 19, 21, 22),\ \tilde{c}' = (c_1', c_2', c_3', c_4') = (0.17, 0.19, 0.21, 0.22)$$

$$\tilde{C}_h = (C_{h1}, C_{h2}, C_{h3}, C_{h4}) = (0.7, 0.9, 1.1, 1.3),\ \tilde{C}_h' = (C_{h1}', C_{h2}', C_{h3}', C_{h4}') = (0.07, 0.09, 0.11, 0.13)$$

$$\tilde{C}_d = (C_{d1}, C_{d2}, C_{d3}, C_{d4}) = (3, 4, 6, 7),\ \tilde{C}_d' = (C_{d1}', C_{d2}', C_{d3}', C_{d4}') = (0.03, 0.04, 0.06, 0.07)$$

$$\tilde{C}_s = (C_{s1}, C_{s2}, C_{s3}, C_{s4}) = (3, 4, 6, 7),\ \tilde{C}_s' = (C_{s1}', C_{s2}', C_{s3}', C_{s4}') = (0.03, 0.04, 0.06, 0.07)$$

$$\tilde{C}_o = (C_{o1}, C_{o2}, C_{o3}, C_{o4}) = (21, 23, 27, 29),$$

Z = 259.372, t^* = 0.841475 and T^* = 0.957242.

Now we used t^* = 0.841475 and T* = 0.957242 for optimize the value of p and we get

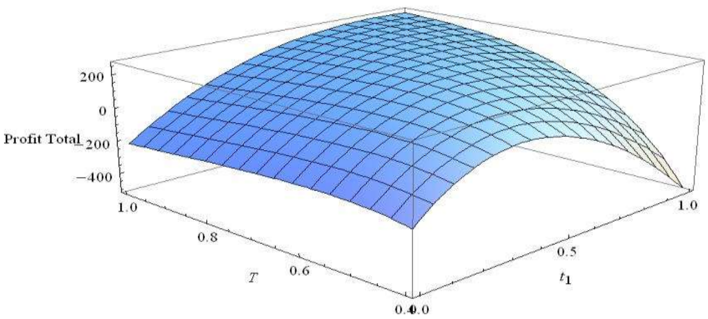

Figure 3 Concavity of fuzzy profit function

p^* = 35.7371 and Z^* =260.764.

5. Sensitivity Analysis

Sensitivity analysis has been carried out for the given examples to study the effect of changes of different parameters like deterioration, price and demand related parameters, and backlogging parameters by changing (increasing and decreasing) -20%, -10%, 10%, and 20% in the parameters at a time in both crisp and fuzzy model. The results of this analysis are shown in the following tables

Table 2: Analysis of ordering cost A

Change in A	Value of A	t_1	T	p	Z
-20%	200	0.728263	0.827605	35. 7346	306.480
-10%	225	0.792057	0.900834	35.7842	277.686
0	250	0.856249	0.974649	35.8332	251.162
10%	275	0.921281	1.049570	35.8818	226.597
20%	300	0.987580	1.12609	35.9301	203.752

Table 3: Analysis for holding cost C_h

Change in C_h	Value of C_h	t_1	T	p	Z
-20%	0.8	0.864708	0.978358	35.8017	253.711
-10%	0.9	0.860463	0.976499	35.8175	252.432
0	1	0.856249	0.974649	35.8332	251.162
10%	1.1	0.852065	0.972808	35.8988	249.899
20%	1.2	0.847912	0.970976	35.8642	248.645

Table 4: Sensitivity analysis for shortage cost Cs

Change in Cs	Value of Cs	t_1	T	p	Z
-20%	4	0.854541	0.974202	35.8316	251.247
-10%	4.5	0.856096	0.974654	35.8331	251.169
0	5	0.856249	0.974649	35.8332	251.162
10%	5.5	0.859089	0.974560	35.8359	251.021
20%	6	0.860530	0.974515	35.8372	250.949

Table 5: Sensitivity analysis for opportunity cost Co

Change in Co	Value of Co	t_1	T	p	Z
-20%	20	0.845563	0.97505	35.8230	251688
-10%	22.5	0.851891	0.974814	35.8291	251.376
0	25	0.856249	0.974649	35.8332	251.162
10%	27.5	0.862808	0.974423	35.8393	250.838
20%	30	0.867551	0.974260	35.8435	250.605

Table 6: Sensitivity analysis for C_d

Change in C_d	Value of C_d	t_1	T	p	Z
-20%	4	0.861556	0.976671	35.8160	252.565
-10%	4.5	0.859581	0.975637	35.8253	251.829
0	5	0.856249	0.974649	35.8332	251.162
10%	5.5	0.855648	0.973577	35.8437	250.361
20%	6	0.853690	0.972551	35.8528	249.630

Table 7: Sensitivity Analysis for C

Change in C	Value of C	t_1	T	p	Z
-20%	16	0.751252	0.836765	33.6775	417.321
-10%	18	0.798361	0.897729	34.7493	329.994
0	20	0.856249	0.974649	35.8332	251.162
10%	22	0.935856	1.076340	36.9397	180.771
20%	24	1.04760	1.2483	38.0780	119.219

Table 8: Sensitivity analysis forζ

Change in ζ	Value of ζ	t_1	T	p	Z
-20%	160	2.2081×10^{22}	8.57063×10^{39}	32.2542	-1.7×10^{39}
-10%	180	1.202320	1.374980	33.5792	97.9563
0	200	0.856249	0.974649	35.8332	251.162
10%	220	0.694767	0.789207	38.2081	456.690
20%	240	0.597203	0.677558	40.6294	713.372

Table 9: Sensitivity analysis forξ

Change in ξ	Value of ξ	t_1	T	p	Z
-20%	3.2	0.650661	0.738697	41.9228	612.223
-10%	3.6	0.733949	0.834127	38.5168	402.941
0	4	0.856249	0.974649	35.8332	251.162
10%	4.4	1.060690	1.210640	33.7095	141.222
20%	4.8	1.51588	1.741480	32.1090	61.0171

Table 10: Sensitivity analysis for □

Change in ℓ	Value of ℓ	t_1	T	p	Z
-20%	-1.176	0.836828	0.952302	35.7980	214.095
-10%	-1.078	0.845378	0.962139	35.8145	232.082
0	-0.98	0.856249	0.974649	35.8332	251.162
10%	-0.882	0.869756	0.990198	35.8545	271.451
20%	-0.784	0.886322	1.00928	35.8788	293.093

Table 11: Sensitivity analysis for d

Change in d	Value of d	t_1	T	p	Z
-20%	0.064	0.874860	0.983020	35.7668	256.542
-10%	0.072	0.865514	0.978831	35.8003	253.839
0	0.08	0.856249	0.974649	35.8332	251.162
10%	0.088	0.847071	0.970478	35.8655	248.510
20%	0.096	0.837982	0.966323	35.8973	245.884

Table 12: Sensitivity analysis for κ

Change in κ	Value of κ	t_1	T	p	Z
-20%	0.56	0.836377	0.975279	35.8193	252.187
-10%	0.63	0.847102	0.974935	35.8268	251.631
0	0.7	0.856249	0.974649	35.8332	251.162
10%	0.77	0.864143	0.974407	35.8388	250.760
20%	0.84	0.871024	0.974201	35.8436	250.411

Table 13: Sensitivity analysis for i

Change in i	Value of i	t_1	T	p	Z
-20%	0.024	0.865306	0.978613	35.8094	254.231
-10%	0.027	0.860758	0.976621	35.8213	252.651
0	0.03	0.856249	0.974649	35.8332	251.162
10%	0.033	0.851779	0.972695	35.8450	249.644
20%	0.036	0.847347	0.970761	35.8567	248.137

5.1 Observation of Sensitivity Analysis

The sensitivity analysis across different cost components demonstrates a generally predictable trend: increases in various costs (such as ordering, holding, and deterioration costs) often lead to a reduction in total cost (Z), with modest impacts on other inventory factors like t_1 and T. Parameters like ζ and ξ, however, show more dramatic effects on inventory performance and cost efficiency, indicating that specific variables play a more crucial role in driving cost and performance outcomes in an inventory system. The analysis examines five different percentages of change in ordering cost (-20%, -10%, 0%, 10%, and 20%). Table-wise observation is given below

1. **Table 2**: As the ordering cost (A) increases, there is a steady rise in both t_1 and p, while total cost Z, experiences a slight upward trend. Conversely, T, likely reflecting some form of inventory or operational cost, shows a consistent decline, indicating that higher ordering costs may reduce overall expenses concerned with this.
2. **Table 3**: An increase in holding cost (C_h) results in a slight reduction in t_1 and Z, while p remains relatively stable. T also decreases marginally, suggesting that rising holding costs might slightly diminish inventory efficiency and total costs.
3. **Table 4**: As the shortage cost (C_s) increases, t_1 and T show minimal fluctuations, while p stays nearly constant. Z, reflecting total cost, shows a slight decline, indicating that higher shortage costs have a minimal but measurable effect on reducing total cost.
4. **Table 5**: With rising opportunity cost (C_o), t_1 increases slightly, while T and p exhibit minimal changes. Z experiences a minor decline, implying that increased opportunity costs marginally lower total costs without significantly affecting other variables.
5. **Table 6**: As the deterioration cost (C_d) increases, both t_1 and T show a very slight decrease, while p experiences a small increase. Z decreases slightly, indicating that higher deterioration costs reduce total costs marginally while slightly affecting inventory performance.
6. **Table 7**: As the cost c increases both t_1 and T significantly increase, while Z experiences a steep decline. This suggests that higher general costs significantly reduce total costs, even though the inventory duration and profit or pricing (p) increases.
7. **Table 8**: An increase in parameter ζ leads to a drastic reduction in t_1 and T, with p showing a steady increment. Z shifts dramatically, from negative values at -20% and soaring to high positive values at +20%. This suggests that ζ heavily influences both total costs and inventory efficiency.
8. **Table 9**: As parameter ξ increases, both t_1 and T increase substantially, while p decreases. Z shows a sharp reduction, implying that ξ significantly impacts inventory performance and total costs.
9. **Table 10**: When ℓ increases, T and p rise slightly, and Z shows a significant increase, indicating that higher values of ℓ result in higher total costs, likely due to extended inventory holding periods.
10. **Table 11**: As parameter d increases, t_1 and T decrease, while p shows a small increase. Z declines, suggesting that the higher values of d reduce total costs and slightly improve pricing or profit.
11. **Table 12**: Increase in the value of the parameter κ leads to a minor increase in t_1, with T remaining almost constant. p shows slight growth, while Z decreases marginally, indicating minimal impact on overall costs.
12. **Table 13**: As parameter i increases, both t_1 and T decrease, while p rises slightly. Z decreases, suggesting that higher values of i reduce total costs while showing a modest effect on inventory variables.

The analysis across multiple tables illustrates how changes in various cost factors, such as ordering, holding, and shortage costs, influence key inventory variables like t_1, T, p, and Z. As costs increase, t_1 and p generally show upward trends, while Z, representing total costs, tends to decline, indicating cost efficiency improvements. For instance, rising ordering costs lead to higher t_1 and p values but lower total expenses

(Z). Similarly, increased holding and deterioration costs cause slight reductions in t_1 and Z, while p remains stable. More substantial changes, like in parameters ζ and ξ, produce dramatic shifts in total costs and performance, revealing their significant impact on the overall inventory system. Overall, the data indicates that adjusting different cost factors can help optimize inventory performance and reduce total costs.

6. Managerial Implication

The significance of taking into account sustainability as an important aspect of decision-making in business operations is the basic idea of this model. This model is a complex yet comprehensive way of managing the real-world problems of the force operations with a critical consideration of carbon emission and ecological harm, by making Stakeholders aware of the crucial impacts on the ecology due to their operations. This helps them to make sure that they adapt according to time and price fluctuations, with the introduction of measures for alleviation of shortages and partial backlogging. Collaboration between force chain and hand training to support sustainability pretensions is inversely important. Striking a balance between profitability and sustainability ensures long-term growth reducing ecological harm. This model is useful for organizations in optimizing force practices reducing environmental footprints and gaining a competitive edge.

7. Conclusion

In conclusion, to serve the needs of an ever-evolving eco-friendly and environmentally conscious world, the development of a sustainable fuzzy inventory model for managing deteriorating goods with time and price-reliant demand, shortages, and partial backlogging, while taking into account the impact of inflation and carbon emission costs plays a crucial role. An important aspect of the model is fuzzy logic which helps in a flexible and adaptive decision-making process that in turn results not just in accurate but also efficient predictions considering the dynamic nature of demands and market volatility. Another aspect is the introduction of the Sustainability Concept by integrating carbon emission costs minimizing the Environmental footprints and giving a holistic approach to modern business operations. Moreover, keeping an account of the Inflation rate makes the model robust and adaptive in dealing with real-world scenarios of variation in economic growth. The cost parameter was given a trapezoidal fuzzy number to make the inventory model more plausible. By employing the graded mean integration method, we optimized the total profit function. Results observation demonstrates that the fuzzy model performs better when there is different cost uncertainty. For businesses operating in dynamic and uncertain environments, this approach provides a thorough foundation for integrated pricing and inventory control. Its implementation has the potential to improve operational efficiency, elevate customer satisfaction, and maximize profitability in many industries. Future advancements in the proposed fuzzy inventory model can be focused on greater adaptability, integration of emerging technologies, and alignment with global sustainability trends. These enhancements will make the model even more robust and versatile in addressing the challenges faced by businesses in an ever-changing economic, environmental, and technological landscape.

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