Mahgoub Transform and Ulam Stability of Logistic Growth Differential Equation

A.Mohanapriya[1], A.Ponmana Selvan[2*]

[1]Assistant Professor, Department of Mathematics, Christ College of Science and Management,Malur, Karnataka, India
[2]Department of Mathematics,Rajalakshmi Engineering College (Autonomous), Thandalam, Chennai, Tamil Nadu, India, Email: selvaharry@yahoo.com
*Corresponding Author

Received: 10.07.2024 Revised: 14.08.2024 Accepted: 11.09.2024

ABSTRACT
In the present work, the main objective is to find the solution of the logistic differential Equation by employing Mahgoub transform and a series powers method.Our analysis confirms the stability of the proposed equation using the Hyers-Ulam and Mittag-Leffer-Hyers-Ulam stability. Applying our results to a population model, we demonstrate the practical significance of the obtained solution for the logistic growth differential equation.

Keywords: Ulam stability, Hyers-Ulam stability, Mittag-Leffler-Hyers-Ulam stability, Logistic differential equation, Mahgoub transform

1. INTRODUCTION

A logistic differential equation (LDE) represents a specific type of differential equation with a logistic function as its solution. These equations find significant utility in modeling systems influenced by limiting factors, such as population growth. With broad applicability across various disciplines, LDEs have emerged as a potent tool, offering more realistic and practical models compared to exponential functions. The well-known differential equation for logistic growth is

$$\dot{U}(t) = U(t)(1 - U(t)). \quad (1)$$

For initial condition $U(0) = U_0$, the analytic solution of (1) can be imparted as

$$U(t) = \frac{U_0}{U_0 + (1 - U_0)e^{-t}}.$$

If $U_0 = 1/2$, then the solution is likely to get logistic function

$$U(t) = \frac{1}{1 + e^{-t}}$$

Abdelrahim Mahgoub introduced the Mahgoub transform [17] as an integral transform method to solve ordinary differential equations, and it has since been widely adopted by researchers for analyzing the solutions of various differential equations, including logistic differential equations [1, 2, 17, 28].

Hyers-Ulam stability has been a popular research topic in the field of functional equations [25], but in recent years, this idea has been applied to differential equations as well. The Hyers-Ulam stability asserts that the difference between an exact and approximate solution is always finite.

Let Y be a normed space and let I be an open interval. Assume that for any function $f : I \to Y$ satisfying the differential inequality

$$\left\| a_n(t)y^{(n)}(t) + \cdots + a_1 y'(t) + a_0 y(t) + h(t) \right\| \le \epsilon$$

for all $t \in I$ and for some $\epsilon > 0$, there exists a solution $f_0 : I \to Y$ of the differential equation

$$a_n(t)y^{(n)}(t) + \cdots + a_1 y'(t) + a_0 y(t) + h(t) = 0$$

such that $\|f(t) - f_0(t)\| \le K(\epsilon)$ for any $t \in I$, where $K(\epsilon)$ is an expression of ϵ only. Then, we say that the above differential equation has the Hyers-Ulam stability.

If the preceding statement is also true when we replace $\in$ and $K(\epsilon)$ by $\varphi(t)$ and $\phi(t)$, where φ, ϕ are appropriate functions not depending on t and t_α explicitly, then we say that the corresponding differential equation has the generalized Hyers-Ulam stability or Hyers-Ulam-Rassias stability.

In 1998, C. Alsina and R. Ger [4] were the first authors who investigated the Hyers-Ulam stability of differential equations. They proved in [4] the following Theorem.

Theorem 1.1. Assume that a differentiable function $f: I \to R$ is a solution of the differentialinequality $\|x'(t) - x(t)\| \le \epsilon$, where I is an open sub interval of R. Then there exists a solution $g: I \to R$ of the differential equation $x'(t) = x(t)$ such that for any $t \in I$, we have $\|f(t) - g(t)\| \le 3\epsilon$.
This result of C. Alsina and R. Ger [4] has been generalized by Takahasi [31]. They proved in [31] that the Hyers-Ulam stability holds true for the Banach Space valued differential equation $y'(t) = \lambda y(t)$.
The Hyers-Ulam stability of differential equations has been studied by several authors, such as [5, 6, 15, 16, 18, 19, 20, 21, 22]. In 2017, Jung et al. [13] used the Mahgoub transform method to demonstrate the H-U stability of linear differential equations. Similarly, Aggarwal et al. [2] employed this method to solve the linear Volterra integral equation. Researchers have also studied the Ulam stability of non-linear differential equations, as evidenced by the literature [6, 14]. The development of the Mittag-Leffler-Hyers-Ulam (MLHU) stability of differential equations began with Wang and Zhou's work in 2012 [33].
Now a days, the Hyers-Ulam stability of differential equations are investigated by number of authors in [5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 30, 32] and the Hyers-Ulam stability of differential equations has been given attention.
Similary, many different methods for solving differential equations have been used to study the Hyers-Ulam stability problem for various differential equation. But using transform techniques are also have more significant advantage for solving differential equations with initial conditions.
In 2014, Alqifiary and Jung [3] investigated the generalized Hyers-Ulam stability of

$$x^{(n)}(t) + \sum_{k=0}^{n-1} \alpha_k x^{(k)}(t) = f(t),$$

by using the Laplace transform method. In 2020, Murali and Selvan [24] established the different forms of Mittag-Leffler-Hyers-Ulam stability of the first order linear differential equation for both homogeneous and non-homogeneous cases by using Laplace transformation(see also [27, 29, 30, 32]).
In 2020, Murali, Selvan and Park [19] investigated the Hyers-Ulam stability of various differential equations using Fourier transform method. Recently, Jung, Selvan and Murali [13] established the various forms of Hyers-Ulam stability of the first-order linear differential equations with constant coefficients by using Mahgoub integral transform (see also [9, 23, 26, 28]).
Very recently, Murali, Selvan, Park and Lee [21] investigated the different forms of Hyers-Ulam stability and Mittag-Leffler-Hyers-Ulam stability of second order linear differential equation of the form $u'' + \mu^2 u = q(t)$ by using Abooth transform method (see also [22]).
We may apply these terminologies to other differential equations also (see []). In this paper, the Mahgoub transform is proposed as a solution to the LDE. While it demonstrates effectiveness in handling linear equations, addressing non-linear equations necessitates a hybrid approach that combines the Mahgoub transform with a series expansion of the dependent variable to achieve a solution. Subsequently, the paper explores different types of H-U and MLHU stability linked to the differential equation governing logistic growth within a population, as represented by the form (1.1).

2. Preliminaries

In this section, we present some notations, definitions and preliminaries which will be needed in this paper.
Definition 2.1. For each mapping $U: (0,\infty) \to R$ of continuousnand exponential order. We consider the set

$$A = \left\{U(t): |U(t)| < Ne^{\frac{|t|}{e_i}}, t \in (-1)^j X\,[0,\infty), i = 1,2\right\},$$

where$N > 0$ and $\in_1, \in_2 > 0$ may be finite and infinite. The Mahgoub transforms of $U \in A$ is is defined by

$$M(U(t)) = \widehat{U}(s) = s\int_0^\infty U(t)t\,dt, t \ge 0, \ \epsilon_1 \le U \le \epsilon_2.$$

Theorem 2.1. If $\widehat{U}(s)$ is the Mahgoub transform of U(t), then

1) $M(1) = 1$
2) $M(t^n) = \frac{n!}{S^n}$
3) $M(e^{at}) = \frac{S}{S-a}$
4) $M(U'(t)) = s(s) - sU(0)$
5) $M(U''(t)) = s^2\widehat{U}(s) - s^2U(0) - sU'(0)$
6) $M(U^{(n)}(t)) = s^n\widehat{U}(s) - \sum_{k=0}^{n-1} s^{n-k} U^{(k)}(0)$

Theorem 2.2. Assume that $U_1(t)$and $U_2(t)$,t≥ 0 are given functions.If $M(U_1(t)) = \widehat{U_1}(s)$ and $U_2(t) = \widehat{U_2}(s)$, then

$$M\{U_1(t) * U_2(t)\} = \frac{1}{S}\widehat{U_1}(s)\widehat{U_2}(s).$$

Definition2.2. If $M(U(t)) = \hat{U}(s)$, then $U(t)$ is called the inverse Mahgoub transform of $\hat{U}(s)$ and is denoted as $U(t) = M^{-1}(\hat{U}(s))$ where M^{-1} is the inverse Mahgoub transform operator.

3. Existence Of The Approximate Solution

In this section, we will look at thelogistic growth model differential equation

$$\frac{dU}{dt} = U - g(U), t \geq 0 \qquad (2)$$

with the initial condition

$$U(0)=U_0, \qquad (3)$$

Where g is a non – linear function of U.Taking Mahgoub transform to (2), we find

$$\hat{U}(s) - sU(0) = \hat{U}(s) - \hat{G}(s)$$

$$\hat{U}(s) = \frac{sU_0}{s-1} - \frac{\hat{G}(s)}{s-1}. \qquad (4)$$

Applying the inverse Mahgoub transform to both sides (4), we get

$$U(t) = U_0 e^t - M^{-1}\left(\frac{\hat{G}(s)}{s-1}\right) \qquad (5)$$

Let $g(U) = U^2$. Then by formal power series

$U(t) = \sum_{n=0}^{\infty} a_n t^n$,

we obtain $g(U) = (\sum_{n=0}^{\infty} a_n t^n)^2 = a_0{}^2 + 2a_0a_1t + (2a_0a_2 + a_1^2)t^2 + (2a_0a_3 + 2a_1a_2)t^3 + \cdots$

$$\hat{G}(s) = M\big(g(U)\big) = a_0{}^2 + \frac{2a_0a_1}{s} + \frac{4a_0a_2+2a_1^2}{s^2} + \frac{12a_0a_3+12a_1a_2}{s^3} + \ldots\ldots$$

However,equation (5) becomes

$$U(t) = U_0e^t - M^{-1}\left(\frac{a_0^2}{s-1} + \frac{2a_0a_1}{s(s-1)} + \frac{4a_0a_2+2a_1^2}{s^2(s-1)} + \frac{12a_0a_3+12a_1a_2}{s^3(s-1)} + \ldots\ldots\right)$$

$$\sum_{n=0}^{\infty} a_n t^n = U_0e^t - M^{-1}\left(\frac{a_0^2}{s} + \frac{a_0^2}{s^2} + \frac{a_0^2}{s^3} + \frac{2a_0a_1}{s^2} + \frac{2a_0a_1}{s} + \frac{4a_0a_2}{s^3} + \frac{2a_1^2}{s^3} + \frac{2a_0a_1}{s^3} + \text{---------}\right)$$

$$= U_0\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) - \left(a_0^2t + \frac{a_0^2t^2}{2!} + \frac{a_0^2t^3}{3!} + \frac{2a_0a_1t}{1!} + \frac{2a_0a_1t^2}{2!} + \frac{2a_0a_1t^3}{3!} + \frac{4a_0a_2t^3}{3!} + \frac{2a_1^2t^3}{3!} + \cdots\right)$$

$$= U_0 + (U_0 - a_0^2)t\left(\frac{U_0}{2} - \frac{a_0^2}{2} - a_0a_1\right)t^2 + \left(\frac{U_0}{6} - \frac{a_0^2}{6} - \frac{2a_0a_1}{3} - \frac{a_1^2}{3}\right)t^3 + \cdots$$

Equating coefficients of powers of t gives

$$a_0 = U_0$$

$$a_1 = U_0 - a_0^2$$

$$a_2 = \frac{U_0}{2} - \frac{a_0^2}{2} - a_0a_1$$

$$a_3 = \frac{U_0}{6} - \frac{a_0^2}{6} - \frac{2a_0a_1}{3} - \frac{a_1^2}{3}$$

…….

For $U_0 = \frac{1}{2}$, we have

$$a_0 = \frac{1}{2}, a_2 = \frac{1}{4}, a_3 = -\frac{1}{48}, \ldots$$

The Solution of (2) is

$$U(t) = \frac{1}{2} + \frac{1}{4}t - \frac{1}{48}t^3 + \cdots$$

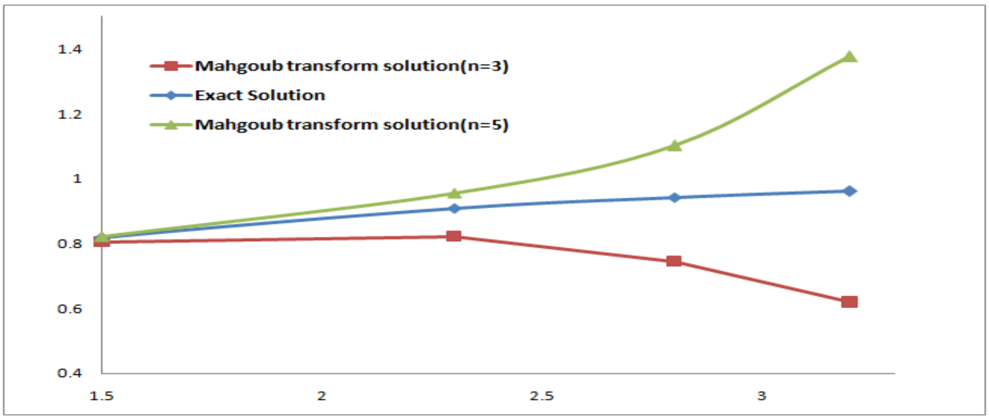

Figure 1: The exact solution (logistic function solution) to LDE (3.1) with initial condition U_0=1/2,in blue, as well approximations of the function by the Mahgoup transform for n=3 in green line and n=5 in red color line.

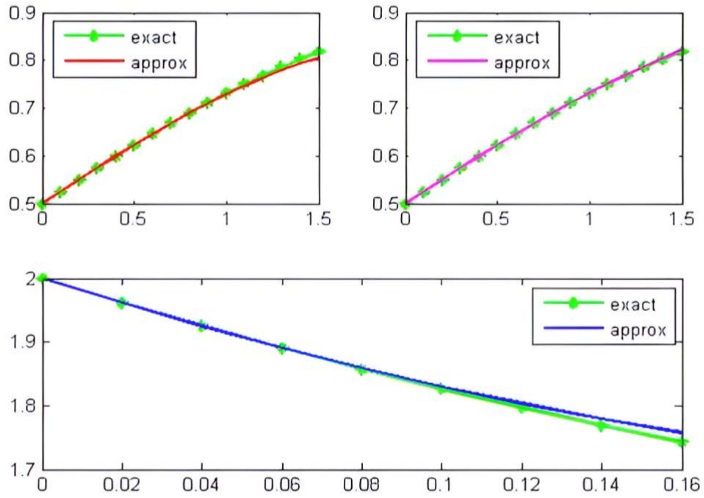

Figure 2: The comparison of logistic function(exact) as well approximation of the function(approximate) obtained by Mahgoub transform. The top two plots are shown for n=3, n=5 at initial condition $U_0=1/2$. The bottom plot for the initial condition $U_0=2$.

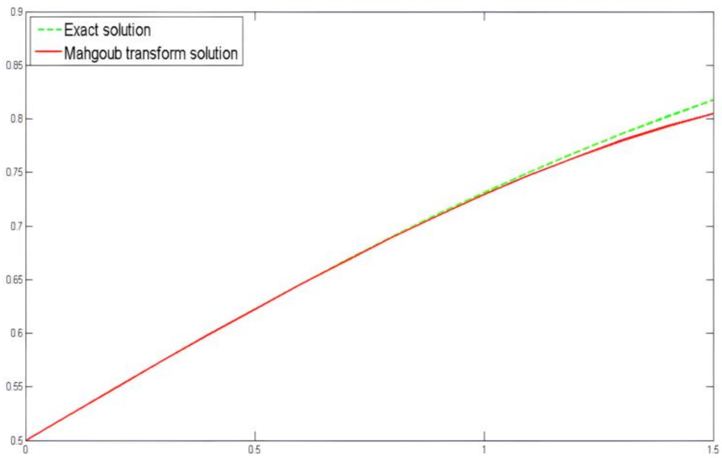

Figure 3: The comparison of exact solution and Mahgoub transform solution of LDE (3.1).

4. Stability Of Solutions

Let $I \in (0,\infty)$,we denote the space of continuously differentiable function on I by C(I).

Theorem 4.1. The logistic differential equation (2) has the Hyers-Ulam stability.

Proof. For every $\varepsilon > 0$, and we consider $U(t) \in C(I)$satisfies the differential inequality

$$|U'(t) - U(t) + g(U)| \le \varepsilon, \quad (6)$$

for all t∈ I. To prove: There is a real number K>0 and U(t) such that

$|U(t) - V(t)| \le K\varepsilon, \varepsilon > 0$

for some V∈ C(I), which satisfies the differential equation $V'(t)=V(t) - g(V)$ for all t∈ I.

Consider a function $p: (0,\infty) \to R$ such that $p(t) = U'(t) - U(t) + g(U)$ for all t>0. By (6), we find$|p(t)| \le \varepsilon$.

Taking Mahgoub transform to p(t), we acquire

$M\{p\} = (s-1)M\{U\} - sU(0) + M\{g(U)\}$

$\widehat{P}(S) = (s-1)\widehat{U}(s) - sU_0 + \widehat{G}(s)$

and

$$M\{U\} = \widehat{U}(s) = \frac{\widehat{P}(s) + sU_0 - \widehat{G}(s)}{s-1}, \quad (7)$$

where, $\widehat{U}(s) = M\{U(t)\}$, $\widehat{P}(s) = M\{P(t)\}$ and $\widehat{G}(s) = M\{g(U)\}$ are the Mahgoub transforms of the functions U(t), P(t) and g(U) respectively.

Define a solution $V(t) = sV_0 e^t + (e^t * g)$, then we find$V(0)=U(0)$. By applying the Mahgoub transform to $V(t)$, we acquire

$M\{U\} = \widehat{U}(s) = \frac{sU_0 - \widehat{G}(s)}{(s-1)}$. (8)

On the other hand, we find

$M\{U'(t) - sV(t) + g(V)\} = (s-1)\widehat{V}(S) - sV(0) + \widehat{G}(s)$.

Using (8), we get

$M\{V'(t) - sV(t) + g(V)\} = 0$.

If M is one-to-one operator and linear, then $V'(t) = V(t) - g(V)$.

That is, V(t) is a solution of (2) with the initial condition (3). From (7) and (8), we obtain

$$M\{U(t) - M\{V(t)\} = \frac{\widehat{P}(s) - \widehat{G}(s) + sU_0}{s-1} - \frac{sU_0 - \widehat{G}(s)}{(s-1)} = \frac{M\{P\}}{(s-1)} = M\{P(t) * e^t\}$$

The above equalities gives $U(t) - V(t) = P(t) * e^t$. Taking modulus on both sides and using $|P(t)| \le \varepsilon$, we get

$$\begin{aligned}|U(t) - V(t)| = |P(t) * e^t| &\le \left|\int_0^t P(t)e^{(t-x)}dx\right| \\ &\le |P(t)|\left|\int_0^t e^{(t-x)}dx\right| \le \varepsilon\int_0^t\left|e^{(t-x)}\right|dx \\ &\le \varepsilon e^t\int_0^t e^{-x}dx = K\varepsilon,\end{aligned}$$

for all t>0. Hence, $|U(t) - V(t)| \le K\varepsilon$, where $K=e^t-1>0$. Therefore, (2) with the condition (3) has the Hyers – Ulam stability.

Theorem 4.2. Let $\varepsilon > 0, U(t) \in C(I)$ and $\emptyset: (0,\infty) \to R$. Then

$\left|U'(t) - U(t) + g(U)\right| \le \emptyset(t)\varepsilon$, (9)

for all $t \in I$. If there exists real number K>0 such that

$|U(t) - V(t)| \le K\emptyset(t)\varepsilon,\ V\in C(I)$

satisfies the differential equation $V'(t) - V(t) + g(V)=0$ for all $t \in I$.

Proof. Consider a mapping $P: (0,\infty) \to R$ by$P(t) = V'(t) - U(t) + g(U)$ for all t>0. By (9), we find $|P(t)| \le \emptyset(t)\ \varepsilon$.By Theorem 4.1, we can reach

$U(t)-V(t) = P(t) *e^t$.

Taking modulus and using $|P(t)| \le \emptyset(t)\varepsilon$, we get

$$\begin{aligned}|U(t) - V(t)| = |P(t) * e^t| &\le \left|\int_0^t P(t)e^{(t-x)}dx\right| \\ &\le |P(t)|\left|\int_0^t e^{(t-x)}dx\right| \\ &\le \varepsilon\emptyset(t)\int_0^t\left|e^{(t-x)}\right|dx \\ &\le K\emptyset(t)\varepsilon,\end{aligned}$$

for all t>0. Then the LDE (2) with the initial condition (3) has the HU – Rassias stability.

Theorem 4.3. The differential equation in a logistic growth model (2) is MLHU stable.

Proof. Given $\varepsilon > 0$, suppose that $U(t) \in C(I)$ satisfies

$\left|U'(t) - U(t) + g(U)\right| \le \varepsilon E_a(t)$, (10)

for all $t\in I$, where $E_a(T)$ is the Mittag – Leffler function.

To prove: There exists real number K>0 and U(t) such that

$|U(t) - V(t)| \le K\varepsilon E_a(t), \varepsilon > 0$,

for some $v\in C(I)$satisfies $V'(t) = V(t) - g(V)$ for all $t \in I$.

Define a function $P: (0,\infty) \to R$by $P(t) =: U'(t) - U(t) + g(U)$, for all t>0. In view of (10), we find$|P(t)| \le \varepsilon E_a(t)$. Taking Mahgoub transform to P(t), we find

$$M\{P\} = (s-1)M\{U\} - sU(0) + M\{g(U)\} \quad (11)$$

$$\hat{P}(s) = (s-1)\hat{U}(s) - sU_0 + \hat{G}(s) \quad (12)$$

and

$$M\{U\} = \hat{U}(s) = \frac{\hat{P}(s) + sU_0 - \hat{G}(s)}{s-1} \quad (13)$$

Set $sV_0e^t + (e^t * g)$, then we have V(0)=U(0). Taking Mahgoub transform to V(t), we acquire

$$M\{V\} = \hat{V}(s) = \frac{sU_0 - \hat{G}(s)}{(s-1)} \quad (14)$$

On the other hand,

$$M\{V'(t) - V(t) + g(V)\} = (s-1)\hat{V}(s) - sV(0) + M(g).$$

Using (14), we get $M\{V'(t) - V(t) + g(V)\} = 0$. Since M is one-to-one operator and linear, then we get $V'(t) = V(t) - g(V)$. This means that V(t) is a solution of (2). It follows from (13) and (14) that

$$U(s) - V(s) = \frac{\widehat{P(s)} - \widehat{G(s)} + sU_0}{s-1} - \frac{sU_0 - \hat{G}(s)}{(s-1)}$$
$$= \frac{M\{P\}}{(s-1)}$$
$$M\{U(t) - M\{V(t)\} = M\{P(t) * e^t\}.$$

The above equalities shows that

$$U(t) - V(t) = P(t)*e^t.$$

Taking modulus and using $|P(t)| \leq \varepsilon E_a(t)$, we get

$$|U(t) - V(t)| = |P(t) * e^t| \leq \left|\int_0^t P(t)e^{(t-x)}dx\right|$$
$$\leq |P(t)| \left|\int_0^t e^{(t-x)}dx\right|$$
$$\leq \varepsilon E_a(t) \left|\int_0^t e^{(t-x)}dx\right|$$

for all t>0, where K=$\left|\int_0^t e^{(t-x)}dx\right|$ exists. Hence, $|U(t) - V(t)| \leq K\varepsilon E_a(t)$.Then (2) with (3) has the MLHU stability.

Theorem 4.4. The LDE (2) with (3) has the MLHU – Rassias stability.

Proof. Given $\varepsilon > 0$. Suppose that $U(t) \in C(I)$ and $\emptyset(t): (0, \infty) \to R$ satisfying

$$\left|U'(t) - U(t) + g(U)\right| \leq \emptyset(t)\varepsilon E_a(t), \qquad (15)$$

for all t $\in I$.

To prove: There exist a real number K>0 and U such that

$$|U(t) - V(t)| \leq K\emptyset(t)\varepsilon E_a(t),$$

for some V$\in C(I)$satisfies $V'(t) - V(t) + g(V) = 0$, for all t$\in I$.

Define $P: (0, \infty) \to R$ such that $P(t) =: U'(t) - U(t) + g(U)$ for all $t > 0$. By (15), we find $|P(t)| \leq \emptyset(t)\ \varepsilon E_a(t)$. Taking Mahgoub transform from P(t), we get

$$M\{P\} = (s-1)M\{U\} - sU(0) + M(g), \qquad (16)$$

and thus

$$M\{U\} = \frac{M\{P\} + sU_0 - M(g)}{s-1}. \qquad (17)$$

By the proof of Theorem 4.3 and using$|P(t)| \leq \emptyset(t)\ \varepsilon E_a(t)$, we get

$$|U(t) - V(t)| \leq \emptyset(t)\varepsilon E_a(t) \left|\int_0^t e^{(t-x)}dx\right|$$

for all t>0, where K=$\left|\int_0^t e^{(t-x)}dx\right|$exists. Hence,$|U(t) - V(t)| \leq K\emptyset(t)\varepsilon E_a(t)$. Thus the LDE (2) has the MLHU-Rassias stability.

5.Application

In this section, we'll go over some applications to help you comprehend the paper's primary findings. First, we study the logistic growth model in a population

$$\frac{dW}{dt} = \xi W\left(1 - \frac{W}{\eta}\right), \qquad (18)$$

Where ξand η are positive constants. Here W=W(t) represents the population of the species at time t and $\xi W\left(1 - \frac{W}{\eta}\right)$,is the per capita growth rate, and ηis the carrying capacity of the environment. Non – dimensionalization of equation (18) by setting

$$U(t) = \frac{W(t)}{\eta}, \quad t = \xi t,$$

results in

$$\frac{dU}{dt} = U(1 - U)\ . \quad (19)$$

If $W(0) = W_0$, then U(0)=$\frac{W_0}{\eta}$, and the analytical solution of the equation (19) follows easily

$$U(t) = \frac{1}{1 + \left(\frac{\eta}{W_{0-1}}\right)e^{-t}}.$$

So, we will apply Theorem 4.1 to the equation (19) to establish the Ullam stability.

6. CONCLUSION

In this paper, we have presented a solution to the differential equation for the logistic growth model. A figure to illustrate the result is given. Comparison with the exact solution and Mahgoub transform solution is plotted. We have also used the Mahgoub transform method to establish various H-U and MLHUtypes stability of these equations. Through the use of the logistic approach, we are able to predict population growth with great accuracy in mathematical modeling.

REFERENCES

[1] S. Aggarwal, N. Sharma, R. Chauhan, A.R. Gupta and A. Khandelwal, A new application of Mahgoub transform for solving linear ordinary differential equations with variable coefficients, J. Computer and Math. Sci., 9(6), 520-525(2018).

[2] S. Aggarwal, N. Sharma and R. Chauhan, Application of Mahgoub transform for solving linear Volterra integral equations of first kind, Global Journal of Engineering Science and Researches, 5 (9), 154-161(2018).

[3] Q.H. Alqifiary and S.M. Jung, Laplace transform and generalized Hyers-Ulam stability of differential equations, Electronic Journal of Differential Equations,2014 (80), 1-11(2014).

[4] C. Alsina and R. Ger, On Some inequalities and stability results related to the exponential function,J.Inequ. Appl. 2, 373–380 (1998).

[5] S. Baskaran, R. Murali, Choonkil Park and PonmanaSelvanArumugam, Sumudu Transform and the Stability of Second Order Linear Differential Equations, J. Math. Inequal. 18 (3), 847–864 (2024). doi:10.7153/jmi-2024-18-46.

[6] S. Baskaran, R. Murali and A. PonmanaSelvan, Mohand Transforms and its Application to Stability of Differential Equation, J. Math. Comp. Sci., 35 (1), 25-34 (2024). DOI: https://dx.doi.org/10.22436/jmcs.035.01.03

[7] A. Buakird and S. Saejung, Ulam stability with respect to a directed graph for some fixed point equations, Carpathian Journal of Mathematics, 35 (1), 23-30(2019).

[8] M. Choubin and H. Jananshiri, A new approach to the Hyers-Ulam-Rassias stability of differential equations, Results in Mathematics,76 (11), 1-14(2021).

[9] A. EmimalKanagaPuhpam and S. Katrin Lydia, Mahgoub transform method for solving linear fractional differential equations, International Journal of Mathematics Trends and Technology, 58 (4), 253-257 (2018).

[10] R. Fukutaka and M. Onitsuka, Best constant in Hyers-Ulam stability of first-order homogeneous linear differential equations with a periodic coefficient, J. Math. Anal. Appl., 473, 1432-1446(2019).

[11] J. Huang, S.M. Jung and Y. Li, On Hyers-Ulam stability of nonlinear differential equations, Bull. Korean Math. Soc., 52 (2) 685-697(2015).

[12] Park, Sehie. "All metric fixed point theorems hold for quasi-metric spaces." Results in Nonlinear Analysis 6.4 (2023): 116-127.

[13] S. M. Jung, Approximate solution of a Linear Differential Equation of Third Order, Bull. Malaysian Math.Soc.,35(4), 1063–1073 (2012).

[14] S.M. Jung, A.PonmanaSelvan and R. Murali, Mahgoub transform and Hyers-Ulam stability of first-order linear differential equation, J. Math. Inequal.,15 (3), 1201-1218(2021).

[15] S.M. Jung, A fixed point approach to the stability of differential equations y = F(x, y), Bull. Malaysian Math. Sci. Soc., 33 (1), 47-56(2010).

[16] S.M. Jung, A. M. Simoes, A. PonmanaSelvan and J.Roh, On the stability of Bessel differential equation, J. Appl. Anal. Comp., 12(5), 2014–2023 (2022).

[17] R. Khandelwal, Y. Khandelwal and P. Chanchal, Mahgoub deterioration method and its application in solving duo-combination of nonlinear partial differential equations, Mathematical Journal of Interdisciplinary Science, 7(1),37-44, 2018.

[18] M.M.A. Mahgoub, The new integral transform Mahgoub transform, Adv. Theoretical and App. Math, 11(4), 391-398 (2016).

[19] R. Murali and A. PonmanaSelvan, Mittag-Leffler-Hyers-Ulam Stability of a linear differential equations of first order using Laplace Transforms, Canadian Journal of Applied Mathematics, 2 (2), 47–59(2020).

[20] R. Murali, A. PonmanaSelvan and Choonkil Park, Ulam stability of linear differential equations using Fourier transform, AIMS Mathematics, 5 (2), 766–780(2019).

[21] R. Murali and A. PonmanaSelvan, Hyers-Ulam Stability of a Free and Forced Vibrations", Kragujevac Journal of Mathematics,44 (2), 299–312(2020).

[22] R. Murali, A. PonmanaSelvan, C. Park and J.R. Lee, Aboodh transform and the stability of second order linear differential equations, Adv. Difference Equation, 2021: 296, 18 pages, (2021).

[23] R. Murali, A. PonmanaSelvan, S. Baskaran, C. Park and J.R. Lee, Hyers-Ulam stability of first-order linear differential equations using Aboodh transform, J.Inequal. Appl., 2021:133, 18 pages, (2021).

[24] R. Murali, A. PonmanaSelvan and S. Baskaran, Stability of linear differential equation higher order using Mahgoub transforms, J.Mathematics. Comp.Sci., 30, 1–9(2023).

[25] A. PonmanaSelvan, G. Ganapathy, M. Saravanan and R. Veerasivaji, Laplace transform and Hyers-Ulam Stability of Differential Equation for Logistic growth in a population Model, Comm. Korean Math. Soc., 38 (4),1163–1173 (2023).

[26] A. PonmanaSelvan, W.G. Park and JaiokRoh, Stability and Instability of an Apollonius-Type Functional Equation, Difference and Differential Equations, Mathematics, 12 (14), 11 pages(2024).
[27] A. PonmanaSelvan and Sandra Pinelas, Mahgoub Transform and Hyers-Ulam Stability of Second-order Linear Differential Equations, Analysis and Applications, Special Issue of Dedicated to Professor Themistocles M. Rassias on the occasion of his 70th Birthday of the Montes Taurus Journal of Pure and Applied Mathematics,4 (3), 263–269(2022).
[28] J.M. Rassias, R. Murali and A. PonmanaSelvan, Mittag-Leffler-Hyers-Ulam Stability of Linear Differential Equations using Fourier Transforms, J. Comp. Anal.Appl., 29, 68-85, (2021).
[29] S. Sabarinathan, D. Muralidharan and A. PonmanaSelvan, Application of Mahgoub integral transform to Bessel's Differential Equations, Comm. in Math.Appl., 12 (4), 919–930 (2021).
[30] A. Selvam and S. Sabarinathan and A. PonmanaSelvan, Mittag-Leffler-Hyers-Ulam Stability for a First and Second Order Nonlinear Differential Equations Using Fourier Transform, DemonstratioMathematica, 57(1), Art. No. 20240033, 13 pages, (2024).
[31] A. Mohanapriya, V.Sivakumar and P. Prakash, A generalized approach of fractional Fourier transform to stability of fractional differential equation,Korean J. Mathematics,29 (4), 749–763 (2021).
[32] S. E. Takahasi, T. Miura, T. and S. Miyajima, On the Hyers-Ulam stability of the Banach space-valued differential equation $y' = \alpha y$, Bull. Korean Math. Soc.,39, 309-315 (2002).
[33] A. Ganesh, V. Govindan,J.R. Lee, A. Mohanapriya and C. Park, Mittag-Leffler-Hyers-Ulam stability of delay fractional differential equation via fractional Fourier transform, Results in Mathematics,76 (4), 180,(2021).
[34] G. Wang, M. Zhou and L. Sun, Hyers-Ulam stability of linear differential equations of first order, Appl. Math.Lett., 21 (10), 1024-1028 (2008).