An EOQ inventory model with fuzzy demand multichannel distribution under effect of learning

Ravindra Kumar[1], Pushpendra Kumar[2*], Purvi J Naik[3], Varsha Parihar[4], Soniya Gupta[5], A. K. Malik[6]

[1,2]Department of Mathematics, Shri Khushal Das University, Hanumangarh (Rajasthan)
[3]Department of Mathematics, Science and Humanities, UPL University of Sustainable Technology,Bharuch, Gujarat, India
[4]Department of Mathematics, BN University, Udaipur (Rajasthan)
[5]Department of Mathematics, Ismail National Mahila (PG) College Meerut (U.P.)
[6]School of Sciences, UP Rajarshi Tandon Open University, Prayagraj (U.P.)
Email: rkumar214053@gmail.com[1], pushpendrasaini22@gmail.com[2], naikpurvi2004@yahoo.co.in[3], varshaparihar1995@gmail.com[4], drsoniyainpg@gmail.com[5], profakmalik@uprtou.ac.in[6]
*Corresponding Author

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ABSTRACT
In this paper, we presentan ordering policies based inventory model with the effect of learning for the retail products during multichannel distribution where the demand of the product is imprecise (uncentain) in nature. The demand of the product is treated as triangular fuzzy number. As we know, the perishable items deteriorate in a short period and as soon as damage due to deterioration rate and deterioration can be ignored. Deterioration of perishable food can try to control with the help of preservation of the environment but extra charge bears to the seller or buyer. The preservation cost added in model this for more profit for the buyer. The effect of learning is holding the cost as well as ordering costand that's why the learning effect minimizes the holding cost and ordering cost.we minimized total inventory cost with respect to cycle length where demand is in imprecise in nature. The numerical example has been given for the justification of the proposed model and sensitivity analysis reflects the results of the proposed model. Few outcomes are useful when learning rate increases, retailer's total cost decreases, and cycle length is almost fixed. In the end, the entire inventory cost keeps down with respect to cycle length. The numbers of examples describe the relevance of the present model.

Keywords: Fuzzy environment, Learning effects, EOQ, Perishable items, Preservation, Deterioration.

1. INTRODUCTION

In this business field, there areseveral issues betweenretailer and customer concerning optimal profit along with total cost from one and other side and this kind of issuesmake efforts to solveby Adad and Jaggi (2003) with the help of an arithmetic model. Thismodel has shown the effects of credit financing policy. Shinn and Hwang (2003) estimate the highest price for the retailer's along with lot size operate with the help of credit financing scheme. In the inventory system,overall cost performs an important role.
Hung and Chung (2003) introduced an inventory model which is the enlarged form of Goyal (1985) describing how to reduce the total cost with the help of credit financing along with payment rebate scheme. An arithmetic model has been spread for best costing and batch sizing in which presume that buying quantity and selling price both are dissimilarbelow credit financing policy when demand is a function of selling price. Hung (2007) recommended progressing the EOQ model in an easy way and specified new suggestions on how to observe the optimal lot size for the retailer. Luo (2007) suggested an inventory model for the cooperation between retailer and customerbelow the credit financing scheme. The research work of Goyal et al. (1985) has been generalized by Su et al. (2007) with the help of credit financing policy and payment rebate.Teng et al. (2006) arranged a two-level credit financing inventory model and in this model the total order is optimized below the permissible delay in amount.
The analysis work of Huang (2003) has been explored by Huang (2006) and upgraded this model with the help of two-level credit financing as well as restricted storage space.Teng and Goyal (2007) introduced a stock model for the buyerswhen the buyer used the credit policy provided by the customerin the industrial sector. Huang (2007) stabilized the establishment of total economic order which was increased with the form of Haung (2003) with the help of a two-level credit financing scheme.

Huang and Hsu (2008) to expand a two-level credit financing schemewith the help of a partial credit scheme. An economic order quantity formulation has been presented by Jaggi et al. (2008) introducing an economic order quantity formulationwith the help of two-level credit policy with credit dependent demand. Huang (2007) research tasks have been modified by Teng and Chang (2009) with the help of a two-level credit financing scheme which is helpful for the consumers. Chen and Kang (2010) expand a model for an industrial system with the help of two-level credit policy with price dependent demand along with a satisfaction. Shah et at. (2010) has provided a large amount of articles for the inventory system with the help of a two-level credit financing scheme. That factor affectedthe cost of airplanes examined by Wright (1936). The learning effect on seller's ordering policy for defective items with trade credit financing was suggested by Jayaswal (2019). The total manufacturing of profit-makingmodels for items with defective quality subjected to the effect of learning described by Jaber et al. (2008). Mandal and Giri (2017) developed an imperfect quality based inventory model for two warehouse system under supply chain model where demand rate varies with respect to stock of the inventory. Sheikh and patel (2017) improved a two-warehouse based model by using of the shortages concept for deteriorating items where the rate of demand varies with time. Some authors shown the idea of the fuzzy in the system of two warehouse. The basic idea of the fuzzy theory is suggested by Zadeh (1965). Zadeh and Bellman (1970) developed a model by applying of the theory of fuzzy concept. Kao and Hsu (2002) derived an inventory model for the single period under fuzzy demand and also treated as a fuzzy number. In this order, De and Rawat (2011) worked on the fuzzy model with the help of triangular fuzzy number. Yadav et al.(2017) presented trade credit inventory model for two-warehouse under deterioration situation. Singha et al.(2019) considered a two-warehouses inventory model with shortages for decaying items under variable demand with respect to selling price. In this direction, Malik et al.(2024) proposed a green supply chain model with inflation and preservation under trade credit policy. Yadav et al. (2022) developed a fuzzy based inventory model for deteriorating items under variable demand rate. Singh and Malik (2010) assumed an EOQ model with variable demand rate under some realistic situations. Kumar (2021) derived the formula for the EOQ under fuzzy environment. Malik and Garg (2021) generalized an inventory model for two-warehouse system under fuzzy environment.Yadav et al.(2022) developed a fuzzy based inventory model for deteriorating items under variable demand.Mandal (2023) proposed an inventory model with fuzzy environment for the deterioration demand under the variable demand with respect to the stock and also variable holding cost.The learning effect is a one type of mathematical too which helps to minimize the total fuzzy cost corresponding to the number of shipment. we included some authors who worked on the learning theory, Jayaswal et al.(2019) developed a learning based model with the effect of trade policy for imperfect quality items under inspection.Jayaswal et al. (2021) presented an imperfect based scenario with trade-credit policy for deteriorating items under inspection process.Jayaswal et al. (2021) assumed an inventory model with preservation technology for the perishable items under learning effect.Alamri et al. (2023) explained an inventory model with fuzzy environment for imperfect items under trade credit policy.Alsaedi et al. (2023) presented a supply chain management with carbon emissions under learning fuzzy theorem. In this direction some authers have contributed likeMalik and Singh (2013), Malik et al. (2017),Maliket al.(2017),Malik and Sharma (2011),Vikram et al.(2016) and Singh and Malik (2010) and these authors gave an inventory model with different approach for diteriorating items under various.In this paper, we developed an inventory model with the combination of two-warehousepolicy and the effect of leaning in ordering cost for the perishable items under fuzzy environment. The total fuzzy cost defuzzyfied with the help of centroid fuzzy method. The behavior of the inventory input parameters also presented in the sensitivity analysis section and future work of this paper briefly explained in the conclusion section.

2. Assumptions and Notations

The mathematical model is derived using following notations and assumptions.

2.1 Assumptions

- Replenishment rate is infinite.
- The rate of demand is imprecise in nature
- The rate of demand is as a triangural fuzzy number
- There are no shortages in this model.
- The lead-time is considered to be zero.
- Unit purchasing cost is less than the unit selling price.
- No replacement policy of perishable items during cycle length.
- Holding cost and ordering cost are the following learning effects.

2.2 Notations

R Annual demand rate of the product (unit /year)

$\tilde{R}$ Annual fuzzy demand of the product(unit/year)

ξ Preservation cost($/unit item)

A Ordering cost which follows the learning effect ($/order)

P Selling price per unit ($)

θ Decaying rate (time)

C Unit purchase cost ($)

h Unit holding cost which follows the learning effect.($/unit/year)

Q Order quantity (unit)

T Cycle length (year)

$q(t)$ The inventory level in the interval, $0 \le t \le T$

$K(T)$ The whole inventory level cost($)

$\tilde{\tilde{K}}(T)$ The whole inventory fuzzy cost($)

3. Mathematical formulation

In the Figure 1, we are considering that, $q(t)$ is the inventory level in the interval $(0 \le t \le T)$. Initially, the stock level is Q. The present stock is reducing due to demand and deterioration and finally, it has finished at $t = T$. The rate of change of inventory stock are followed as differential equation as given below:

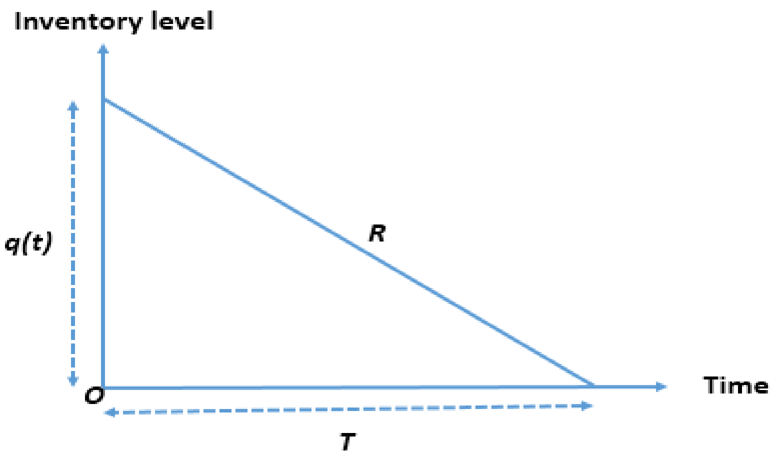

Figure 1: Representation of inventory model

$$\frac{dq(t)}{dt} + \theta q(t) = -R,\ t \in [0, T], \qquad (1)$$

With initial and boundary conditions, $q(0) = Q$ and $q(T) = 0$.

The solution of eq. (1) is given below;

$$q(t) = \frac{R}{\theta}\left(e^{\theta(T-t)} - 1\right) \qquad (2)$$

We calculated the order quantity by using of boundary condition from theequation (2), then, we get

$$Q = q(0) = \frac{R}{\theta}\left(e^{\theta T} - 1\right) \qquad (3)$$

Now, the component of cost concern this scenario which is given below

Ordering cost per cycle, $OC = \frac{1}{T}\left(C_1 + \frac{C_2}{n^{\beta}}\right)$ (4)

Holding cost per cycle, $IHC = \frac{\left(h_1 + \frac{h_2}{n^{\beta}}\right)R}{\theta^2 T}\left(e^{\theta T} - \theta T - 1\right)$ (5)

Deterioration cost per unit time $CD = C\left(Q - RT\right) = \frac{CR}{\theta T}\left(e^{\theta T} - \theta T - 1\right)$ (6)

Preservation cost $PV = \xi T$ (7)

Now, the total inventory cost per unit time is

$K(T) = \frac{1}{T}\left[IHC + OC + CD + PV\right]$(8

3.2 Formulation of model under fuzzy environment

As per assumption, it assumed that demand rate of the product is followed as a triangular fuzzy number and the total fuzzy cost has been defuzzified by the application of the centroid method (CM) and from the equation (8),
the total inventory fuzzified cost per unit time

$$\tilde{K}(T) = \frac{1}{T}\left[\frac{\left(h_1 + \frac{h_2}{n^{\beta}}\right)\tilde{R}}{\theta^2 T}\left(e^{\theta T} - \theta T - 1\right) + \frac{1}{T}\left(C_1 + \frac{C_2}{n^{\beta}}\right) + \frac{C\tilde{R}}{\theta T}\left(e^{\theta T} - \theta T - 1\right) + \xi T\right] \quad (9)$$

The total defuzzfied inventory cost by using of the centroid method, from the equation (9), we

$$\tilde{\tilde{K}}(T) = \frac{1}{T}\left[\frac{\left(h_1 + \frac{h_2}{n^{\beta}}\right)\left(\frac{(d_1 + d_2 + d_3)}{3}\right)}{\theta^2 T}\left(e^{\theta T} - \theta T - 1\right) + \frac{1}{T}\left(C_1 + \frac{C_2}{n^{\beta}}\right) + \frac{C\left(\frac{(d_1 + d_2 + d_3)}{3}\right)}{\theta T}\left(e^{\theta T} - \theta T - 1\right) + \xi T\right] \quad (10)$$

The equation (10) represents the total defuzzified cost for the multichannel of the retail product.

4. Solution Metho

In this section, we are calculating the decsion variable by using of maxima and minima property. We calculate the optimal cycle time and for the optimal cycle time T, we set $\frac{d\tilde{\tilde{K}}(T)}{dT} = 0$ which give th cycle time

$$T = T_1(\text{say}) = \sqrt{\frac{2\left(C_1 + \frac{C_2}{n^{\beta}}\right)}{\left(\frac{(d_1 + d_2 + d_3)}{3}\right)\left\{h_1 + \frac{h_2}{n^{\beta}} + C(\theta)\right\}}} \quad (11)$$

Now, we calculates the second derivatives

$$\frac{d\tilde{\tilde{K}}(T)}{dT} = -\frac{\left(C_1 + \frac{C_2}{n^{\beta}}\right)}{T^2} + \frac{\left(h_1 + \frac{h_2}{n^{\beta}}\right)\left(\frac{(d_1 + d_2 + d_3)}{3}\right)}{2} + \frac{C\left(\frac{(d_1 + d_2 + d_3)}{3}\right)\theta}{2} \quad (12)$$

and $\frac{d^2\tilde{\tilde{K}}(T)}{dT^2} = \frac{2\left(C_1 + \frac{C_2}{n^{\beta}}\right)}{T^3}$ (13)

which gives , $\dfrac{d^2\tilde{\tilde{K}}(T)}{dT^2}=\dfrac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{T_1^{\,3}}>0$ (14)

From (14), shows the convexity of total inventory cost, hence the optimal cycle length is that;

$$T=T_1(\text{say})=\sqrt{\frac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{\left(\dfrac{(d_1+d_2+d_3)}{3}\right)\left\{h_1+\dfrac{h_2}{n^\beta}+C(\theta)\right\}}} \quad (15)$$

The value of optimal cycle time from the equation (15) replaced in the equation (10), we get;

$$\tilde{\tilde{K}}(T)=\frac{1}{\sqrt{\dfrac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{\left(\dfrac{(d_1+d_2+d_3)}{3}\right)\left\{h_1+\dfrac{h_2}{n^\beta}+C(\theta)\right\}}}}\left[\begin{array}{l}\dfrac{\left(h_1+\dfrac{h_2}{n^\beta}\right)\left(\dfrac{(d_1+d_2+d_3)}{3}\right)}{\theta^2\left(\sqrt{\dfrac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{\left(\dfrac{(d_1+d_2+d_3)}{3}\right)\left\{h_1+\dfrac{h_2}{n^\beta}+C(\theta)\right\}}}\right)}\left(e^{\theta T}-\theta T-1\right)\\ +\dfrac{1}{\sqrt{\dfrac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{\left(\dfrac{(d_1+d_2+d_3)}{3}\right)\left\{h_1+\dfrac{h_2}{n^\beta}+C(\theta)\right\}}}}\left(C_1+\dfrac{C_2}{n^\beta}\right)\\ +\dfrac{C\left(\dfrac{(d_1+d_2+d_3)}{3}\right)}{\theta T}\left(e^{\theta T}-\theta T-1\right)+\xi\left(\sqrt{\dfrac{2\left(C_1+\dfrac{C_2}{n^\beta}\right)}{\left(\dfrac{(d_1+d_2+d_3)}{3}\right)\left\{h_1+\dfrac{h_2}{n^\beta}+C(\theta)\right\}}}\right)\end{array}\right] \quad (16)$$

4.1 Numerical example

Table 1: Inventory parameters and decesion variable

Fuzzy demand rate	$\tilde{R}$ =(400,500,600) (unit)
Fixed holding cost	$h_1 = 2$(\$)and $h_2 = 1$(\$)
Fixed ordering cost	$C_1 = 30$(\$) and $C_2 = 10$(\$)
Learning rate	$\beta = 0.23$
Decaying rate	$\theta = 0.23$
Preservation cost	$\varepsilon = 0.15$ (\$)
Number of shipment	$n = 5$
Purchasing cost	$C = 50$ (\$)
Optimal cycle time	$T^* = 0.2643$ (year)
Minimum inventory fuzzy cost	$\tilde{\tilde{K}}(T^*) = 3246$ (\$)

5. Sensitive analysis

In this section, we analysed the impact of inventory parameters on the optimal cycle time and total fuzzy inventory cost. the impact of learning rate, shipments, deterioration rate, preservation cost and fuzzy demand rate have been presented from the Table 2 to Table 6.

Table 2: Impact of learning rate under cycle time and whole fuzzy cost per cycle.

Learning rate β	Cycle length T (Year)	Retailer's totalfuzzy cost $\tilde{\tilde{K}}(T^*)$ (\$)
0.23	0.2643	3246
0.24	0.2643	3254
0.25	0.2643	3268
0.26	0.2643	3274
0.27	0.2643	3287

Table 3: Impact of the number of shipments on cycle time and whole fuzzy cost per cycle

Number of shipments (n)	Cycle length T (year)	Retailer's total fuzzy cost $\tilde{\tilde{K}}(T^*)$ ($)
1	0.2610	3204
2	0.2628	3212
3	0.2632	3225
4	0.2638	3238
5	0.2643	3246

Table 4: Impact of the decaying rate on cycle time and whole cost.

Deterioration rate θ	Cycle time T (Year)	Retailer's total fuzzycost $\tilde{\tilde{K}}(T^*)$ ($)
0.10	0.2754	2544
0.15	0.2730	2687
0.20	0.2643	3246
0.25	0.2340	3432
0.30	0.2010	3656

Table 5: Impact of the preservation cost on retailer's cycle time and whole cost

Preservation cost ξ /items	Cycle length T (year)	Retailer's total fuzzy cost $\tilde{\tilde{K}}(T^*)$ ($)
0.15	0.2643	3246
1.15	0.2643	3154
2.15	0.2643	3355
3.15	0.2643	3456

Table 6: Impact of the fuzzy demand rate on retailer's cycle time and whole cost

Fuzzy demand rate $\tilde{R}\,(d_1, d_2, d_3)$	Cycle length T (year)	Retailer's total fuzzy cost $\tilde{\tilde{K}}(T^*)$ ($)
$(400,,500,600)$	0.2643	3246
$(500,,600,700)$	0.2630	3358
$(700,,800,900)$	0.2594	3398
$(900,,1000,1100)$	0.2434	3456

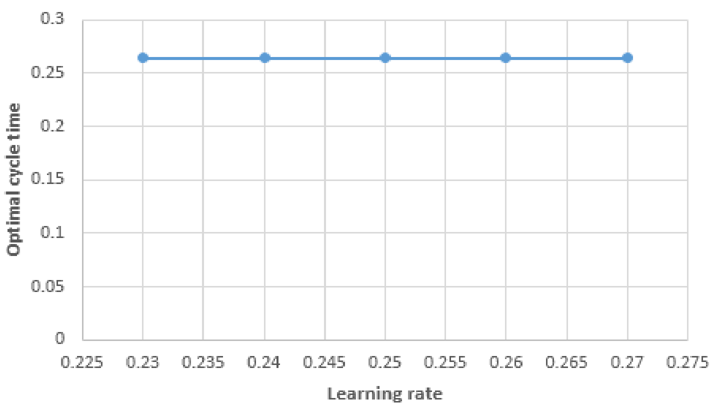

Figure 2: Impact of learning rate on optimal cycle time

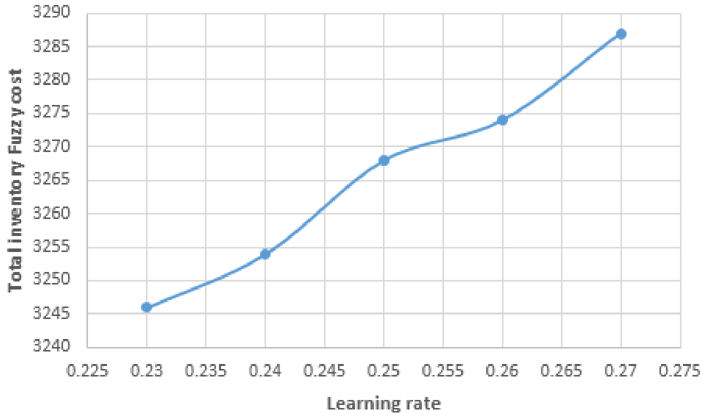

Figure 3: Impact of learning rate on total inventory fuzzy cost

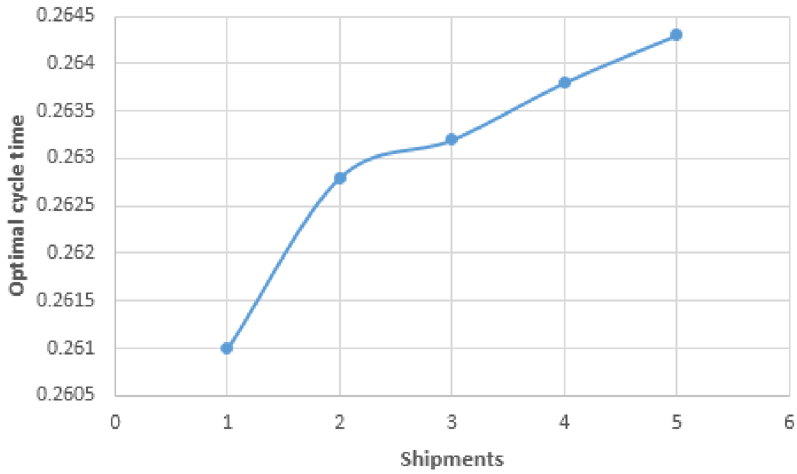

Figure 4: Impact of shipments on optimal cycle time

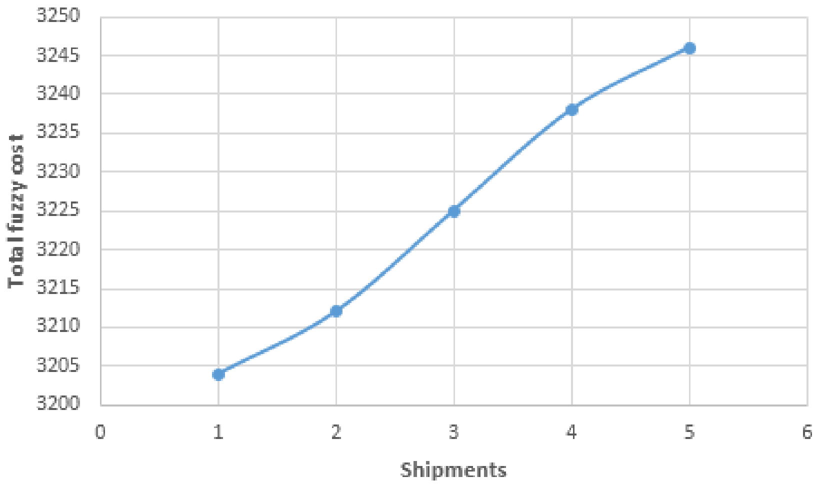

Figure 5: Impact of shipments on the total fuzzy cost

Observations and Managerial insights

➢ From Table-2,Figure 2 and 3, it is found that the learning rate is increased from 0.23 to 0.27, retailer's total fuzzy cost decreases when learning rate is increased and the cycle length is almost fixed. This study informs decision-makers to take account of the learning effect while making decisions which helps them to earn more profit for the organization. Hence the retailer gets more information for the exercise of shipments.

➢ From Table-3,Figure 4 and 5, it is found that the number of shipments increases, from 1-5, and the cycle length is increased up to 5th shipment. Retailer's total cost decreases when the number of shipments increases while cycle length is steadily increased.

- From table-4, it is found that the deterioration rate is increased slowly and the cycle time initially decreases while the seller's overall cost is increased with respect to deterioration rate.
- From table-5, it is found that the preservation cost is increased up to 0.15 to 3.15 and cycle length is fixed while the seller's overall fuzzy cost is increased.
- From Table-6, if the rate of fuzzy demand increases then, cycle length decreases and retailer's total fuzzy cost increases.

CONCLUSION

This article has tried to develop an arithmetic formula to decide cycle length and the correlation withoverall fuzzy cost with the effect of learning put in over the holding cost as well as the ordering cost where demand rate is treated as triangular fuzzy number. Eventually, we have come to an end and the consequences of this model appear that the seller's whole cost comes down as learning rate increases. When items are easily spoiled then preservation ought to be essential to control the deterioration rate however, the total cost increases. In this framework the items disclose that learning notion it is very helpful to get less inventory value. The mathematical analysis detected together clearly suggested that the existence of preservation and the effect of learning had a positive effect on the whole cost. Present work improves for more sensible positions such as supply reliant and cloudy environments etc.

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