Variable holding cost Inventory model under the permissible delay in payments

Anchal[1], Pushpendra Kumar[2*], Varsha Parihar[3], Soniya Gupta[4], A. K. Malik[5]

[1,2]Department of Mathematics, Shri Khushal Das University, Hanumangarh (Rajasthan)
[3]Department of Mathematics, BN University, Udaipur (Rajasthan)
[4]Department of Mathematics, Ismail National Mahila (PG) College Meerut(U.P.)
[5]School of Sciences, UP Rajarshi Tandon Open University, Prayagraj (U.P.)
Email: beniwal678@gmail.com[1], pushpendrasaini22@gmail.com[2], varshaparihar1995@gmail.com[3], drsoniyainpg@gmail.com[4], profakmalik@uprtou.ac.in[5]
*Corresponding Author

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ABSTRACT
In competitive business environment, product demand is often positively influenced by the displayed stock level, which is why suppliers frequently allow retailers to delay payments. This study examines an inventory model that explores the favorable conditions where time-varying holding costs, permissible delays in payment, and multi-level demand prove advantageous for manufacturers and businesses. In the proposed decision model, during the allowable delay period, retailers are permitted to postpone payments for purchased products without incurring interest charges. This study focuses on analyzing the optimal replenishment policy for products that deteriorate gradually and have variable demand. The results, supported by a numerical example, demonstrate the model's effectiveness in maximizing profit and optimizing replenishment time. Additionally, a sensitivity analysis is conducted to further illustrate the model's practical applications.

Keywords: Inventory, Multivariable demand, Permissible delay in payments, Inflation, variable holding cost.

1. INTRODUCTION

In today's world, displaying products in large quantities in malls, big bazaars, and supermarkets attracts more customers and increases demand. As a result, the effects of inflation and varying demand must be considered when developing the best inventory policy for businesses. However, most researchers studying inventory models often overlook the combined impact of trade credit and inflation. Both the option to delay payments and inflation have a significant influence on the optimal ordering policy and product demand. The first Economic Order Quantity (EOQ) model that included inflation was introduced by [1].A lot of researchers assumed the inventory models under the condition of permissible delay in payment. First an EOQ inventory model with constant demand assuming the permissible delay in payments [2]. Expanded the inventory model [2],assuming an inventory model using fix deterioration rate [4].

The first mathematical model aimed at optimizing total inventory cost, while accounting for stock-dependent demand in a utilization environment, was explored in [3]. Over the past two decades, numerous researchers worldwide have published studies on inventory models with demand dependent on inventory levels, incorporating other crucial factors such as deterioration, demand, permissible delay in payment, shortage, inflation;some of such articles are by [5-8].In production and inventory management literature, stock-dependent level is an appropriate methodology to deal customers to increase the interest level to purchase more items and inventory models with demand and deteriorate rate were discussed [9-19]. Some important optimization techniques conditions and supply chain models are discussed by [64-68]. Some studies were also conducted with two storage capacity with inventory model and deteriorate rate by [20-23].Inventory modeling is a well-established field within operations management, receiving significant attention in management science, operational research, and practitioner-focused journals. However, the literature on this topic is fragmented, and there is a lack of comprehensive, up-to-date reviews. When studying deteriorating inventory systems, factors like demand and deterioration rate must be carefully considered. Demand drives the inventory system, while the deterioration rate represents the characteristics of the items. Additionally, factors like price discounts,

shortages, inflation, and time-value of money are crucial in shaping inventory models. By combining these various elements, different inventory models can be developed to address specific scenarios.
A mathematical model for deteriorating inventory systems, with a focus on models where demand is quadratic types [24-28].Traditionally, most inventory models assume that products have an infinite shelf life while in storage, meaning they remain unchanged and fully usable to meet future demand. While the impact of deterioration can often be ignored when the rate of decay is minimal, there are many cases where deterioration significantly affects inventory management, necessitating its explicit consideration. Inventory models with stock-dependent demand are examined by the [29-36]. A particularly intriguing area within inventory theory is the mathematical modeling of deteriorating items with variable demand rate by [37-53]. Some optimum inventory model with fuzzy environment are discussed by [54-63]. The deterioration rate is another crucial factor in the study of deteriorating item inventories, as it characterizes the nature of how items deteriorate over time. When examining deterioration rates, several different scenarios can arise.
Deteriorating items are common in everyday life; however, there is no clear consensus in academia regarding their precise definition. Beyond demand and deterioration rate, other important factors such as price discounts, allowance for shortages, inflation, and the time-value of money also play significant roles in inventory management. Price discounts, for example, are a commonly used strategy by sellers to encourage bulk purchasing, and many researchers have incorporated this into models of deteriorating inventory.

2. Assumptions and Notations

For the developed inventory model, we use the following notations and assumptions:

- The demand rate is $D(t) = a + bt + cI_{nv}(t)$, Where $(a, b, c) \geq 0$ are positive constants and $I_{nv}(t)$ is the inventory level at time t.
- θ is the deterioration rate
- C_1 is the ordering cost per order
- C_3 is the deteriorating cost per unit
- C_4 is the purchasing cost per unit
- C_5 is the sales revenue cost per unit
- $C_h=h_1+h_2t$ is the inventory holding cost per unit time
- t_m is the permissible delay in payment offered by supplier in months
- R is the net discount rate of inflation; R = r-i, where r is the discount rate representing the time value of money, and i is the inflation rate
- Z_p is the interest charges per month
- $Z_e$is the interest earned per in stocks per month
- TVP_1 is the total optimum inventory profit per unit time of the developed system ($t_1 \leq t_m \leq T$).
- TVP_2 is the total optimum inventory profit per unit time of the developed system ($t_m \geq T$).
- $T=t_1+t_2$ total time horizon for the developed inventory system.

3. Mathematical Model

The initial concept behind developing the inventory model was that the retailer could generate revenue and earn interest before paying the purchasing cost. This is based on the assumption that the retailer benefits from the payment delay period offered by the supplier, allowing them to earn returns during this interval. During the time interval [0, t_1], the inventory level (I_{nv1}) decreases due to the multivariable demand rate. The inventory level drops to zero due to demand and the deterioration in the items during the interval is [t_1, T]; in time ($0 \leq t \leq t_1$) items has no deterioration and in time ($t_1 \leq t \leq T$) the items has deterioration. Thus, the examined model, the inventory level at any time t can be represented by the following differential equations:

$$\frac{dI_{nv}(t)}{dt} = \begin{cases} -D(t), & 0 \leq t \leq t_1 \\ -D(t) - \theta I_{nv}(t), & t_1 \leq t \leq T \end{cases} \quad \text{.... (1)}$$

with the boundary conditions $I_1(0) = L$, $I_2(t_1 + t_2 = T) = 0$ respectively, solving the above system of equations (1), we get

$$I_{nv}(t) = \begin{cases} (L + x_1)e^{-ct} - x_1 - \frac{b}{c}t, & 0 \leq t \leq t_1 \\ (x_2 - tx_3) + e^{(c+\theta)(T-t)}(Tx_3 - x_2), & t_1 \leq t \leq T \end{cases}$$

where $x_1=\left(\dfrac{ac-b}{c^2}\right),\ x_2=\left(\dfrac{b-a\left(c+\theta\right)}{\left(c+\theta\right)^2}\right),\ x_3=\left(\dfrac{b}{c+\theta}\right)$ .... (2)

Due to continuity of $I_{nv}(t)$ at $t=t_1$, it follows from the above system of equations (2) we have

$$L=\left(\frac{b}{c}t_1+x_1+x_2-t_1x_3\right)e^{ct_1}-x_1-e^{(c+\theta)T}\left(x_2-Tx_3\right) \quad ...(3)$$

The total optimum inventory profit per cycle contains the following components:

The inventory ordering cost (ICO) is CO= C_1. ...(4)

The inventory holding cost (ICH) is

$$ICH=C_h\left(\int_0^{t_1}e^{-Rt}\,I_{nv}(t)dt+\int_{t_1}^{T}e^{-Rt}\,I_{nv}(t)dt\right)$$

$$=h_1\left[\begin{array}{l}\left(\dfrac{L+x_1}{R+c}\right)\left(1-e^{-(R+c)t_1}\right)+\left(e^{-Rt_1}-1\right)\left(\dfrac{cRx_1+b}{cR^2}\right)\\+\dfrac{b}{cR}t_1e^{-Rt_1}+\dfrac{\left(x_3-x_2R\right)}{R^2}\left(e^{-RT}-e^{-Rt_1}\right)\\+\dfrac{x_3}{R}\left(Te^{-RT}-t_1e^{-Rt_1}\right)+\left(e^{(c+\theta)t_2-Rt_1}-e^{-RT}\right)\left(\dfrac{Tx_3-x_2}{R+c+\theta}\right)\end{array}\right]$$

$$+h_2\left[\begin{array}{l}\left(\dfrac{L+x_1}{(R+c)^2}\right)\left(1-e^{-(R+c)t_1}\left\{t_1\left(R+c\right)+1\right\}\right)\\-\dfrac{x_1}{R^2}\left(1-e^{-Rt_1}\left\{t_1R+1\right\}\right)\\-\dfrac{b}{R^3c}\left(2-e^{-Rt_1}\left\{t_1{}^2R^2+2t_1R+2\right\}\right)\\+\dfrac{\left(x_2-tx_3\right)}{R^2}\left(e^{-Rt_1}\left(Rt_1+1\right)-e^{-RT}\left(RT+1\right)\right)\\+\dfrac{\left(Tx_3-x_2\right)}{\left(R+c+\theta\right)^2}e^{(c+\theta)T}\left\{\begin{array}{l}e^{-(R+c+\theta)t_1}\left(\left(R+c+\theta\right)t_1+1\right)\\-e^{-(R+c+\theta)T}\left(\left(R+c+\theta\right)T+1\right)\end{array}\right\}\end{array}\right] \quad ...(5)$$

The inventory deterioration cost (ICD) is

$$ICD=C_d\int_{t_1}^{T}\theta\,e^{-Rt}\,I_{nv}(t)dt$$

$$=C_d\theta\left[\begin{array}{l}\dfrac{\left(x_3-x_2R\right)}{R^2}\left(e^{-RT}-e^{-Rt_1}\right)+\dfrac{x_3}{R}\left(Te^{-RT}-t_1e^{-Rt_1}\right)\\+\left(e^{(c+\theta)t_2-Rt_1}-e^{-RT}\right)\left(\dfrac{Tx_3-x_2}{R+c+\theta}\right)\end{array}\right] \quad ...(6)$$

The inventory purchasing cost (ICP) is

$$ICP=C_p\times L=C_p\left[\left(\frac{b}{c}t_1+x_1+x_2-t_1x_3\right)e^{ct_1}-x_1-e^{(c+\theta)T}\left(x_2-Tx_3\right)\right] \quad ...(7)$$

The inventory sales revenue cost (ISRC) is

$$ISRC=C_5\int_0^{T}e^{-Rt}\,D(t)dt$$

$$=C_5\left[\begin{array}{l}\left(\dfrac{L+x_1}{R+c}\right)\left(1-e^{-(R+c)t_1}\right)+\left(e^{-Rt_1}-1\right)\left(\dfrac{cRx_1+b}{cR^2}\right)\\+\dfrac{b}{cR}t_1e^{-Rt_1}+\dfrac{\left(x_3-x_2R\right)}{R^2}\left(e^{-RT}-e^{-Rt_1}\right)\\+\dfrac{x_3}{R}\left(Te^{-RT}-t_1e^{-Rt_1}\right)+\left(e^{(c+\theta)t_2-Rt_1}-e^{-RT}\right)\left(\dfrac{Tx_3-x_2}{R+c+\theta}\right)\\+\left(1-e^{-RT}\right)\left(\dfrac{aR+b}{R^2}\right)-\dfrac{bT}{R^2}e^{-RT}\end{array}\right] \quad ...(8)$$

In this paper we have considered two cases with permissible delay in payment:

Case-1: $t_1 \le t_m \le T$, the interest payable is

$$IP_1 = C_p Z_p \int_{t_m}^{T} I_{nv}(t)\,dt$$

$$= C_p Z_p \left[x_2 \left(T - t_m\right) - \frac{x_3}{2}\left(T^2 - t_m{}^2\right) - \left(\frac{Tx_3 - x_2}{c+\theta}\right)\left(1 - e^{(c+\theta)(T-t_m)}\right) \right] \quad \text{... (9)}$$

The interest earned is

$$IE_1 = C_s Z_e \left[\int_0^{t_1} t.\left(a + bt + cI_{nv}(t)\right)dt + \int_{t_1}^{t_m} t.\left(a + bt + cI_{nv}(t)\right)\,dt \right]$$

$$= C_5 Z_e \left[\begin{array}{l} \dfrac{at_m{}^2}{2} + \dfrac{bt_m{}^3}{3} + \left(\dfrac{L + x_1}{c}\right)\left(1 - e^{-ct_1}(ct_1 + 1)\right) \\ +c\left(\dfrac{x_2 t_m{}^2}{2} - \dfrac{x_3 t_m{}^3}{3} - \dfrac{x_2 t_1^2}{2} + \dfrac{x_3 t_1^3}{3}\right) - \dfrac{x_1 c t_1^2}{2} - \dfrac{b t_1^3}{3} \\ +c(Tx_3 - x_2)\left\{ \dfrac{e^{(c+\theta)t_2}}{c+\theta}\left(t_1 + \dfrac{1}{c+\theta}\right) - \dfrac{e^{(c+\theta)(T-t_m)}}{c+\theta}\left(t_m + \dfrac{1}{c+\theta}\right) \right\} \end{array} \right] \quad \text{.... (10)}$$

The total optimum inventory profit (TVP_1 for $t_1 \le t_m \le T$) per unit time is

$$TVP_1 = \frac{1}{T}\left[ISRC - ICO - ICH - ICD - ICP - IP_1 + IE_1 \right] \quad \text{... (11)}$$

For the profit function TVP_1 is maximum, and the necessary and sufficient conditions are $\frac{dTVP_1}{dt_2} = 0$ and

$\frac{d^2 TVP_1}{dt_2^2} < 0.$

Case-I1: $t_m \ge T$, there is no interest charges are paid for the products, *i.e.,*

$$IP_2 = 0 \quad \text{.... (12)}$$

The interest earned is

$$IE_2 = C_5 Z_e \left[\int_0^{t_1} t.\left(a + bt + cI_{nv}(t)\right)dt + \int_{t_1}^{t_m} t.\left(a + bt + cI_{nv}(t)\right)\,dt + D(t)T(t_m - T) \right]$$

$$= C_5 Z_e \left[\begin{array}{l} \dfrac{at_m{}^2}{2} + \dfrac{bt_m{}^3}{3} + \left(\dfrac{L + x_1}{c}\right)\left(1 - e^{-ct_1}(ct_1 + 1)\right) \\ +c\left(\dfrac{x_2 t_m{}^2}{2} - \dfrac{x_3 t_m{}^3}{3} - \dfrac{x_2 t_1^2}{2} + \dfrac{x_3 t_1^3}{3}\right) - \dfrac{x_1 c t_1^2}{2} - \dfrac{b t_1^3}{3} + aT(t_m - T) \\ +c(Tx_3 - x_2)\left\{ \dfrac{e^{(c+\theta)t_2}}{c+\theta}\left(t_1 + \dfrac{1}{c+\theta}\right) - \dfrac{e^{(c+\theta)(T-t_m)}}{c+\theta}\left(t_m + \dfrac{1}{c+\theta}\right) \right\} \end{array} \right] \quad \text{.... (13)}$$

The total optimum inventory cost (TVP_2 for $t_m \ge T$)per unit time is

$$TVP_2 = \frac{1}{T}\left[ISRC - ICO - ICH - ICD - ICP - IP_2 + IE_2 \right] \quad \text{...(14)}$$

For the profit function TVP_2 is maximum, and the necessary and sufficient conditions are $\frac{dTVP_2}{dt_2} = 0$ and

$\frac{d^2 TVP_2}{dt_2^2} < 0.$

4. Numerical Example and Sensitivity Analysis

To demonstrate the optimum solution procedure, discussed the following examples:

Ex.1. Let C_1=450, R=0.015, C_5=150, C_4=55, t_m=0.50 month, Z_p= 0.1, Z_e=0.090, h_1= 0.40, h_2= 0.40, C_3= 0.060, θ=0.45, a=450, b=0.40 and c=0.20. From Table 1, we observe that the inventory profitfunction (TVP_1=35117.1794) is maximum when t_1=1/2 and t_2=0.7720 month.

Table 1: Sensitivity analysis of various parameters with TVP_1

Change in		t_2	L	TVP_1	$\frac{d^2TVP_1}{dt_2^2} > 0$
C_1	495	0.7753	1057.8904	35081.8471	-8376.5263
	450	0.7720	1054.4771	35117.1794	-8381.3383
	405	0.7686	1051.0553	35152.6040	-8386.2348
R	0.0165	0.7648	1047.0952	35046.0533	-8499.8961
	0.015	0.7720	1054.4771	35117.1794	-8381.3383
	0.0135	0.7793	1062.0279	35188.8334	-8263.2899
C_5	165	0.9229	1217.7692	43385.0154	-5828.1382
	150	0.7720	1054.4771	35117.1794	-8381.3383
	135	0.6729	955.6716	26972.2756	-11012.4106
C_4	60.5	0.6791	961.6425	30536.6947	-11813.3819
	55	0.7720	1054.4771	35117.1794	-8381.3383
	49.5	0.9456	1243.8352	39829.1970	-5031.2868
t_m	0.55	0.7729	1056.4600	35360.3725	-8250.3899
	0.50	0.7720	1054.4771	35117.1794	-8381.3383
	0.45	0.7703	1052.8007	34878.3364	-8514.9970
Z_p	0.11	0.7556	1037.6924	35049.2515	-8728.7262
	0.10	0.7720	1054.4771	35117.1794	-8381.3383
	0.09	0.7893	1072.3762	35187.4801	-8035.8942
Z_e	0.099	0.7673	1049.6736	35199.3357	-8371.5780
	0.090	0.7720	1054.4771	35117.1794	-8381.3383
	0.081	0.7766	1059.2450	35035.2041	-8391.2696
h_1	0.44	0.7702	1052.7075	35098.2061	-8416.8892
	0.40	0.7720	1054.4771	35117.1794	-8381.3383
	0.36	0.7737	1056.2584	35136.1776	-8345.8063
h_2	0.044	0.7718	1054.3516	35116.4799	-8384.3060
	0.040	0.7720	1054.4771	35117.1794	-8381.3383
	0.036	0.7721	1054.6027	35117.8790	-8378.3702
C_3	0.066	0.7719	1054.3952	35116.8440	-8383.0117
	0.060	0.7720	1054.4771	35117.1794	-8381.3383
	0.054	0.7720	1054.5610	35117.5148	-8379.6648
θ	0.495	0.6774	988.9160	34104.5586	-10883.2665
	0.45	0.7720	1054.4771	35117.1794	-8381.3383
	0.405	0.8975	1147.4613	36212.9866	-6114.9088
a	495	0.7688	1156.3535	38661.5130	-9226.0403
	450	0.7720	1054.4771	35117.1794	-8381.3383
	405	0.7758	952.5936	31572.9453	-7536.7480
b	0.44	0.7721	1054.6277	35119.9870	-8379.6750
	0.40	0.7720	1054.4771	35117.1794	-8381.3383
	0.36	0.7719	1054.3266	35114.3719	-8383.0021
c	0.22	0.8462	1143.8538	36266.3128	-6801.4926
	0.20	0.7720	1054.4771	35117.1794	-8381.3383
	0.18	0.7199	994.6305	33997.2325	-9911.7873

The following graph (Fig. 1) shows the relation between optimal inventory profit (TVP_1) and time period t_1 and t_2.

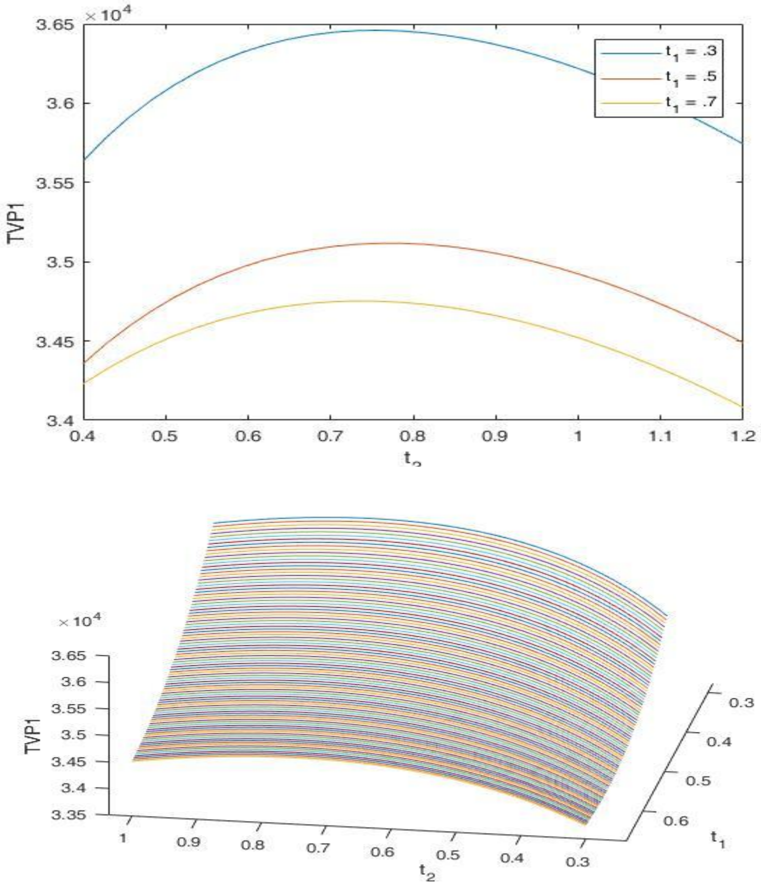

Fig 1: Graphical representation and 3-D view of optimal inventory profit TVP_1 and t_2, TVP_1 v/s t_2 and t_1

Ex.2. Let C_1=450, R=0.015, C_5=150, C_4=55, t_m=1.150 month, Z_p= 0.1, Z_e=0.090, h_1= 0.40, h_2= 0.40, C_3= 0.060, θ=0.45, a=450, b=0.40 and c=0.20. From Table 2, we observe that the inventory profit function(TVP_1=40223.1506) is maximum when t_1=1/2 and t_2=0.1784 month.

Table 2: Sensitivity analysis of various parameters with TVC_2

Change in		t_2	L	TVP_2	$\frac{d^2TP}{d^2t}$
C_1	495	0.1825	550.2881	40157.0188	-24105.8690
	450	0.1784	547.4704	40223.1506	-24234.6739
	405	0.1744	544.6417	40289.6769	-24365.9109
R	0.0165	0.1762	545.9287	40185.8314	-24477.9951
	0.015	0.1784	547.4704	40223.1506	-24234.6739
	0.0135	0.1807	549.0328	40260.6139	-23991.5973
C_5	165	0.1352	517.5923	48784.3072	-24417.1238
	150	0.1784	547.4704	40223.1506	-24234.6739
	135	0.1807	573.2636	31700.4762	-24443.8930
C_4	60.5	0.1352	568.8963	35797.8377	-26905.7189
	55	0.1784	547.4704	40223.1506	-24234.6739
	49.5	0.2149	517.2991	44680.9612	-21906.9744
t_m	1.165	0.1706	542.0256	40457.8299	-24474.1090
	1.15	0.1784	547.4704	40223.1506	-24234.6739
	1.135	0.1861	552.8061	39991.4230	-24007.0432
Z_p	0.11	0.1785	547.4704	40223.1506	-24234.6739
	0.1	0.1784	547.4704	40223.1506	-24234.6739
	0.09	0.1784	547.4704	40223.1506	-24234.6739
Z_e	0.099	0.1234	509.6225	41143.3774	-28037.6931
	0.09	0.1784	547.4704	40223.1506	-24234.6739

	0.081	0.2363	588.7592	39380.0356	-20942.6935
h_1	0.44	0.1782	547.2960	40210.6070	-24284.9683
	0.40	0.1784	547.4704	40223.1506	-24234.6739
	0.36	0.1787	547.6454	40235.6985	-24184.3784
h_2	0.044	0.1784	547.4606	40222.8566	-24237.4079
	0.040	0.1784	547.4704	40223.1506	-24234.6739
	0.036	0.1785	547.4802	40223.4447	-24231.9398
C_3	0.066	0.1784	547.4619	40223.1212	-24237.0557
	0.060	0.1784	547.4704	40223.1506	-24234.6739
	0.054	0.1785	547.4788	40223.1801	-24232.2921
θ	0.495	0.1665	549.9918	39563.2188	-27231.2198
	0.45	0.1784	547.4704	40223.1506	-24234.6739
	0.405	0.1888	542.8605	40920.2422	-21481.8712
a	495	0.1744	599.4077	44309.2019	-26793.7834
	450	0.1784	54704704	40223.1506	-24234.6739
	405	0.1829	495.5235	40920.2422	-21678.0130
b	0.44	0.1784	547.4526	40225.9211	-24230.1543
	0.40	0.1784	547.4704	40223.1506	-24234.6793
	0.36	0.1785	547.4882	40220.2812	-242391935
c	0.22	0.1744	543.3080	41136.2360	-22245.6397
	0.20	0.1784	547.4704	40223.1506	-24234.6739
	0.18	0.1843	553.0998	39228.1517	-26085.6331

The following graph (Fig. 2) shows the relation between optimal inventory cost (TVP_2) and time period t_1 and t_2.

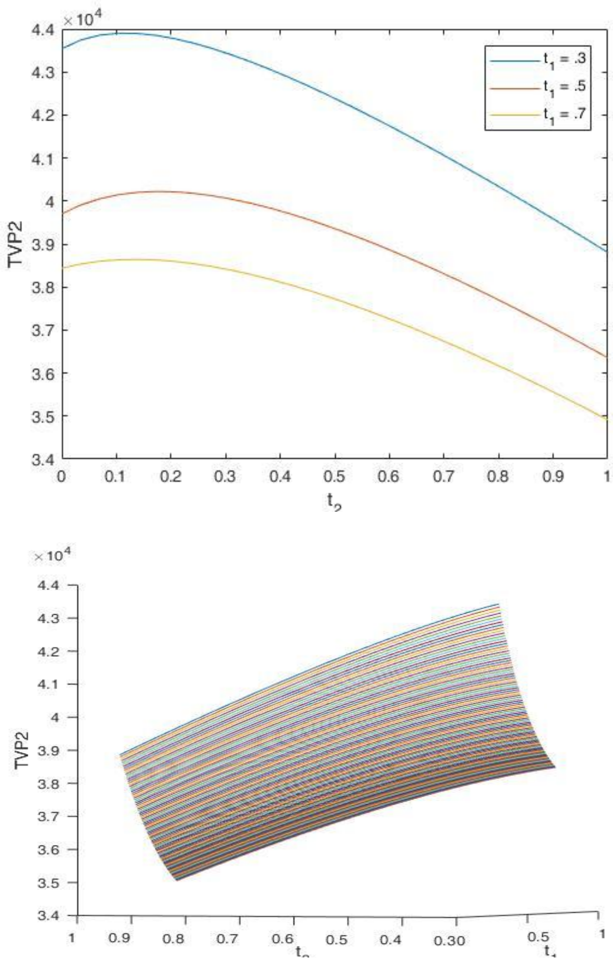

Fig 2: Graphical representation and 3-D view of optimal inventory cost TVP_2 and t_2, TVP_2 v/s t_2 and t_1

5. CONCLUSION

This paper examines an inventory system for non-instantaneously deteriorating items, taking into account the effects of inflation and trade credit terms. In real-world scenarios, most products deteriorate as they reach the end of their shelf life. The proposed inventory model includes two numerical examples to highlight key features of the results, along with a sensitivity analysis of the various parameters to the determine the optimal solution. It is recommended that more researchers focus on studying deteriorating item inventory problems within supply chains using fuzzy, stochastic, and dynamic research methods. This approach will help ensure that the findings can be effectively applied in practice. It is hoped that this paper provides a comprehensive overview of recent developments in deteriorating item inventory management, serving as a foundation for future research in this field. Additionally, future research directions could involve the development of inventory models that incorporate production-dependent factors, partial backlogging, and the use of two warehouses, among other considerations.

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