Control Chart Based On Six Sigma For Range With Robust Scale Estimator

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ABSTRACT
Good planning is a top priority for every organization, whether it is in the industrial or service sectors, since it will affect how well the company performs in the marketplace. Good planning makes it simpler to develop, maintain, and upgrade products in an economical manner. The amount of planning needed depends on whether the product is new or not. This study proposes a new six-sigma based control chart that outperforms the Shewhart (1931) chart when the underlying normalcy assumption is not met. The new control chart uses the interquartile range (IQR) under normal and exponential distributions for range.

Keywords: Range, Robust control chart, Six sigma.

1. INTRODUCTION
The root cause of all quality problems is process. The root cause is the one who bears the most of the blame for the quality problem. If the root cause is eliminated, the procedure or product quality ought to go close to perfection. Therefore, perfect procedures lead to perfect outcomes. The output of the process must be perfect in the short and long periods in order for its clientele to be continually satisfied.

Statistical process control, or SPC, is a quality control method that employs statistical approaches to ensure process quality. SPC is used to monitor and control a process. Monitoring and management of the process is necessary to ensure that it operates to its fullest capacity. The process produces the most compliant product with the least amount of waste when it is running at peak efficiency. Any process that yields quantifiable results can use SPC. Process quality is ensured by SPC, which also ensures product quality. SPC stands for statistical process control. It is a preventative method as opposed to a detection approach like inspection. SPC prioritizes problem prevention and early detection above post-mortem problem repair.The single tool utilized in SPC is the control chart. W.A. Shewhart designed the control chart to oversee and evaluate all repeated tasks. It is based on the premise that there is some degree of variation in all repeated procedures. Radhakrishnan (2009) suggested a single sample plan that was indexed to six sigma quality levels (SSQLs) using the weighted Poisson distribution and the intervening random effect Poisson distribution as the basis line distributions. Radhakrishnan and Sivakumaran (2009) used the six-sigma idea to build mixed sample plans with Chsp-2 plan and attribute plan that are indexed via six sigma quality levels. They created single, double, and repeated group sampling plans that were indexed by six sigma quality levels (SSQLs) utilizing the six-sigma concept, employing the Poisson distribution as the foundation line distribution.

2. Control chart
A control chart is one statistical technique used to determine whether or not the process is under control. In 1931, W.A. Shewhart invented it. Quantifiable quality parameters like part dimensions, height, weight, voltage, and so on are addressed by variable control charts. These charts are used to monitor process variation over time, assess process effectiveness, and distinguish between process variation that results from random causes and variation that arises from assignable reasons. Variable control charts fall into one of three groups, depending on the type of data.

- Control chart for averages called $\overline{X}$ -chart.
- Control chart for ranges called R-chart.
- Control chart for standard deviation called S-chart.

3. Control procedure with charts

Data collection, control limit computation, and control chart creation are the first steps in the process. The next step is to interpret the control charts for process control. Examine the assignable cause(s) if the process is out of control, eliminate it(them) via corrective action, and then allow the process to finish. Determining the assignable causes requires keeping account of the production sequence, time, and sources, which include the machine and tool used, the operator, and the batch of material employed.
Whether or not the process is under control will determine whether or not it meets the standard. If the process meets the criteria, sampling will continue for process monitoring and trend analysis, if any. If a process trend is observed, it is necessary to determine when to reset the machine. In particular, Radhakrishnan and Balamurugan (2012) developed a control chart based on six sigma efforts with a variable sample size for fraction defectives for the organizations implementing six sigma initiatives in the product/service sectors.

4. Process capability (C_P)

Process capability uses capability indices to compare an in-control process's output to the specification limitations. The ratio of the spread of the process values, as measured by six process standard deviation units, to the spread of the process specifications is formed to make the comparison.

$$Cp=\frac{\text{Upper Specification limit-Lower Specification limit}}{6\sigma}$$

5. Interquartile range (IQR)

A measure of variability based on quartile division of a data set is called the interquartile range (IQR). A rank-ordered data collection is divided into four equal pieces by these quartiles. The first, second, and third quartiles are the values that split each portion; they are represented by the letters Q_1, Q_2, and Q_3, respectively (Adekeye and Azubuike, 2012).

- Q_1 is the "middle" value in the first half of the rank-ordered data set
- Q_2 is the median value in the set
- Q_3 is the "middle" value in the second half of the rank-ordered data set

The interquartile range is equal to Q_3 minus Q_1.

6. Conditions for Application

- Less human involvement in the entire process/system is desired;
- The organization applies Six Sigma with strong measures quality efforts in its operations.

7. Methods and materials

To construct the control limits, we need an estimate of the standard deviation ‘s’. We may estimate ‘s’ from either the standard deviations orthe ranges of the ‘m’ samples. For the present, we will use the range method. If $X_1, X_2, ...X_n$ is a sample of size ‘n’, then the range of the sample is the difference between the largest andsmallest observations; that is,

$$R = X_{\max} - X_{\min}$$

Let $R_1, R_2, ...R_m$ be the ranges of the ‘m’ samples. The average range is

$$\bar{R} = \frac{R_1, +R_2 + ... + R_m}{m}$$

Process variability may be monitored by plotting values of the sample range ‘R’ on a control chart. The centre line and control limits of the ‘R’ chart are as follows:

Case (i): σ_X known

The control limits for range are given by

$$E(R) \pm 3\sigma_R = d_2\sigma_X \pm 3\sigma_R$$

$$E(R) \pm 3\sigma_R = \sigma_X\left(d_2 \pm 3d_3\right), \text{ where } d_2 + 3d_3 = D_2 \text{ and } d_2 - 3d_3 = D_1$$

Hence, the control limits on the R-chart if σ_x is known are as follows:

$$UCL_R = D_2\sigma_x \text{ and } LCL_R = D_1\sigma_x$$

Case (ii): σ_x unknown and estimated by $\frac{\bar{R}}{d_2}$

The control limits are given by

$$UCL_R = E(R) + 3\sigma_R = \bar{R} + 3d_3\sigma_X \text{ and } LCL_R = E(R) - 3\sigma_R = \bar{R} - 3d_3\sigma_X$$

The development of the equationsfor computing the control limits on the and R control charts is relatively easy. We observed that there is a well-known relationship between therange of a sample from a normal distribution and the standard deviation of that distribution. The random variable $W = \frac{R}{\sigma}$ is called the relative range. The parameters of thedistribution of 'W' are a function of the sample size 'n'. The mean of 'W' is 'd_2'. Therefore, it is the average range of the 'm' preliminary samples, wemay use $\hat{\sigma} = \frac{\bar{R}}{d_2}$.

Now, substituting $\frac{\bar{R}}{d_2}$ for $d_3\sigma_x$ in the above equation, we get the control limits as

$$UCL_R = \bar{R} + 3d_3\frac{\bar{R}}{d_2} = \bar{R}\left(1 + 3\frac{d_3}{d_2}\right) \text{ and } LCL_R = \bar{R} - 3d_3\frac{\bar{R}}{d_2} = \bar{R}\left(1 - 3\frac{d_3}{d_2}\right)$$

where

$$1 + 3\left(\frac{d_3}{d_2}\right) = D_4 \text{ and } 1 - 3\left(\frac{d_3}{d_2}\right) = D_3.$$

Hence, the control limits on the R-chart if σ_x is unknown are as follows:

$$UCL_R = D_4\bar{R} \text{ and } LCL_R = D_3\bar{R}$$

where d_2, d_3, D_1, D_2, D_3 and D_4 are quality control constants which are given in Appendix-I.

The construction of range control chart based on 3σ with the three-sigma probability level under normal distribution $p(z \le z_{3\sigma}) = 1 - \alpha_1, \alpha_1 = 0.0027$ is as follows:

$$UCL_R^{3\sigma} = \bar{R}_R + 3\left(\frac{d_3\bar{R}}{d_2}\right)$$

$$CL_R^{3\sigma} = \bar{R}_R$$

$$LCL_R^{3\sigma} = \bar{R}_R - 3\left(\frac{d_3\bar{R}}{d_2}\right)$$

where d_2 and d_3 are the quality control constants.

The standard deviation $\left(\sigma_{N.R}^{3\sigma-IQR}\right)$ for constructing control limits with three-sigma by using IQR for range under normal distribution as follows:

$$UCL_{N.R}^{3\sigma.IQR} = \bar{R}_R + 3\left(\frac{\sigma_{N.R}^{3\sigma-IQR}}{\sqrt{n}}\right)$$

$$CL_{N.R}^{3\sigma.IQR} = \bar{R}_R$$

$$LCL_{N.R}^{3\sigma.IQR} = \bar{R}_R - 3\left(\frac{\sigma_{N.R}^{3\sigma-IQR}}{\sqrt{n}}\right)$$

The standard deviation $\left(\sigma_{E.R}^{3\sigma-IQR}\right)$ for constructing control limits with three-sigma by using IQR for range under exponential distribution as follows:

$$UCL_{E.R}^{3\sigma.IQR} = \bar{R}_R + 3\left(\frac{\sigma_{E.R}^{3\sigma-IQR}}{\sqrt{n}}\right)$$

$$CL_{E.R}^{3\sigma.IQR} = \bar{R}_R$$

$$LCL_{E.R}^{3\sigma.IQR} = \bar{R}_R - 3\left(\frac{\sigma_{E.R}^{3\sigma-IQR}}{\sqrt{n}}\right)$$

The range control chart uses the average of subgroup Downton estimator $\left(Dn_R^{3\sigma}\right)$ to define control boundaries and the centre line based on the average of range.
The process mean will be determined by taking the average of range of the subgroup range, and the process standard deviation $\left(Dn_R^{3\sigma}\right)$ will be derived from the subgroup to lessen the impact of outliers on the control limits under normal distribution.

$$UCL_R^{3\sigma} = \bar{R}_R + 3\left(\frac{Dn_R^{3\sigma}}{\sqrt{n}}\right)$$

$$CL_R^{3\sigma} = \bar{R}_R$$

$$LCL_R^{3\sigma} = \bar{R}_R - 3\left(\frac{Dn_R^{3\sigma}}{\sqrt{n}}\right)$$

Balamuruganand Uma (2021) proposed the six-sigma probability level under normal distribution $p(\mathrm{z} \le \mathrm{z}_{6\sigma}) = 1 - \alpha_1, \alpha_1 = 3.4 \times 10^{-6}$ for constructing a range control chart based on 6σ.

$$UCL_R^{6\sigma} = \bar{R}_R + \phi_{6\sigma}\left(\frac{\sigma_R^{6\sigma}}{\sqrt{n}}\right)$$

$$CL_R^{6\sigma} = \bar{R}_R$$

$$LCL_R^{6\sigma} = \bar{R}_R - \phi_{6\sigma}\left(\frac{\sigma_R^{6\sigma}}{\sqrt{n}}\right)$$

The six-sigma range control chart defines control limits and the centre line based on the process standard deviation $\left(D_{n.MR}^{6\sigma}\right)$ by taking the average of the subgroup Downton estimator.

$$UCL_{Dn.R}^{6\sigma} = \bar{R}_R + \phi_{6\sigma}\left(\frac{D_{n.R}^{6\sigma}}{\sqrt{n}}\right)$$

$$CL_{Dn.R}^{6\sigma} = \bar{R}_R$$

$$LCL_{Dn.R}^{6\sigma} = \bar{R}_R - \phi_{6\sigma}\left(\frac{D_{n.R}^{6\sigma}}{\sqrt{n}}\right)$$

The suggested process standard deviation $\left(\tau_{N.6\sigma}^{R-IQR}\right)$ for constructing range control limits with six sigma by using interquartile range (IQR) under normal distribution is used to establish the tolerance level (TL) and process capability. Using the process standard deviation $\left(\tau_{N.6\sigma}^{R-IQR}\right)$ instead of the standard deviation (σ) from the control chart for rangeyields:

$$UCL_{N.R}^{6\sigma.IQR} = \bar{R}_R + \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{R-IQR}}{\sqrt{n}}\right)$$

$$CL_{N.R}^{6\sigma.IQR} = \bar{R}_R$$

$$LCL_{N.R}^{6\sigma.IQR} = \bar{R}_R - \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{R-IQR}}{\sqrt{n}}\right)$$

The proposed process standard deviation $\left(\tau_{E.6\sigma}^{R-IQR}\right)$ for generating range control limits with six sigma using IQR under exponential distribution is used to determine the tolerance level (TL) and process capabilities. To calculate range yields, use the process standard deviation $\left(\tau_{E.6\sigma}^{R-IQR}\right)$ rather than the control chart's standard deviation (σ).

$$UCL_{E.R}^{6\sigma.IQR} = \bar{R}_R + \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{R-IQR}}{\sqrt{n}}\right)$$

$$CL_{E.R}^{6\sigma.IQR} = \bar{R}_R$$

$$LCL_{E.R}^{6\sigma.IQR} = \bar{R}_R - \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{R-IQR}}{\sqrt{n}}\right)$$

8.Illustration

The example provided by Douglas C. Montgomery (2008, Page No.273) is considered here. The net weight (in oz) of a dry bleach product is tobe monitored using a samplesize of n = 5. Data for 20 preliminary samples are shown below.

(i) The control chartfor rangebased on 3σ, then the control limits are as follows:

$$UCL_R^{3\sigma} = \bar{R}_R + 3\left(\frac{d_3\bar{R}}{d_2}\right) = 0.475 + 3\left(\frac{0.864\times 0.475}{2.326}\right) = 1.004$$

$$CL_R^{3\sigma} = \bar{R}_R = 0.475$$

$$LCL_R^{3\sigma} = \bar{R}_R - 3\left(\frac{d_3\bar{R}}{d_2}\right) = 0.475 - 3\left(\frac{0.864\times 0.475}{2.326}\right) = -0.054 \square 0$$

(ii) The control chartfor range by using interquartile range (IQR) based on 3σ under normal distribution, then the control limits are as follows:

$$UCL_{N.R}^{3\sigma.IQR} = \bar{R}_R + 3\left(\frac{\sigma_{N.R}^{3\sigma-IQR}}{\sqrt{n}}\right) = 0.475 + 3\left(\frac{0.174}{\sqrt{5}}\right) = 0.709$$

$$CL_{N.R}^{3\sigma.IQR} = \bar{R}_R = 0.475$$

$$LCL_{N.R}^{3\sigma.IQR} = \bar{R}_R - 3\left(\frac{\sigma_{N.R}^{3\sigma-IQR}}{\sqrt{n}}\right) = 0.475 - 3\left(\frac{0.174}{\sqrt{5}}\right) = 0.241$$

Table 1: Measurement on net weight (in oz) of a dry bleach product

Sample number	a	b	c	d	e	Range	Standard deviation	IQR_{Norm}	IQR_{Exp}	Downton Statistic
1	15.80	16.30	16.20	16.10	16.60	0.800	0.292	0.148	0.182	0.319
2	16.30	15.90	15.90	16.20	16.40	0.500	0.230	0.297	0.364	0.248
3	16.10	16.20	16.50	16.40	16.30	0.400	0.158	0.148	0.182	0.177
4	16.30	16.20	15.90	16.40	16.20	0.500	0.187	0.074	0.091	0.195
5	16.10	16.10	16.40	16.50	16.00	0.500	0.217	0.222	0.273	0.230
6	16.10	15.80	16.70	16.60	16.40	0.900	0.370	0.371	0.455	0.408
7	16.10	16.30	16.50	16.10	16.50	0.400	0.200	0.297	0.364	0.213
8	16.20	16.10	16.20	16.10	16.30	0.200	0.084	0.074	0.091	0.089
9	16.30	16.20	16.40	16.30	16.50	0.300	0.114	0.074	0.091	0.124
10	16.60	16.30	16.40	16.10	16.50	0.500	0.192	0.148	0.182	0.213
11	16.20	16.40	15.90	16.30	16.40	0.500	0.207	0.148	0.182	0.213

12	15.90	16.60	16.70	16.20	16.50	0.800	0.327	0.297	0.364	0.355
13	16.40	16.10	16.60	16.40	16.10	0.500	0.217	0.222	0.273	0.230
14	16.50	16.30	16.20	16.30	16.40	0.300	0.114	0.074	0.091	0.124
15	16.40	16.10	16.30	16.20	16.20	0.300	0.114	0.074	0.091	0.124
16	16.00	16.20	16.30	16.30	16.20	0.300	0.122	0.074	0.091	0.124
17	16.40	16.20	16.40	16.30	16.20	0.200	0.100	0.148	0.182	0.106
18	16.00	16.20	16.40	16.50	16.10	0.500	0.207	0.222	0.273	0.230
19	16.40	16.00	16.30	16.40	16.40	0.400	0.173	0.074	0.091	0.160
20	16.40	16.40	16.50	16.00	15.80	0.700	0.303	0.297	0.364	0.319
Average						**0.475**	**0.196**	**0.174**	**0.214**	**0.210**

Source: Douglas C. Montgomery (2008)

(iii) The control chartforrange by using interquartile range (IQR) based on 3σ under exponential distribution, then the control limits are as follows:

$$UCL_{E.R}^{3\sigma.IQR} = \bar{R}_R + 3\left(\frac{\sigma_{E.R}^{3\sigma-IQR}}{\sqrt{n}}\right) = 0.475 + 3\left(\frac{0.214}{\sqrt{5}}\right) = 0.762$$

$$CL_{E.R}^{3\sigma.IQR} = \bar{R}_R = 0.475$$

$$LCL_{E.R}^{3\sigma.IQR} = \bar{R}_R - 3\left(\frac{\sigma_{E.R}^{3\sigma-IQR}}{\sqrt{n}}\right) = 0.475 - 3\left(\frac{0.214}{\sqrt{5}}\right) = 0.188$$

(iv) The control chartforrange by using Downton statistic based on 3σ, then the control limits are as follows:

$$UCL_R^{3\sigma} = \bar{R}_R + 3\left(\frac{Dn_R^{3\sigma}}{\sqrt{n}}\right) = 0.475 + 3\left(\frac{0.210}{\sqrt{5}}\right) = 0.757$$

$$CL_R^{3\sigma} = \bar{R}_R = 0.475$$

$$LCL_R^{3\sigma} = \bar{R}_R - 3\left(\frac{Dn_R^{3\sigma}}{\sqrt{n}}\right) = 0.475 - 3\left(\frac{0.210}{\sqrt{5}}\right) = 0.193$$

(v) The six-sigma based control chartforrange, then the control limits are as follows:

$$UCL_R^{6\sigma} = \bar{R}_R + \phi_{6\sigma}\left(\frac{\sigma_R^{6\sigma}}{\sqrt{n}}\right) = 0.475 + \left(\frac{4.831 \times 0.024}{\sqrt{5}}\right) = 0.527$$

$$CL_R^{6\sigma} = \bar{R}_R = 0.475$$

$$LCL_R^{6\sigma} = \bar{R}_R - \phi_{6\sigma}\left(\frac{\sigma_R^{6\sigma}}{\sqrt{n}}\right) = 0.475 - \left(\frac{4.831 \times 0.024}{\sqrt{5}}\right) = 0.423$$

(vi) The six-sigma based control chartforrange by using Downton statistic, then the control limits are as follows:

$$UCL_{Dn.R}^{6\sigma} = \bar{R}_R + \phi_{6\sigma}\left(\frac{D_{n.R}^{6\sigma}}{\sqrt{n}}\right) = 0.475 + \left(\frac{4.831 \times 0.027}{\sqrt{5}}\right) = 0.532$$

$$CL_{Dn.R}^{6\sigma} = \bar{R}_R = 0.475$$

$$LCL_{Dn.R}^{6\sigma} = \bar{R}_R - \phi_{6\sigma}\left(\frac{D_{n.R}^{6\sigma}}{\sqrt{n}}\right) = 0.475 - \left(\frac{4.831 \times 0.027}{\sqrt{5}}\right) = 0.418$$

(vii) The six-sigma based control chartfor range by using interquartile range (IQR) under normal distribution, then the control limits are as follows:

$$UCL_{N.R}^{6\sigma.IQR} = \bar{R}_R + \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{R-IQR}}{\sqrt{n}}\right) = 0.475 + \left(\frac{4.831\times 0.025}{\sqrt{5}}\right) = 0.528$$

$$CL_{N.R}^{6\sigma.IQR} = \bar{R}_R = 0.475$$

$$LCL_{N.R}^{6\sigma.IQR} = \bar{R}_R - \left(\frac{\phi_{6\sigma}\tau_{N.6\sigma}^{R-IQR}}{\sqrt{n}}\right) = 0.475 - \left(\frac{4.831\times 0.025}{\sqrt{5}}\right) = 0.422$$

(viii) The six-sigma based control chartfor range by using interquartile range (IQR) under exponential distribution, then the control limits are as follows:

$$UCL_{E.R}^{6\sigma.IQR} = \bar{R}_R + \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{R-IQR}}{\sqrt{n}}\right) = 0.475 + \left(\frac{4.831\times 0.030}{\sqrt{5}}\right) = 0.541$$

$$CL_{E.R}^{6\sigma.IQR} = \bar{R}_R = 0.475$$

$$LCL_{E.R}^{6\sigma.IQR} = \bar{R}_R - \left(\frac{\phi_{6\sigma}\tau_{E.6\sigma}^{R-IQR}}{\sqrt{n}}\right) = 0.475 - \left(\frac{4.831\times 0.030}{\sqrt{5}}\right) = 0.409$$

Table 2: Summary of three-sigma and six-sigma control charts for range

Sl.No	Control chart	UCL	LCL	CLI
1	Shewhart three-Sigma	1.004	-0.054	1.059
2	Three-sigma with IQR under Exponential distribution	0.762	0.188	0.574
3	Three-sigma with IQR under Normal distribution	0.709	0.241	0.467
4	Three-sigma using Downton	0.757	0.193	0.564
5	Six-sigma using Downton	0.532	0.418	0.115
6	Six-sigma	0.527	0.423	0.103
7	Six-sigma with IQR under Exponential distribution	0.541	0.409	0.131
8	Six-sigma with IQR under Normal distribution	0.528	0.422	0.107

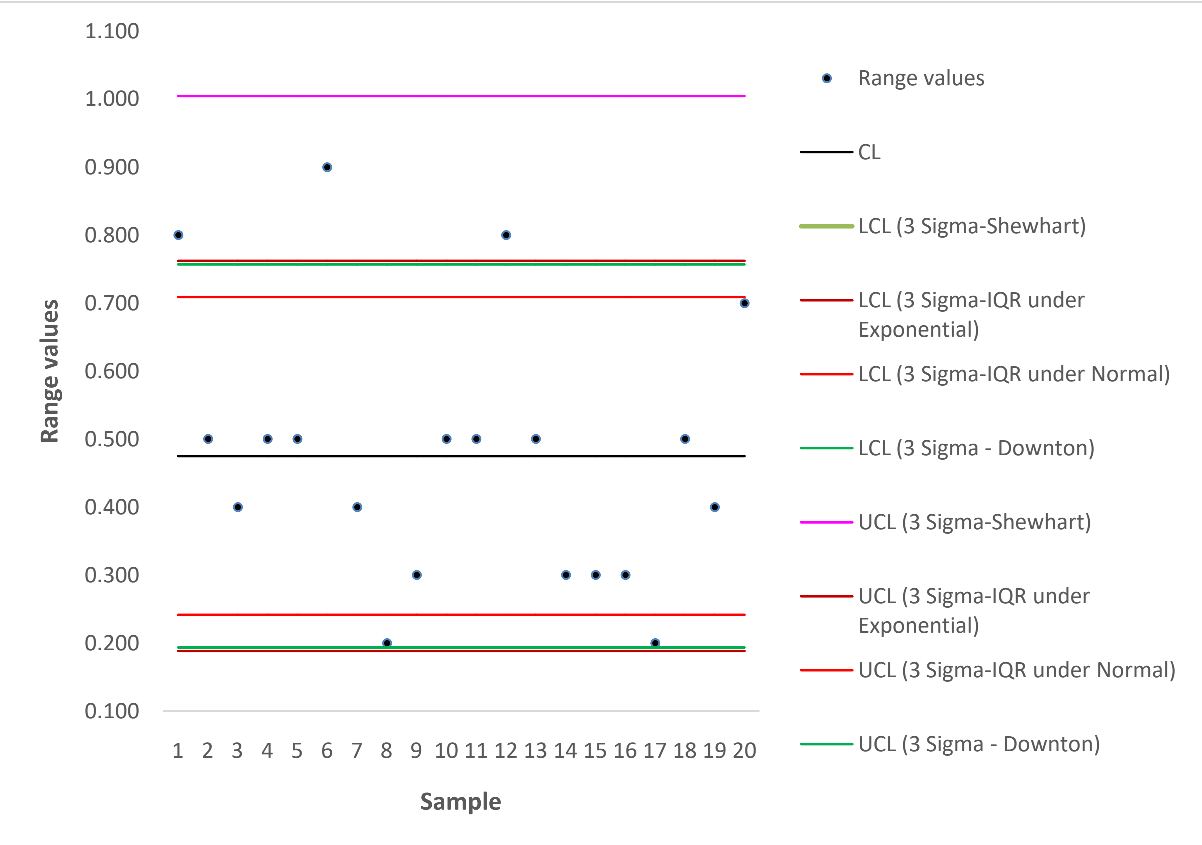

Figure 1: Three-sigma control charts for range

It is clearly stated that only seven sample points fall inside the control limits, with a control limit interval (CLI) of 0.103 compared with other control limits for the sample size n=5, indicating that the process is out of control based on six sigma for range, as shown by the subsequent Table-2, Figure-1, and Figure-2. The Table-3, Figure-3, and Figure-4 show that the current three-sigma and other control charts are less successful than the six-sigma control chart for range based on average run duration.

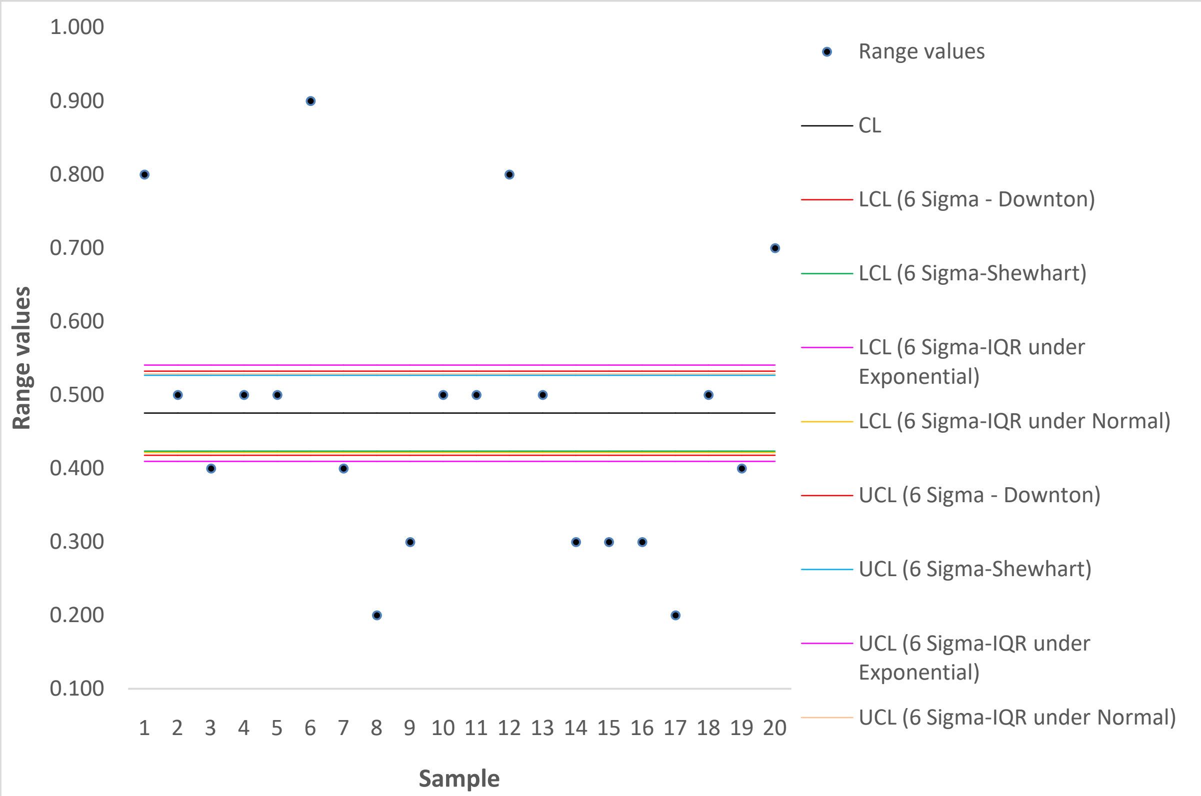

Figure 2: Six-sigma control charts for range

Table 3: ARL of three-sigma and six-sigma control charts for range

Sl.No	Multiple of σ / Control chart	0.030	0.031	0.032	0.033	0.034	0.035
1	Shewhart three-Sigma	129	129	128	127	126	125
2	Three-sigma with IQR under Exponential distribution	246	240	234	228	223	217
3	Three-sigma with IQR under Normal distribution	208	201	195	189	183	177
4	Three-sigma using Downton	242	237	231	225	220	214
5	Six-sigma using Downton	95	77	62	50	41	34
6	Six-sigma	46	37	30	24	20	17
7	Six-sigma with IQR under Exponential distribution	228	184	149	122	100	82
8	Six-sigma with IQR under Normal distribution	58	47	38	31	25	21

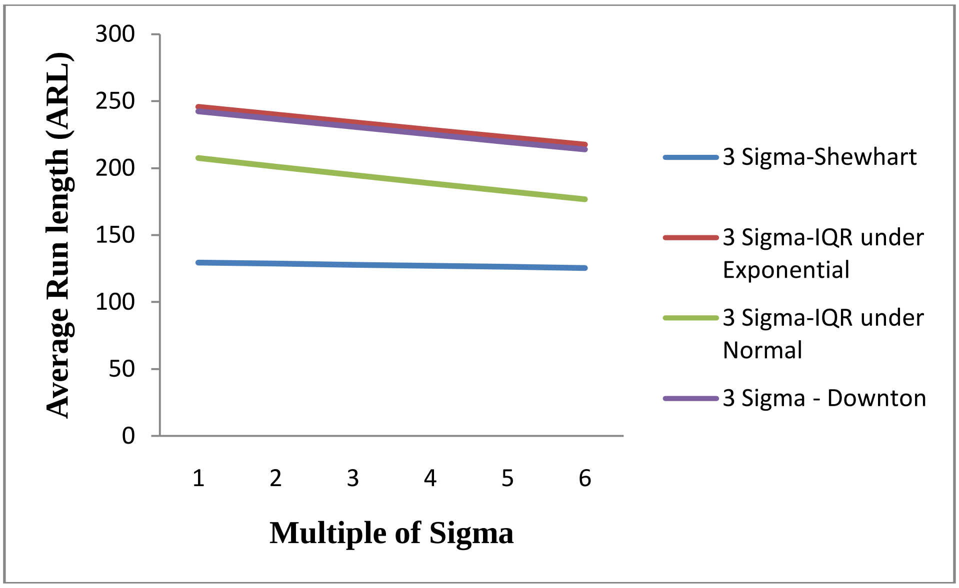

Figure 3: ARL for Three-sigma control charts for range

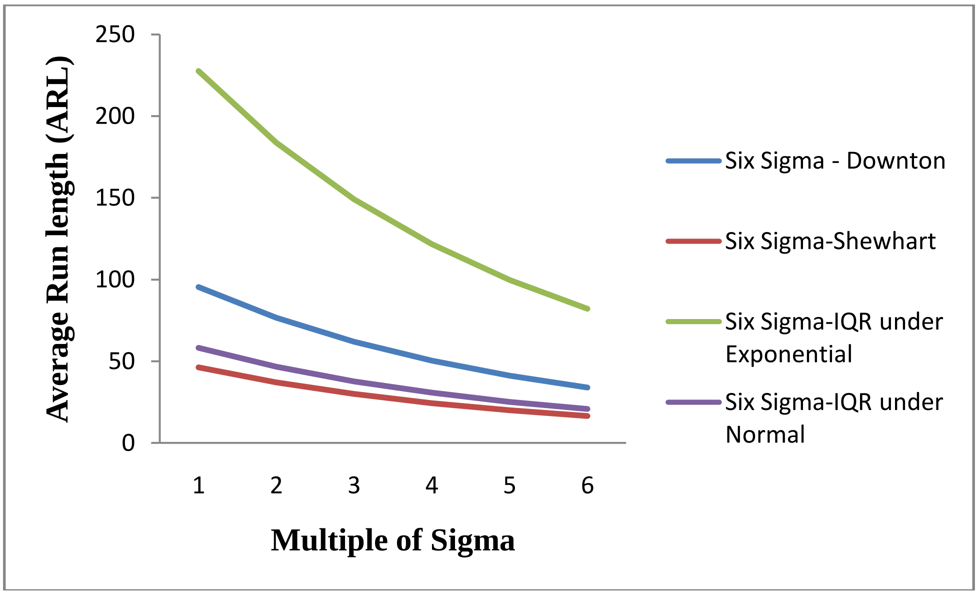

Figure 4: ARL for Six-sigma control charts for range

9. CONCLUSION

The aforementioned results show that when Shewhart 3-Sigma, IQR, and six-sigma based IQR control limits are used under normal and exponential distributions for range, the process is not statistically controlled. Additionally, the interval between the six-sigma based control chart is smaller than the interval between the Shewhart and IQR control limits. It is evident that the product or service is not up to par, and as a result, the system or process has to be improved.

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