Characterization of Homomorphism and anti Homomorphism of Pythagorean fuzzy ideas in semiring

R.Raghu[1], J.Jayaraj[2]

[1]Guest Lecturer, Department of Mathematics, Thiru. A. Govindasamy Govt. Arts College,Tindivanam, 604307, Email: c.r.raghu89@gmail.com

[2]Assistant Professor,Department of Mathematics,Dr. Puratchithalaivar M.G.R Government Arts and Science College, Kudavasal, 612601& Annamalai University,Annamalainagar, 608002, Email: joe.jayaraj@gmail.com

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ABSTRACT

In this paper, we introduce the notion Homomorphism of Pythagorean fuzzy ideas in semiring and anti homomorphism of Pythagorean fuzzy ideals in semiring, some interesting properties, results are discussed in this paper.

Keywords : Pythagorean, Fuzzy set, semiring

1. INTRODUCTION

The concept of semirings was introduced by H.S.Vandiver in 1935 and has since then been studied by many authors[1, 2, 6, 8]. Zadeh[12] studied the notion of fuzzy set theory. Atanassov[4] introduced intuitionistic fuzzy sets as a generalization of fuzzy sets. In intuitionistic fuzzy sets, the sum of membership degree and non-membership degree should not exceed one. Yager[10] initially introduced the concept of Pythagorean fuzzy sets. In a Pythagorean fuzzy sets, the sum of the squared membership and non-membership degrees satisfies the condition. More recently, Yager[10, 11] proposed Pythagorean fuzzy sets as a powerful tool for effectively managing uncertainty or imprecise information in real-world scenarios. These sets enforce a constraint where the sum of squares of membership and non-membership degrees is less than or equal to 1.

Pythagorean fuzzy sets have showcased remarkable efficacy in navigating uncertainties, prompting a surge of scholarly exploration across diverse research avenues, resulting in significant progress. The conceptualization of Pythagorean fuzzy sets facilitates a more comprehensive and accurate portrayal of uncertain information when juxtaposed with intuitionistic fuzzy sets. Across various disciplines, academics have meticulously examined the algebraic attributes of Pythagorean fuzzy sets, shedding light on their practical applications and foundational theoretical constructs. Many authors studied the algebraic structures of Pythagorean fuzzy sets.

This paper is structured into three sections. The first and second sections serve as the introduction and cover basic results related to this paper's topic. In the third section, we introduce Homomorphism of Pythagorean fuzzy bi-ideal, interior ideal in semirings is also a Pythagorean fuzzy bi-ideal and Pythagorean fuzzy interior ideal. Section 4 deals with anti homomorphism of Pythagorean fuzzy bi-ideal, interior ideal in semirings is also a Pythagorean fuzzy bi-ideal and Pythagorean fuzzy interior ideal.

2. Preliminaries

In this section we present the basic concepts related to this paper.

Definition 2.1 A nonempty set S is said to be a semi-ring with respect to two binary compositions, addition and multiplication defined on it, if the following conditions are satisfied:

1. $(S, +)$ is a commutative semigroup with zero.
2. $(S, .)$ is a semigroup.
3. for any three elements $a, b, c \in S$, the left distributive law

$a.(b + c) = a.b + a.c$ and the right distributive law $(b + c).a = b.a + c.a$.

4. $s.0 = 0.s$, for all $s \in S$.

Definition 2.2 A nonempty subset $\mathcal{I}$ of a semi-ring S is called an ideal if

1. $a, b \in \mathcal{I}$ implies $a + b \in \mathcal{I}$

2. $a \in \mathcal{I}, s \in S$ implies $s.a \in \mathcal{I}$ and $a.s \in \mathcal{I}$

Definition 2.3 Let μ be a nonempty fuzzy subset of a semi-ring S. Then μ is called a fuzzy left(right) ideal of S if for all $i, j \in S$.
1. $\mu(i + j) \geq \min\{\mu(i), \mu(j)\}$
2. $\mu(ij) \geq \mu(j)(\text{resp.}, \mu(ij) \geq \mu(i))$

A fuzzy ideal of a semi-ring S is a nonempty fuzzy subset of S which is both a fuzzy left ideal and a fuzzy right ideal of S.

3. Homomorphism of Pythagorean fuzzy ideals in semiring

Definition 3.1 Let $\phi: R \to S$ be a semiring homomorphism. Assume that P is a Pythagorean fuzzy ideal of R with sup property and $\phi(P)$ be the image of P under ϕ. Given $\phi(x), \phi(y), \phi(u) \in \phi(R)$. Let $x_0 \in \phi^{-1}(\phi(x)), y_0 \in \phi^{-1}(\phi(y)), u_0 \in \phi^{-1}(\phi(u))$ such that

$\mu(x_0) = \sup_{z\in\phi^{-1}(\phi(x))} \mu(z),\ \vartheta(x_0) = \inf_{z\in\phi^{-1}(\phi(x))} \vartheta(z)$

$\mu(y_0) = \sup_{z\in\phi^{-1}(\phi(y))} \mu(z),\ \vartheta(y_0) = \inf_{z\in\phi^{-1}(\phi(y))} \vartheta(z)$

$\mu(u_0) = \sup_{z\in\phi^{-1}(\phi(u))} \mu(z),\ \vartheta(u_0) = \inf_{z\in\phi^{-1}(\phi(u))} \vartheta(z).$

Theorem 3.2 Let $\phi: R \to S$ be an onto homomorphism of semiring if P is a Pythagorean fuzzy left ideal in semiring R. with sup property then $\phi(P)$ is a Pythagorean fuzzy left ideal in semiring S.

Proof. Let P be a Pythagorean fuzzy left ideal of R, $x, y, u \in R$.

$$\begin{aligned}\mu(\phi(x+y)) &= \sup_{z\in\phi^{-1}(\phi(x+y))} \mu(z)\\ &\geq \mu(x_0 + y_0)\\ &\geq \min\{\mu(x_0), \mu(y_0)\}\\ &= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \mu(z), \sup_{z\in\phi^{-1}(\phi(y))} \mu(z)\}\\ &= \min\{\mu(\phi(x)), \mu(\phi(y))\}\end{aligned}$$

and

$$\begin{aligned}\vartheta(\phi(x+y)) &= \inf_{z\in\phi^{-1}(\phi(x+y))} \vartheta(z)\\ &\leq \vartheta(x_0 + y_0)\\ &\leq \max\{\vartheta(x_0), \vartheta(y_0)\}\\ &= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \inf_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}\\ &= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}\end{aligned}$$

and

$$\begin{aligned}\mu(\phi(xy)) &= \sup_{z\in\phi^{-1}(\phi(xy))} \mu(z)\\ &\geq \mu(x_0 y_0)\\ &\geq \min\{\mu(x_0), \mu(y_0)\}\\ &= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \mu(z), \sup_{z\in\phi^{-1}(\phi(y))} \mu(z)\}\\ &= \min\{\mu(\phi(x)), \mu(\phi(y))\}\\ \vartheta(\phi(xy)) &= \inf_{z\in\phi^{-1}(\phi(xy))} \vartheta(z)\\ &\leq \vartheta(x_0 y_0)\\ &\leq \max\{\vartheta(x_0), \vartheta(y_0)\}\\ &= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \inf_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}\\ &= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}\end{aligned}$$

Finally

$$\begin{aligned}\mu(\phi(xy)) &= \sup_{z\in\phi^{-1}(\phi(xy))} \mu(z)\\ &\geq \mu(x_0 y_0)\\ &\geq \mu(y_0)\\ &= \sup_{z\in\phi^{-1}(\phi(y))} \mu(z)\\ &= \mu(\phi(y))\\ \vartheta(\phi(xy)) &= \inf_{z\in\phi^{-1}(\phi(xy))} \vartheta(z)\end{aligned}$$

$$\leq \vartheta(x_0 y_0)$$
$$\leq \vartheta(y_0)$$
$$= \inf_{z \in \phi^{-1}(\phi(y))} \vartheta(z)$$
$$= \vartheta(\phi(y))$$

Therefore $\phi(P)$ is a Pythagorean fuzzy left ideal in semiring S.

Theorem 3.3 Let $\phi: R \to S$ be an onto homomorphism of semiring if P is a Pythagorean fuzzy bi-ideal in semiring R. with sup property then $\phi(P)$ is a Pythagorean fuzzy bi-ideal in semiring S.

Proof. Let P be a Pythagorean fuzzy bi-ideal of R, $x, y, u \in R$.

$$\mu(\phi(x+y)) = \sup_{z \in \phi^{-1}(\phi(x+y))} \mu(z)$$
$$\geq \mu(x_0 + y_0)$$
$$\geq \min\{\mu(x_0), \mu(y_0)\}$$
$$= \min\{\sup_{z \in \phi^{-1}(\phi(x))} \mu(z), \sup_{z \in \phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(x+y)) = \inf_{z \in \phi^{-1}(\phi(x+y))} \vartheta(z)$$
$$\leq \vartheta(x_0 + y_0)$$
$$\leq \max\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \max\{\inf_{z \in \phi^{-1}(\phi(x))} \vartheta(z), \inf_{z \in \phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

and

$$\mu(\phi(xy)) = \sup_{z \in \phi^{-1}(\phi(xy))} \mu(z)$$
$$\geq \mu(x_0 y_0)$$
$$\geq \min\{\mu(x_0), \mu(y_0)\}$$
$$= \min\{\sup_{z \in \phi^{-1}(\phi(x))} \mu(z), \sup_{z \in \phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(xy)) = \inf_{z \in \phi^{-1}(\phi(xy))} \vartheta(z)$$
$$\leq \vartheta(x_0 y_0)$$
$$\leq \max\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \max\{\inf_{z \in \phi^{-1}(\phi(x))} \vartheta(z), \inf_{z \in \phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

Finally

$$\mu(\phi(xuy)) = \sup_{z \in \phi^{-1}(\phi(xuy))} \mu(z)$$
$$\geq \mu(x_0 u_0 y_0)$$
$$\geq \min\{\mu(x_0), \mu(y_0)\}$$
$$= \min\{\sup_{z \in \phi^{-1}(\phi(x))} \mu(z), \sup_{z \in \phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(xuy)) = \inf_{z \in \phi^{-1}(\phi(xuy))} \vartheta(z)$$
$$\leq \vartheta(x_0 u_0 y_0)$$
$$\leq \max\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \max\{\inf_{z \in \phi^{-1}(\phi(x))} \vartheta(z), \inf_{z \in \phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

Therefore $\phi(P)$ is a Pythagorean fuzzy bi-ideal in semiring S.

Theorem 3.4 Let R and S are semiring homomorphism image of a Pythagorean fuzzy interior ideal posse-sing the sup property is a Pythagorean fuzzy interior ideal.

Proof. Let P be a Pythagorean fuzzy interior ideal of R, $x, y, u \in R$.

$$\mu(\phi(x+y)) = \sup_{z \in \phi^{-1}(\phi(x+y))} \mu(z)$$
$$\geq \mu(x_0 + y_0)$$
$$\geq \min\{\mu(x_0), \mu(y_0)\}$$

$$= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \mu(z), \sup_{z\in\phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(x+y)) = \inf_{z\in\phi^{-1}(\phi(x+y))} \vartheta(z)$$
$$\leq \vartheta(x_0 + y_0)$$
$$\leq \max\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \inf_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

and

$$\mu(\phi(xy)) = \sup_{z\in\phi^{-1}(\phi(xy))} \mu(z)$$
$$\geq \mu(x_0y_0)$$
$$\geq \min\{\mu(x_0), \mu(y_0)\}$$
$$= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \mu(z), \sup_{z\in\phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(xy)) = \inf_{z\in\phi^{-1}(\phi(xy))} \vartheta(z)$$
$$\leq \vartheta(x_0y_0)$$
$$\leq \max\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \inf_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

Finally

$$\mu(\phi(xuy)) = \sup_{z\in\phi^{-1}(\phi(xuy))} \mu(z)$$
$$\geq \mu(x_0u_0y_0)$$
$$= \mu(u_0)$$
$$= \sup_{z\in\phi^{-1}(\phi(u))} \mu(z)$$
$$= \mu(\phi(u))$$
$$\vartheta(\phi(xuy)) = \inf_{z\in\phi^{-1}(\phi(xuy))} \vartheta(z)$$
$$\leq \vartheta(x_0u_0y_0)$$
$$= \vartheta(u_0)$$
$$= \inf_{z\in\phi^{-1}(\phi(u))} \vartheta(z)$$
$$= \vartheta(\phi(u))$$

Therefore $\phi(P)$ is a Pythagorean fuzzy interior ideal in semiring S.

Definition 3.5 Let ϕ be a mapping form $R \to S$ and Let P_1 and P_2 be Pythagorean fuzzy sets in R and S respectively. $\phi^{-1}(P_2)$ is a Preimage of P_2 under ϕ, is a Pythagorean fuzzy set in R. $\phi^{-1}(\mu(x)) = \mu(\phi(x))$ and $\phi^{-1}(\vartheta(x)) = \vartheta(\phi(x))$, $\forall x \in R$.

Theorem 3.6 Let ϕ mapping from $R \to S$ is a homomorphism of semiring and let P_2 be a Pythagorean fuzzy left ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean fuzzy left ideal of semiring R.
Proof. Let $x, y, u \in R$, then we have

$$\phi^{-1}\mu(x+y) = \mu(\phi(x+y))$$
$$= \mu(\phi(x) + \phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(x+y) = \vartheta(\phi(x+y))$$
$$= \vartheta(\phi(x) + \phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

and

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$

$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Finally

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\geq \mu(\phi(y))$$
$$= \phi^{-1}(\mu(y))$$
$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\leq \vartheta(\phi(y))$$
$$= \phi^{-1}(\vartheta(y))$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean fuzzy left ideal of semiring R.

Theorem 3.7 Let ϕ mapping from $R \rightarrow S$ is a homomorphism of semiring and let P_2 be a Pythagorean fuzzy bi-ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean fuzzy bi-ideal of semiring R.
Proof. Let $x, y, u \in R$, then we have

$$\phi^{-1}\mu(x+y) = \mu(\phi(x+y))$$
$$= \mu(\phi(x) + \phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(x+y) = \vartheta(\phi(x+y))$$
$$= \vartheta(\phi(x) + \phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

and

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Finally

$$\phi^{-1}\mu(xuy) = \mu(\phi(xuy))$$
$$= \mu(\phi(x)\phi(u)\phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(xuy) = \vartheta(\phi(xuy))$$
$$= \vartheta(\phi(x)\phi(u)\phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean fuzzy bi-ideal of semiring R.

Theorem 3.8 Let ϕ mapping from $R \rightarrow S$ is a homomorphism of semiring and let P_2 be a Pythagorean fuzzy interior ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean fuzzy interior ideal of semiring R.
Proof. Let $x, y, u \in R$, then we have

$$\phi^{-1}\mu(x+y) = \mu(\phi(x+y))$$
$$= \mu(\phi(x) + \phi(y))$$
$$\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(x+y) = \vartheta(\phi(x+y))$$
$$= \vartheta(\phi(x) + \phi(y))$$
$$\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

and

$$\begin{aligned}\phi^{-1}\mu(xy) &= \mu(\phi(xy))\\ &= \mu(\phi(x)\phi(y))\\ &\geq \min\{\mu(\phi(x)), \mu(\phi(y))\}\\ &= \min\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}\\ \phi^{-1}\vartheta(xy) &= \vartheta(\phi(xy))\\ &= \vartheta(\phi(x)\phi(y))\\ &\leq \max\{\vartheta(\phi(x)), \vartheta(\phi(y))\}\\ &= \max\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}\end{aligned}$$

Finally

$$\begin{aligned}\phi^{-1}\mu(xuy) &= \mu(\phi(xuy))\\ &= \mu(\phi(x)\phi(u)\phi(y))\\ &\geq \mu(\phi(u))\\ &= \phi^{-1}(\mu(u))\\ \phi^{-1}\vartheta(xuy) &= \vartheta(\phi(xuy))\\ &= \vartheta(\phi(x)\phi(u)\phi(y))\\ &\leq \vartheta(\phi(u))\\ &= \phi^{-1}(\vartheta(u))\end{aligned}$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean fuzzy interior ideal of semiring R.

4. Anti Homomorphism of Pythagorean fuzzy ideal in semiring

Theorem 4.1 Let $\phi: R \to S$ be an onto homomorphism of semiring if P is a Pythagorean anti fuzzy bi-ideal in semiring R. with sup property then $\phi(P)$ is a Pythagorean anti fuzzy bi-ideal in semiring S.

Proof. Let P be a Pythagorean fuzzy bi-ideal of R, $x, y, u \in R$.

$$\begin{aligned}\mu(\phi(x+y)) &= \inf_{z\in\phi^{-1}(\phi(x+y))} \mu(z)\\ &\leq \mu(x_0 + y_0)\\ &\leq \max\{\mu(x_0), \mu(y_0)\}\\ &= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \mu(z), \inf_{z\in\phi^{-1}(\phi(y))} \mu(z)\}\\ &= \max\{\mu(\phi(x)), \mu(\phi(y))\}\\ \vartheta(\phi(x+y)) &= \sup_{z\in\phi^{-1}(\phi(x+y))} \vartheta(z)\\ &\geq \vartheta(x_0 + y_0)\\ &\geq \min\{\vartheta(x_0), \vartheta(y_0)\}\\ &= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \sup_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}\\ &= \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}\end{aligned}$$

and

$$\begin{aligned}\mu(\phi(xy)) &= \inf_{z\in\phi^{-1}(\phi(xy))} \mu(z)\\ &\leq \mu(x_0y_0)\\ &\leq \max\{\mu(x_0), \mu(y_0)\}\\ &= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \mu(z), \inf_{z\in\phi^{-1}(\phi(y))} \mu(z)\}\\ &= \max\{\mu(\phi(x)), \mu(\phi(y))\}\\ \vartheta(\phi(xy)) &= \sup_{z\in\phi^{-1}(\phi(xy))} \vartheta(z)\\ &\geq \vartheta(x_0y_0)\\ &\geq \min\{\vartheta(x_0), \vartheta(y_0)\}\\ &= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \sup_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}\\ &= \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}\end{aligned}$$

Finally

$$\begin{aligned}\mu(\phi(xuy)) &= \inf_{z\in\phi^{-1}(\phi(xuy))} \mu(z)\\ &\leq \mu(x_0u_0y_0)\\ &\leq \max\{\mu(x_0), \mu(y_0)\}\\ &= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \mu(z), \inf_{z\in\phi^{-1}(\phi(y))} \mu(z)\}\\ &= \max\{\mu(\phi(x)), \mu(\phi(y))\}\end{aligned}$$

$$\vartheta(\phi(xuy)) = \sup_{z\in\phi^{-1}(\phi(xuy))} \vartheta(z)$$
$$\geq \vartheta(x_0u_0y_0)$$
$$\geq \min\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \sup_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

Therefore $\phi(P)$ is a Pythagorean anti fuzzy bi-ideal in semiring S.

Theorem 4.2 Let R and S are semiring homomorphism image of Pythagorean anti fuzzy interior ideal possesing the sup property is a Pythagorean anti fuzzy interior ideal.
Proof. Let P be a Pythagorean anti fuzzy interior ideal of R, $x, y, u \in R$.

$$\mu(\phi(x+y)) = \inf_{z\in\phi^{-1}(\phi(x+y))} \mu(z)$$
$$\leq \mu(x_0 + y_0)$$
$$\leq \max\{\mu(x_0), \mu(y_0)\}$$
$$= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \mu(z), \inf_{z\in\phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(x+y)) = \sup_{z\in\phi^{-1}(\phi(x+y))} \vartheta(z)$$
$$\geq \vartheta(x_0 + y_0)$$
$$\geq \min\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \sup_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

and

$$\mu(\phi(xy)) = \inf_{z\in\phi^{-1}(\phi(xy))} \mu(z)$$
$$\leq \mu(x_0y_0)$$
$$\leq \max\{\mu(x_0), \mu(y_0)\}$$
$$= \max\{\inf_{z\in\phi^{-1}(\phi(x))} \mu(z), \inf_{z\in\phi^{-1}(\phi(y))} \mu(z)\}$$
$$= \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$\vartheta(\phi(xy)) = \sup_{z\in\phi^{-1}(\phi(xy))} \vartheta(z)$$
$$\geq \vartheta(x_0y_0)$$
$$\geq \min\{\vartheta(x_0), \vartheta(y_0)\}$$
$$= \min\{\sup_{z\in\phi^{-1}(\phi(x))} \vartheta(z), \sup_{z\in\phi^{-1}(\phi(y))} \vartheta(z)\}$$
$$= \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$

Finally

$$\mu(\phi(xuy)) = \inf_{z\in\phi^{-1}(\phi(xuy))} \mu(z)$$
$$\leq \mu(x_0u_0y_0)$$
$$\leq \mu(u_0)$$
$$= \inf_{z\in\phi^{-1}(\phi(u))} \mu(z)$$
$$= \mu(\phi(u))$$
$$\vartheta(\phi(xuy)) = \sup_{z\in\phi^{-1}(\phi(xuy))} \vartheta(z)$$
$$\geq \vartheta(x_0u_0y_0)$$
$$\geq \vartheta(u_0)$$
$$= \sup_{z\in\phi^{-1}(\phi(u))} \vartheta(z)$$
$$= \vartheta(\phi(u))$$

Therefore $\phi(P)$ is a Pythagorean anti fuzzy interior ideal in semiring S.

Theorem 4.3 Let ϕ mapping from $R \to S$ is a homomorphism of semiring and let P_2 be a Pythagorean anti fuzzy left ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy left ideal of semiring R.
Proof. Let $x, y, u \in R$, then we have

$$\phi^{-1}\mu(x+y) = \mu(\phi(x+y))$$
$$= \mu(\phi(x) + \phi(y))$$

$$\leq \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(x+y) = \vartheta(\phi(x+y))$$
$$= \vartheta(\phi(x) + \phi(y))$$
$$\geq \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

and

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\leq \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\geq \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Finally

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\leq \mu(\phi(y))$$
$$= \phi^{-1}(\mu(y))$$
$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\geq \vartheta(\phi(y))$$
$$= \phi^{-1}(\vartheta(y))$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy left ideal of semiring R.

Theorem 4.4 Let ϕ mapping from $R \rightarrow S$ is a homomorphism of semiring and let P_2 be a Pythagorean anti fuzzy bi-ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy bi-ideal of semiring R.
Proof. Let $x, y, u \in R$, then we have

$$\phi^{-1}\mu(x+y) = \mu(\phi(x+y))$$
$$= \mu(\phi(x) + \phi(y))$$
$$\leq \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(x+y) = \vartheta(\phi(x+y))$$
$$= \vartheta(\phi(x) + \phi(y))$$
$$\geq \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

and

$$\phi^{-1}\mu(xy) = \mu(\phi(xy))$$
$$= \mu(\phi(x)\phi(y))$$
$$\leq \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(xy) = \vartheta(\phi(xy))$$
$$= \vartheta(\phi(x)\phi(y))$$
$$\geq \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Finally

$$\phi^{-1}\mu(xuy) = \mu(\phi(xuy))$$
$$= \mu(\phi(x)\phi(u)\phi(y))$$
$$\leq \max\{\mu(\phi(x)), \mu(\phi(y))\}$$
$$= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\}$$
$$\phi^{-1}\vartheta(xuy) = \vartheta(\phi(xuy))$$
$$= \vartheta(\phi(x)\phi(u)\phi(y))$$
$$\geq \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\}$$
$$= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\}$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy bi-ideal of semiring R.

Theorem 4.5 Let ϕ mapping from $R \rightarrow S$ is a homomorphism of semiring and let P_2 be a Pythagorean anti fuzzy interior ideal of S. Then preimage $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy interior ideal of semiring R.

Proof. Let $x, y, u \in R$, then we have

$$\begin{aligned} \phi^{-1}\mu(x+y) &= \mu(\phi(x+y)) \\ &= \mu(\phi(x)+\phi(y)) \\ &\le \max\{\mu(\phi(x)), \mu(\phi(y))\} \\ &= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\} \\ \phi^{-1}\vartheta(x+y) &= \vartheta(\phi(x+y)) \\ &= \vartheta(\phi(x)+\phi(y)) \\ &\ge \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\} \\ &= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\} \end{aligned}$$

and

$$\begin{aligned} \phi^{-1}\mu(xy) &= \mu(\phi(xy)) \\ &= \mu(\phi(x)\phi(y)) \\ &\le \max\{\mu(\phi(x)), \mu(\phi(y))\} \\ &= \max\{\phi^{-1}(\mu(x)), \phi^{-1}(\mu(y))\} \\ \phi^{-1}\vartheta(xy) &= \vartheta(\phi(xy)) \\ &= \vartheta(\phi(x)\phi(y)) \\ &\ge \min\{\vartheta(\phi(x)), \vartheta(\phi(y))\} \\ &= \min\{\phi^{-1}(\vartheta(x)), \phi^{-1}(\vartheta(y))\} \end{aligned}$$

Finally

$$\begin{aligned} \phi^{-1}\mu(xuy) &= \mu(\phi(xuy)) \\ &= \mu(\phi(x)\phi(u)\phi(y)) \\ &\le \mu(\phi(u)) \\ &= \phi^{-1}(\mu(u)) \\ \phi^{-1}\vartheta(xuy) &= \vartheta(\phi(xuy)) \\ &= \vartheta(\phi(x)\phi(u)\phi(y)) \\ &\ge \vartheta(\phi(u)) \\ &= \phi^{-1}(\vartheta(u)) \end{aligned}$$

Therefore $\phi^{-1}(P_2)$ is a Pythagorean anti fuzzy interior ideal of semiring R.

REFERENCES

[1] J. Ahsan and M. Shabir, Semirings with projective ideals, Math. Japonica 38 (1993), 271- 276.(1).

[2] P. J. Allen, A fundamental theorem of homomorphisms for semirings, Proc. Amer. Math.Soc. 21 (1969), 412 - 416.(2)

[3] Ahsan, J. Saifullah, K. and Khan, M.F.(1993), Fuzzy Semirings, Fuzzy Sets and Systems, 302-309.

[4] Atanassov, K. T. (1986) Intuitionistic fuzzy sets. Fuzzy Sets and Systems,20, 87-96.

[5] Bhargavi Y and Eswarlal T (2015), Fuzzy Γ-semirings, international Journal of Pure and Applied Mathematics,98,339-349.

[6] L. Dale, Direct sums of semirings and the Krull-Schmidt theorem, Kyungpook Math. J. 17(5), 135 - 141.

[7] Dutta, T. K., Sardar, S. K. and Goswami, S. (2011),Operations on fuzzy ideals of Γ semirings, Proceedings of National seminar on Algebra, Analysis and Discrete Mathematics arXiv:1101.4791v1 [math.GM].

[8] H. S. Kim, On quotient semiring and extension of quotient halfring, Comm. Korean Math. Soc. 4 (1989), 17-22.(10)

[9] Rosenfeld, A.(1971), Fuzzy Groups, J. Math Analysis Applications 35, 512-519.

[10] Yager, R.R, Pythagorean fuzzy subsets, in: Proceedings of Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton. Canada, 57-61(2013).

[11] Yager, R.R.: Pythagorean membership grades in multicriteria decision making. IEEE Trans. Fuzzy Syst. 22(4), 958-965(2014).

[12] L. A Zadeh, Fuzzy sets. Inform and control. 8 (1965) 338-353.