Fuzzy Emad-Falih Transform

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ABSTRACT
This study suggested a novel fuzzy transform based on the Emad-Falih transform and used it to derive accurate solutions to first-order fuzzy differential equations. To clarify this methodology, pertinent properties and theorems are shown in detail, and the approach is demonstrated by resolving specific issues.

Keywords: fuzzy number; fuzzy differential equation; strongly generalized differentiable; fuzzy Emad-Falih transforms; fuzzy first-order differential equation.

1. INTRODUCTION

In recent decades, fuzzy differential equations have been employed in various disciplines due to their widespread and substantial applications in a wide range of businesses. We provided this work, which includes a novel approach for solving this kind of problem, in order to stay up with the field of fuzzy differential equations' quick growth and advancement. Our study will be restricted to solving first-order fuzzy differential equations. Following the introduction of the fuzzy derivative by Chang and Zadeh [1], Kandel and Byatt [2] introduced the idea of fuzzy differential equations, and Abbasbandy and Allahviranloo [3] presented the numerical solution approach for solving fuzzy differential equations over time. A extension of the Hukuhara derivative, the fuzzy derivative was described by Seikkala [4]. Bede and Gal [5] provide an excellent introduction to generalized differential, which Bede et al. [6] investigate. In order to solve these kinds of equations, a new fuzzy transform based on the Emad-Falih transform will be built in this study. On the other hand, a number of researchers have studied the "fuzzification" of several techniques that are often used in the crisp scenario and have developed fuzzy versions of these methods, including fuzzy Laplace and fuzzy Abood (see to [7,8] and the cited work therein). Samer et al. used fuzzy systems in the second dimension (system research) to estimate costs [9].

2. Fundamental Preliminaries

For the interest of completeness, the following basic ideas and theorems related to our work in this area are given

(2.1) Definition [10]

By $\mathbb{R}$, the set of all real numbers is represented as, the mapping $\digamma: \mathbb{R} \to [0,1]$is fuzzy number if it fulfills
1. $\digamma$ is upper semi-continuous.
2. $\digamma$ is fuzzy convex, i. e., $\digamma(\zeta\chi + (1-\zeta)\Upsilon) \geq \min\{\digamma(\chi), \digamma(\Upsilon)\}$,for all $\chi, \Upsilon \in \mathbb{R}$ and $\zeta \in [0,1]$.
3. $\digamma$ is normal i. e., $\exists\, \chi_0 \in \mathbb{R}$ for which $\digamma(\chi) = 1$.
4. $\mathrm{supp}(\digamma) = \{\chi \in \mathbb{R}; \digamma(\chi) > 0\}$, and $\mathrm{cl}(\mathrm{Supp}(\digamma))$ is compact.

Let ϖ be the set of all fuzzy number on $\mathbb{R}$. The ζ-level set of a fuzzy number $\digamma \in \varpi, 0 \leq \zeta \leq 1$ denoted by $[\digamma]_\zeta$is defined as

$$[\digamma]_\zeta = \begin{cases} \{\chi \in \mathbb{R}, \digamma(\chi) \geq \zeta\}, & \text{if } 0 \leq \zeta \leq 1 \\ \mathrm{cl}(\mathrm{Supp}(\digamma)) & \text{if } \zeta = 0 \end{cases}$$

Done $[\digamma]_\zeta = [\underline{\digamma}(\zeta), \overline{\digamma}(\zeta)]$, so the$\zeta$–level set$[\digamma]_\zeta$ is a bounded and closed interval for all $\zeta \in [0,1]$.

Zadeh's extension principle states that the operation of addition on ϖ is given by

$$(\digamma \oplus \Omega)(\chi) = \sup_{\Upsilon\in\mathbb{R}} \min\{\digamma(\Upsilon), \Omega(\chi-\Upsilon)\}, \chi \in \mathbb{R}$$

and a fuzzy number's scalar multiplication is provided by

$$(\rho \odot Ғ)(\chi) = \begin{cases} Ғ(\frac{\chi}{\rho}), & \text{if} \rho > 0 \\ \hat{0} & \text{if } \rho = 0 \end{cases} \text{ where } \hat{0} \in \varpi$$

The following characteristics are widely acknowledged to be true at all levels:
$[Ғ \oplus \Omega]_\zeta = [Ғ]_\zeta + [\Omega]_\zeta$, $[\rho \odot Ғ]_\zeta = \rho[Ғ]_\zeta$.

(2.2) Definition [11]

A pair that is sorted parametrically is a fuzzy number, $(\underline{Ғ}, \overline{Ғ})$ of functions $\overline{Ғ}(\zeta), \underline{Ғ}(\zeta)$, $\zeta \in [0,1]$, which fulfills:

1. $\underline{Ғ}(\zeta)$ is a continuous function with a right function of 0 and a left function of (0,1] that is non-decreasing.
2. $\overline{Ғ}(\zeta)$ is a bounded, non-increasing function with 0 continuous right and (0,1] continuous left.
3. $\underline{Ғ}(\zeta) \leq \overline{Ғ}(\zeta)$, $\zeta \in [0,1]$.

For arbitrary $Ғ = (\underline{Ғ}(\zeta), \overline{Ғ}(\zeta)), f = (\underline{f}(\zeta), \bar{f}(\zeta)), 0 \leq \zeta \leq 1$ and $\rho > 0$ we define:

1. Addition $Ғ \oplus \Omega = (\underline{Ғ}(\zeta) + \underline{\Omega}(\zeta), \overline{Ғ}(\zeta) + \overline{\Omega}(\zeta))$.
2. Subtraction $Ғ \ominus_h \Omega = (\underline{Ғ}(\zeta) - \overline{\Omega}(\zeta), \overline{Ғ}(\zeta) - \underline{\Omega}(\zeta))$.
3. Multiplication $Ғ \odot f =$

$$(\min\left\{\underline{Ғ}(\zeta)\overline{\Omega}(\zeta), \underline{Ғ}(\zeta)\underline{f}(\zeta), \overline{Ғ}(\zeta)\overline{\Omega}(\zeta), \overline{Ғ}(\zeta)f\underline{\Omega}(\zeta)\right\}, \max\left\{\underline{Ғ}(\zeta)\overline{\Omega}(\zeta), \underline{Ғ}(\zeta)\underline{\Omega}(\zeta), \overline{Ғ}(\zeta)\overline{\Omega}(\zeta), \overline{Ғ}(\zeta)\underline{\Omega}(\zeta)\right\}$$

4. Scalar multiplication $\rho \odot Ғ = \begin{cases} (\rho\underline{Ғ}, \rho\overline{Ғ}) & \rho \geq 0, \\ \left(\rho\overline{Ғ}, \rho\underline{Ғ}\right) & \rho < 0. \end{cases}$. If $\rho = 1$ then $\rho \odot Ғ = -Ғ$

(2.3) Definition [6]

Let $Ғ$ and Ω are fuzzy numbers, the Hausdorff distance between fuzzy numbers is provided by:
$Ҕ: \varpi \times \varpi \to [0, +\infty]$
$Ҕ(Ғ, \Omega) = \sup_{\zeta \in [0,1]} \max\left\{\left|\underline{Ғ}(\zeta) - \underline{\Omega}(\zeta)\right|, \left|\overline{Ғ}(\zeta) - \overline{\Omega}(\zeta)\right|\right\}$,
Where $Ғ = \left(\underline{Ғ}(\zeta), \overline{Ғ}(\zeta)\right), f = \left(\underline{\Omega}(\zeta), \overline{\Omega}(\zeta)\right) \subset Ɽ$ and following properties are well known:
1. $Ҕ(Ғ \oplus л, \Omega \oplus Ғ) = Ҕ(Ғ, \Omega), \forall Ғ, \Omega, л \in \varpi$.
2. $Ҕ(\rho \odot Ғ, \rho \odot \Omega) = |\rho| Ҕ(Ғ, \Omega), \forall Ғ, \Omega \in \varpi, \rho \in Ɽ$.
3. $Ҕ(Ғ \oplus \Omega, л \oplus ђ) \leq Ҕ(Ғ, \Omega) + Ҕ(л, ђ), \forall Ғ, \Omega, л, ђ \in \varpi$.
4. $(Ҕ, \varpi)$ is a complete metric space.

(2.4) Definition [11]

Let $\phi: Ɽ \longrightarrow \varpi$ be a function with fuzzy values. Assuming a random fixed point $\chi_0 \in Ɽ$ and $\epsilon > 0$ there exists $\delta > 0$ such that $|\chi - \chi_0| < \delta \Longrightarrow Ҕ\left(\phi(\chi), \phi(\chi_0)\right) < \epsilon$, $Ғ$ is said to be a continuous fuzzy-valued function.

(2.5) Definition [12]

A mapping $\phi: Ɽ \times \varpi \longrightarrow \varpi$ is referred to as continuous at one point $(\tau_0, \chi_0) \in Ɽ \times \varpi$ supplied for any fixed $\zeta_0 \in [0,1]$ and arbitrary $\varepsilon > 0$ there exists an $\delta(\varepsilon, \zeta)$ such that $Ҕ\left([\phi(\tau, \chi)]_\zeta, [\phi(\tau_0, \chi_0)]_\zeta\right) < \varepsilon$ whenever $|\tau - \tau_0| < \delta(\varepsilon, \zeta)$ and $Ҕ([\chi]_\zeta, [\chi_0]_\zeta < \delta(\varepsilon, \zeta)$ for all $\tau \in Ɽ, \chi \in \varpi$

(2.1) Theorem [13]

Assume $\Phi(\chi)$ function with fuzzy values on $[e, \infty)$ and it is embodied by $((\underline{\Phi}(\chi, \zeta), \overline{\Phi}(\chi, \zeta))$. For any fixed $\zeta \in [0,1]$, let $\underline{\Phi}(\chi, \zeta)$ and $\overline{\Phi}(\chi, \zeta)$ are Riemann-integrals on $[\wp, q]$. For every $q \geq \wp$, if there are two positive functions $\underline{\theta}(\zeta)$ and $\overline{\theta}(\zeta)$ such that $\int_{\wp}^{q} |\underline{\Phi}(\chi, \zeta)| d\chi \leq \underline{\theta}(\zeta)$ and $\int_{\wp}^{q} |\overline{\Phi}(\chi, \zeta)| d\chi \leq \overline{\theta}(\zeta)$, then the fuzzy number is the improper fuzzy Riemann-integrable, and $\Phi(\chi)$ is said to be improper fuzzy Riemann-integrable on $[\wp, \infty]$, i.e.

$$\int_{\wp}^{\infty} \Phi(\chi)\, d\chi = \left[\int_{\wp}^{\infty} (\underline{\Phi}(\chi, \zeta) d\chi, \int_{\wp}^{\infty} \overline{\Phi}(\chi, \zeta) d\chi\right]$$

(2.6) Definition [12]
Assume Ƒ, ƒ ∈ ϖ.There is л ∈ ϖsuch that $Ƒ = ƒ \oplus л$then л is known the H-differential of Ƒ and ƒ andit is represented by$Ƒ \ominus ƒ$. In this paper, the sign " ⊖ " always stands for H-difference, and also note that $\ominus \neq \ominus_h$ and $Ƒ \ominus ƒ \neq Ƒ + (-1)\, ƒ$

(2.7) Definition [14]
A function$\Phi: (\wp, q): \longrightarrow \varpi$ and $\chi_0 \in (\wp, q)$. We say that Φis strongly generalized differentiable at χ_0If such an element exists $(\chi_0) \in \varpi$, such that:

i. $\forall\, Q > 0$ *that is adequately little, there are*$\Phi(\chi_0 + Q) \ominus \Phi(\chi_0), \Phi(\chi_0) \ominus \Phi(\chi_0 - Q)$,
where $lim_{Q \to 0} \frac{\Phi(\chi_0 + Q) \ominus \Phi(\chi_0)}{Q} = lim_{Q \to 0} \frac{\Phi(\chi_0) \ominus \Phi(\chi_0 - Q)}{Q} = \Phi'(\chi_0)$

or

ii. $\forall\, Q > 0$ *that is adequately little, there are*$\Phi(\chi_0) \ominus \Phi(\chi_0 + Q), \Phi(\chi_0 - Q) \ominus \Phi(\chi_0)$
where $lim_{Q \to 0} \frac{\Phi(\chi_0) \ominus \Phi(\chi_0 + Q)}{-Q} = lim_{Q \to 0} \frac{\Phi(\chi_0 - Q) \ominus \Phi(\chi_0)}{-Q} = \Phi'(\chi_0)$

or

iii. $\forall\, Q > 0$ *that is adequately little, there are*$\Phi(\chi_0 + Q) \ominus \Phi(\chi_0), \Phi(\chi_0 - Q) \ominus \Phi(\chi_0)$
where $lim_{Q \to 0} \frac{\Phi(\chi_0 + Q) \ominus \Phi(\chi_0)}{Q} = lim_{\tau \to 0} \frac{\Phi(\chi_0 - Q) \ominus \Phi(\chi_0)}{-Q} = \Phi'(\chi_0)$

or

iv. $\forall\, Q > 0$ *that is adequately little, there are*$\Phi(\chi_0) \ominus \Phi(\chi_0 + Q), \Phi(\chi_0) \ominus \Phi(\chi_0 - Q)$
where $lim_{Q \to 0} \frac{\Phi(\chi_0) \ominus \Phi(\chi_0 + Q)}{-Q} = lim_{Q \to 0} \frac{\Phi(\chi_0) \ominus \Phi(\chi_0 - Q)}{Q} = \Phi'(\chi_0)$.

(2.2) Theorem [15]
Let $\Phi(\chi): Ʀ \longrightarrow \varpi$be a function and represents $\Phi(\chi) = ((\underline{\Phi}(\chi, \zeta), \overline{\Phi}(\chi, \zeta))$in every instance for $\zeta \in [0,1]$. Then:

1. If $\Phi(\chi)$ is differentiable form i, then $(\underline{\Phi}(\chi, \zeta)$ and $\overline{\Phi}(\chi, \zeta)$ are differentiable functions and $\Phi'(\chi) = (\underline{\Phi}'(\chi, \zeta), \overline{\Phi}'(\chi, \zeta))$.
2. If $\Phi(\chi)$ is differentiable form ii, then $(\underline{\Phi}(\chi, \zeta)$ and $\overline{\Phi}(\chi, \zeta)$ are differentiable functions and $\Phi'(\chi) = (\overline{\Phi}'(\chi, \zeta), \underline{\Phi}'(\chi, \zeta))$.

3. Fuzzy Emad-Falih Transform

The fuzzy start and boundary value difficulties that go along with fuzzy differential equations are resolved by the fuzzy Aboodh transform technique. Fuzzy Aboodh transformations simplify the challenge of solving afuzzy differential equation by reducing it to an algebraic problem. Operational calculus, a crucial branch of practical mathematics, is this transition from calculus operations to algebraic operations on transformations.

(3.1) Emad- Falih Integral Transform[16]
Emad A. Kuffi and Sara F. Maktoof introduced a novel integral transform known as the (Emad-Falih) integral transform. This Transform is defined for the function $\Phi(\chi)$ as:
$\mathbb{EF}\,[\Phi(\chi)] = \frac{1}{r}\int_0^{\infty} \Phi(\chi) e^{-r^2}\, d\chi = \mathbb{T}(r)$, Where $\chi \geq 0, \mu_1 \leq r \leq \mu_1$ andin the function Φ, the variable r is used as a factor to the variable χ

(3.2) Definition
Let $\Phi(\chi)$ be a fuzzy-valued continuous function. Suppose that$\frac{1}{r}\Phi(\chi) \odot e^{-r^2}$is an inappropriate fuzzy Integrable at Rimann on $[0, \infty)$, then $\frac{1}{r}\int_0^{\infty} \Phi(\chi) \odot e^{-r^2}\, d\chi$is being called fuzzy Emad- Falih transform and is known as

$$\widehat{\mathbb{EF}}[\Phi(\chi)] = \frac{1}{r}\int_0^{\infty} \Phi(\chi) \odot e^{-r^2}\, d\chi \;, (r > 0 \text{ and integer})$$

$$\frac{1}{r}\int_0^{\infty} \Phi(\chi) \odot e^{-r^2}\, d\chi = \left(\frac{1}{r}\int_0^{\infty} \underline{\Phi}(\chi, \zeta) e^{-r^2}\, d\chi, \frac{1}{r}\int_0^{\infty} \overline{\Phi}(\chi, \zeta) e^{-r^2}\, d\chi\right).$$

Using the definition of classical Emad- Falih transform, to get:
$\mathbb{EF}[\underline{\Phi}(\chi, \zeta)] = \frac{1}{r}\int_0^{\infty} \underline{\Phi}(\chi, \zeta) e^{-r^2}\, d\chi$ *and* $\mathbb{EF}[\overline{\Phi}(\chi, \zeta)] = \frac{1}{r}\int_0^{\infty} \overline{\Phi}(\chi, \zeta) e^{-r^2}\, d\chi$, then:
$\widehat{\mathbb{EF}}[\Phi(\chi)] = (\mathbb{EF}[\underline{\Phi}(\chi, \zeta)], \mathbb{EF}[\overline{\Phi}(\chi, \zeta)]$

(3.3) Theorem

Let $\Phi(\chi)$,$¥(\chi)$ be continuous fuzzy-valued functions, $\mathbb{d}_1$ and $\mathbb{d}_2$ are constants, then

(1). $\widehat{\mathbb{EF}}[\mathbb{d}_1 \odot \Phi(\chi)] = \mathbb{d}_1 \odot \widehat{\mathbb{EF}}[\Phi(\chi)]$.

(2). $\widehat{\mathbb{EF}}[(\mathbb{d}_1 \odot \Phi(\chi)) \oplus (\mathbb{d}_2 \odot ¥(\chi))] = (\mathbb{d}_1 \odot \widehat{\mathbb{EF}}[\Phi(\chi)]) \oplus (\mathbb{d}_2 \odot \widehat{\mathbb{EF}}[¥(\chi)])$

Proof

$$\widehat{\mathbb{EF}}[\mathbb{d}_1 \odot \Phi(\chi)] = \left(\mathbb{EF}[\mathbb{d}_1 \underline{\Phi}(\chi,\zeta)], \mathbb{EF}[\mathbb{d}_1 \overline{\Phi}(\chi,\zeta)]\right) = \left(\frac{1}{r}\int_0^\infty \mathbb{d}_1 \underline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi, \frac{1}{r}\int_0^\infty \mathbb{d}_1 \overline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi\right)$$

$$= \left(\frac{\mathbb{d}_1}{r}\int_0^\infty \underline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi, \frac{\mathbb{d}_1}{r}\int_0^\infty \overline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi\right)$$

$$= \mathbb{d}_1\left(\frac{1}{r}\int_0^\infty \underline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi, \frac{1}{r}\int_0^\infty \overline{\Phi}(\chi,\zeta) e^{-r^2}\, d\chi\right) = \mathbb{d}_1\left(\mathbb{EF}[\underline{\Phi}(\chi,\zeta)], \mathbb{EF}[\overline{\Phi}(\chi,\zeta)]\right)$$

$$= \mathbb{d}_1 \odot \widehat{\mathbb{EF}}[\Phi(\chi)]$$

(2). Suppose $\Phi(\chi) = (\underline{\Phi}(\chi,\zeta), \overline{\Phi}(\chi,\zeta)$ and $¥(\chi) = (\underline{¥}(\chi,\zeta), \overline{¥}(\chi,\zeta)$

$$\widehat{\mathbb{EF}}[(\mathbb{d}_1 \odot \Phi(\chi)) \oplus (\mathbb{d}_2 \odot ¥(\chi))] = \left(\mathbb{EF}\,[\mathbb{d}_1 \underline{\Phi}(\chi,\zeta) + \mathbb{d}_2 \underline{¥}(\chi,\zeta)], \mathbb{EF}[\mathbb{d}_1 \overline{\Phi}(\chi,\zeta) + \mathbb{d}_2 \overline{¥}(\chi,\zeta)]\right)$$

$$= \left(\frac{1}{r}\int_0^\infty e^{-r^2}\left(\mathbb{d}_1 \underline{\Phi}(\chi,\zeta) + \mathbb{d}_2 \underline{¥}(\chi,\zeta)\right) d\chi, \frac{1}{r}\int_0^\infty e^{-r^2}\left(\mathbb{d}_1 \overline{\Phi}(\chi,\zeta) + \mathbb{d}_2 \overline{¥}(\chi,\zeta)\right) d\chi\right)$$

$$= \left(\frac{1}{r}\int_0^\infty e^{-r^2} \mathbb{d}_1 \underline{\Phi}(\chi,\zeta) d\chi, \frac{1}{r}\int_0^\infty \mathbb{d}_1 \overline{\Phi}(\chi,\zeta) e^{-r^2} d\chi)\right)$$

$$+ \left(\frac{1}{r}\int_0^\infty e^{-r^2} \mathbb{d}_2 \underline{¥}(\chi,\zeta) d\chi, \frac{1}{r}\int_0^\infty \mathbb{d}_2 \overline{¥}(\chi,\zeta) e^{-r^2} d\chi)\right)$$

$$= \mathbb{d}_1\left(\frac{1}{r}\int_0^\infty e^{-r^2} \underline{\Phi}(\chi,\zeta) d\chi, \frac{1}{r}\int_0^\infty \overline{\Phi}(\chi,\zeta) e^{-r^2} d\chi)\right)$$

$$+ \mathbb{d}_2\left(\frac{1}{r}\int_0^\infty e^{-r^2} \underline{¥}(\chi,\zeta) d\chi, \frac{1}{r}\int_0^\infty \overline{¥}(\chi,\zeta) e^{-r^2} d\chi)\right)$$

$$= \mathbb{d}_1\left(\mathbb{EF}[\underline{\Phi}(\chi,\zeta)], \mathbb{EF}[\overline{\Phi}(\chi,\zeta)]\right) + \mathbb{d}_2\left(\mathbb{EF}[\underline{¥}(\chi,\zeta)], \mathbb{EF}[\overline{¥}(\chi,\zeta)]\right)$$

$$= (\mathbb{d}_1 \odot \widehat{\mathbb{EF}}[\Phi(\chi)]) \oplus (\mathbb{d}_2 \odot \widehat{\mathbb{EF}}[¥(\chi)])$$

4. Fuzzy Emad- FalihTransform for First -Order Fuzzy Differential Equation

Examining the fuzzy Emad-Falih transform of the derivative of a first order under generalized H-differentiability is essential to solving fuzzy differential equations for high-order fuzzy differential equations.

(4.1) Theorem

Let $\Phi(\chi)$ is the primitive of $\Phi'(\chi)$ on $[0,\infty)$ and $\Phi(\chi)$is a fuzzy-valued function that is integrable, then:

a. $\Phi(\chi)$is (i)-differentiable then $\widehat{\mathbb{EF}}[\Phi'(\chi)] = r^2 \odot \widehat{\mathbb{EF}}[\Phi(\chi)] \ominus \frac{1}{r} \odot \Phi(0)$.

b. $\Phi(\chi)$is (ii)-differentiable then $\widehat{\mathbb{EF}}[\Phi'(\chi)] = \left(-\frac{1}{r} \odot \Phi(0)\right) \ominus \left(-r^2 \odot \widehat{\mathbb{EF}}[\Phi(\chi)]\right)$

Proof (a)

For a fixed, arbitrary$0 \le \zeta \le 1$,

$$r^2 \odot \widehat{\mathbb{EF}}[\Phi(\chi)] \ominus \frac{1}{r} \odot \Phi(0) = \left(r^2 \mathbb{EF}[\underline{\Phi}(\chi,\zeta)] - \frac{1}{r}\underline{\Phi}(0,\zeta), r^2 \mathbb{EF}[\overline{\Phi}(\chi,\zeta)] - \frac{1}{r}\overline{\Phi}(0,\zeta)\right)$$

Since

$$\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)] = r^2 \mathbb{EF}[\underline{\Phi}(\chi,\zeta)] - \tfrac{1}{r}\underline{\Phi}(0,\zeta), \mathbb{EF}\left[\overline{\Phi}'(\chi,\zeta)\right] = r^2 \mathbb{EF}[\overline{\Phi}(\chi,\zeta)] - \tfrac{1}{r}\overline{\Phi}(0,\zeta).$$

Since $\Phi(\chi)$ is differentiable of form i using Theorem (2.2):

$\underline{\Phi}'(\chi,\zeta)=\underline{\Phi}'(\chi,\zeta), \overline{\Phi'}(\chi,\zeta)=\overline{\Phi}'(\chi,\zeta)$

$\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)]=\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)]=r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]-\frac{1}{r}\underline{\Phi}(0,\zeta)$

$\mathbb{EF}\left[\overline{\Phi'}(\chi,\zeta)\right]=\mathbb{EF}\left[\overline{\Phi}'(\chi,\zeta)\right]=r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]-\frac{1}{r}\overline{\Phi}(0,\zeta)$

$r^2\widehat{\mathbb{EF}}[\Phi(\chi)] \ominus \frac{1}{r}\Phi(0)=\left(\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)], \mathbb{EF}\left[\overline{\Phi'}(\chi,\zeta)\right]\right)=\widehat{\mathbb{EF}}[\Phi'(\chi)]$

(b)

$\left(-\frac{1}{r}\odot\Phi(0)\right)\ominus\left(-r^2\odot\widehat{\mathbb{EF}}[\Phi(\chi)]\right)=\left(-\frac{1}{r}\overline{\Phi}(0,\zeta)+r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)], -\frac{1}{r}\underline{\Phi}(0,\zeta)+r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]\right)$

Since

$\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)]=r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]-\frac{1}{r}\underline{\Phi}(0,\zeta), \mathbb{EF}\left[\overline{\Phi}'(\chi,\zeta)\right]=r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]-\frac{1}{r}\overline{\Phi}(0,\zeta).$

Since $\Phi(\chi)$ is differentiable of form ii using Theorem (2.2):

$\underline{\Phi}'(\chi,\zeta)=\overline{\Phi}'(\chi,\zeta), \overline{\Phi'}(\chi,\zeta)=\underline{\Phi}'(\chi,\zeta)$

$\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)]=\mathbb{EF}\left[\overline{\Phi}'(\chi,\zeta)\right]=r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]-\frac{1}{r}\overline{\Phi}(0,\zeta)$

$\mathbb{EF}\left[\overline{\Phi'}(\chi,\zeta)\right]=\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)]=r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]-\frac{1}{r}\underline{\Phi}(0,\zeta)$

$\left(-\frac{1}{r}\odot\Phi(0)\right)\ominus\left(-r^2\odot\widehat{\mathbb{EF}}[\Phi(\chi)]\right)=\left(\mathbb{EF}[\underline{\Phi}'(\chi,\zeta)], \mathbb{EF}\left[\overline{\Phi'}(\chi,\zeta)\right]\right)=\widehat{\mathbb{EF}}[\Phi'(\chi)]$

(4.1) Example: Consider a fuzzy initial value problem:

$\Phi'(\chi)=\Phi(\chi)$, $\Phi(0,\zeta)=(\zeta-1,1-\zeta), 0\le\zeta\le 1$.

Solution:

Apply both sides' fuzzy Emad-Falih transforms to get

$\widehat{\mathbb{EF}}[\Phi'(\chi)]=\widehat{\mathbb{EF}}[\Phi(\chi)]$

Case (1)

$\Phi(\chi)$ be (i)-differentiable,

$r^2\odot\widehat{\mathbb{EF}}[\Phi(\chi)] \ominus \frac{1}{r}\odot\Phi(0)=\widehat{\mathbb{EF}}[\Phi'(\chi)]$

Using upper and lower functions, to have

$r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]-\frac{1}{r}\underline{\Phi}(0,\zeta)=\mathbb{EF}[\underline{\Phi}(\chi,\zeta)], r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]-\frac{1}{r}\overline{\Phi}(0,\zeta)=[\overline{\Phi}(\chi,\zeta)]$

$(r^2-1)\ \mathbb{EF}[\underline{\Phi}(\chi,\zeta)]=\frac{1}{r}(\zeta-1), (r^2-1)\ \mathbb{EF}[\overline{\Phi}(\chi,\zeta)]=\frac{1}{r}(1-\zeta)$

$\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]=\frac{1}{(r^2-1)}\frac{1}{r}(\zeta-1), \mathbb{EF}[\overline{\Phi}(\chi,\zeta)]=\frac{1}{(r^2-1)}\frac{1}{r}(1-\zeta)$

$\underline{\Phi}(\chi,\zeta)=(\mathbb{EF})^{-1}\left(\frac{1}{(r^2-1)}\frac{1}{r}(\zeta-1)\right), \overline{\Phi}(\chi,\zeta)=(\mathbb{EF})^{-1}\left(\frac{1}{(r^2-1)}\frac{1}{r}(1-\zeta)\right)$

Using inverse Emad- Falih transform

$\underline{\Phi}(\chi,\zeta)=(\zeta-1)e^{\chi}, \overline{\Phi}(\chi,\zeta)=(1-\zeta)e^{\chi}$

Case (2)

$\Phi(\chi)$ be (ii)-differentiable,

$\widehat{\mathbb{EF}}[\Phi'(\chi)]=\left(-\frac{1}{r}\odot\Phi(0)\right)\ominus\left(-r^2\odot\widehat{\mathbb{EF}}[\Phi(\chi)]\right)$

Using upper and lower functions, to have

$r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]-\frac{1}{r}\underline{\Phi}(0,\zeta)=\mathbb{EF}[\overline{\Phi}(\chi,\zeta)], r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]-\frac{1}{r}\overline{\Phi}(0,\zeta)=\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]$

$r^2\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]=\frac{1}{r}(\zeta-1)+\ \mathbb{EF}[\overline{\Phi}(\chi,\zeta)], r^2\mathbb{EF}[\overline{\Phi}(\chi,\zeta)]=\frac{1}{r}(1-\zeta)+\mathbb{EF}[\underline{\Phi}(\chi,\zeta)]$

With simple calculation and Using inverse Emad- Falih transform obtained the solution of case (2)

$\underline{\Phi}(\chi,\zeta)=(\zeta-1)e^{-\chi}, \overline{\Phi}(\chi,\zeta)=(1-\zeta)e^{-\chi}$

5. CONCLUSION

We have developed the fuzzy Emad-Falih transform to solve fuzzy initial-value issues for first-order linear fuzzy differential equations, which can be understood via the use of the highly extended differentiability idea. This might result in solutions whose support varies with time.

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