A Glimpse on Homogeneous Ternary Quadratic $3x^2+2y^2=275z^2$

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ABSTRACT
This paper aims at determining non-zero distinct integer solutions to the homogeneous cone represented by the second-degree equations with three unknowns $3x^2+2y^2=275z^2$ Different patterns of solutions in integer are given by applying factorizationmethod and also, applying substitution technique.

Keywords: Ternary second degree, Homogeneous second degree, solutions in Integers

Launch
The subject of quadratic Diophantine equations has plenty of interesting problems. The equal or unequal equations of second degree having three unknowns are rich in variety and attracted many mathematicians.
For an extensive review of sizable literature and various problems, one may refer [1-16]. This paper focuses on finding various choices of non-zero distinct integer solutions to the homogeneous cone represented by the ternary quadratic Diophantine equation $3x^2+2y^2=275z^2$ .Different sets of integer solutions are obtained by applying the method of factorization and also, through employing the linear transformations.

Technical procedure
The homogeneous ternary quadratic equation representing cone under consideration is

$$3x^2+2y^2=275z^2 \tag{1}$$

The procedure to determine varieties of solutions in integer for (1) is analysed as follows:
Set 1
Assume

$$x=5X, y=5Y \tag{2}$$

The given equation is reduced to

$$3X^2+2Y^2=11z^2 \tag{3}$$

Take

$$z=3a^2+2b^2 \tag{4}$$

Write integer 11in(3)as shown below

$$11=\left(\sqrt{3}+i2\sqrt{2}\right)\left(\sqrt{3}-i2\sqrt{2}\right) \tag{5}$$

Using (4) and (5) in (3) and applying factorization, we have consider

$$\sqrt{3}X+i\sqrt{2}Y=\left(\sqrt{3}+i2\sqrt{2}\right)\left(\sqrt{3}\,a+i\sqrt{2}b\right)^2$$

Comparing the corresponding terms, one has

$$X = 3a^2 - 2b^2 - 8ab$$
$$Y = 2(3a^2 - 2b^2) + 6ab \quad (6)$$

Substituting (6) in(2) ,

$$x = 15a^2 - 10b^2 - 40ab$$
$$y = 30a^2 - 20b^2 + 30ab \quad (7)$$

Thus, (7) & (4) gives the integer solution of (1)

Set 2

Write (3) as

$$11z^2 - 2Y^2 = 3X^2 \quad (8)$$

Take

$$X = 11a^2 - 2b^2 \quad (9)$$

Consider the integer 3 in (8) as

$$3 = (\sqrt{11} + 2\sqrt{2})(\sqrt{11} - 2\sqrt{2}) \quad (10)$$

Substituting (10), (9) in (8) and in view of factorization, consider

$$\sqrt{11}z + \sqrt{2}Y = (\sqrt{11} + 2\sqrt{2})(\sqrt{11}a + \sqrt{2b})^2 \quad (11)$$

Equating the corresponding terms in the above equation

$$z = 11a^2 + 2ab^2 + 8ab$$
$$Y = 22AB + 22a^2 + 4b^2 \quad (12)$$

Substituting (12), (9) in (2) ,we obtain

$$x = 55a^2 - 10b^2$$
$$y = 110ab + 110a^2 + 20b^2 \quad (13)$$

Thus,(13)&(12) gives the integer solutions to (1)

Set 3

Rewrite (3) as

$$11z^2 - 3X^2 = 2Y^2 \quad (14)$$

Assume

$$Y = 11a^2 - 3b^2 \quad (15)$$

Write integer2 in (14) as

$$2 = \frac{(\sqrt{11} + \sqrt{3})(\sqrt{11} - \sqrt{3})}{4} \quad (16)$$

Using (16), (15) in (14) and applying factorization, consider

$$\sqrt{11}z + \sqrt{3}X = \left(\frac{\sqrt{11 + \sqrt{3}}}{2}\right)(\sqrt{11}a + \sqrt{3}b)^2 \quad (17)$$

Equating the corresponding terms in (17) and writing a =2A, b=2B, one has

$$z = 22A^2 + 6B^2 + 12AB \quad (18)$$

and $X = 22A^2 + 6B^2 + 44AB$

$$Y = 44A^2 - 12B^2$$

In view of (2),we have

$$x = 110A^2 + 30B^2 + 220AB$$
$$y = 220A^2 - 60B^2 \quad (19)$$

Thus,(18)&(19) satisfy (1).

Set 4

Introducing

$$X = \alpha + 11\beta$$
$$z = \alpha + 3\beta \quad (20)$$
$$Y = 2p$$
in (14),itleads to
$$\alpha^2 = p^2 + 33\beta^2 \quad (21)$$

which is satisfied by
$\beta = 2rs, p = 33r^2 - s^2, \alpha = 33r^2 + s^2.$
In this case, we obtain
$x = 5(33r^2 + s^2 + 22rs), y = 10(33r^2 - s^2), z = 33r^2 + s^2 + 6rs.$
In addition to (20), there are other solutions to (21) that are illustrated below:
Express (21) as pair of equations as follows in Table 1:

Table 1. pair of equations

Pair	1	2	3	4	5	6
$\alpha + p$	$33\beta^2$	$3\beta^2$	$11\beta^2$	33β	3β	11β
$\alpha - p$	1	11	3	β	11β	3β

Solving each of the above pair of equations, the values of α, p & β are found.
For the sake of simplicity and brevity, the corresponding solutions to (1) are shown as follows:
Solutions from pair1
$$x = 330k^2 + 440k + 140, y = 660k^2 + 660k + 160, z = 66k^2 + 72k + 20$$
Solutions from pair2
$$x = 30k^2 + 140k + 90, y = 60k^2 + 60k - 40, z = 6k^2 + 12k + 10$$
Solutions from pair3
$$x = 110k^2 + 220k + 90, y = 220k^2 + 220k + 40, z = 22k^2 + 28k + 10$$
Solutions from pair4
$$x = 280k^2 + 140, y = 320k + 160, z = 40k + 20$$
Solutions from pair5
$$x = 180k + 90, y = -80k - 40, z = 20k + 10$$
Solutions from pair6
$$x = 180k + 90, y = 80k + 40, z = 20k + 10$$

CONCLUSION
An attempt has been made to obtain various sets of integer solutions to the homogeneous cone $3x^2 + 2y^2 = 275z^2$.As the ternary quadratic equations are plenty, the readers and researchers may search for other forms of second degree euqtions with three unknowns equations to determine their integer solutions.

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