Solving Symmetric Octagonal Intuitionistic Fuzzy Assignment Problem using Modified Best Candidate Method

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ABSTRACT
The concept of an Intuitionistic Fuzzy Number (IFN) is of importance for representing an ill-known quantity. Ranking fuzzy numbers plays a very important role in the decision process, data analysis and applications. An assignment problem is a type of linear programming problem which deals with assigning various jobs to equal number of machines in such a way that the total cost is minimized by maximizing the profit. In this paper, the cost of the Fuzzy Assignment Problem is considered as Symmetric Octagonal Intuitionistic Fuzzy number and it is defuzzified by using a ranking procedure. The classical assignment problem expounded is solved by adopting Modified Best Candidate method which optimizes the cost efficiency.The procedure is illustrated with a numerical Example.

Keywords: Best candidate method, Symmetric Octagonal Intuitionistic Fuzzy number fuzzy number, Defuzzification technique.

1. INTRODUCTION

Assignment problems is an impressive subject, and is employed all the time in solving problems from engineering and management science, and have been widely applied in both manufacturing and service systems. In an assignment problem, n jobs are to be performed by n machines, depending on their efficiency to do the job. In an assignment problem, Cij denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one machine can be assigned exactly one job.

In this paper, we have solved Octagonal Intuitionistic Fuzzy Assignment Problem Using Modified Best Candidate Method. The Symmetric Octagonal Intuitionistic Fuzzy Number (SOIFN) is defuzzified by Magnitude ranking method and the obtained classic AP is solved by Modified BCM with an numerical example.

2.Preliminaries

Literature Review

Zadeh (1965) [9] introduced the concept of fuzzy sets to handle the problem of uncertainty in the evaluation of many real-life situations. Atanassov. K. (1986) [3] introduced Intuitionistic fuzzy set (IFS) as an extension of the fuzzy set, where the degree of membership denoting a non-belongingness to a set is explicitly specified along with the degree of membership of belongingness to the set. In 2017. Menaka.G [7] used a new ranking technique to defuzzify the symmetric octagonal Intuitionistic fuzzy number. [4] HlayelAbdallah Ahmad proposed a new method called the Best Candidate Method (BCM) of electing the candidates among the other candidates and finding the solution for the optimization problems. This method increases solution optimality comparatively more than other classic methods. [1] Furthermore, with slight modifications, Hlayel extended the method and proposed the Modified Best Candidate method, which reduces the time complexity and size scalability.

Basic Definitions

2.1 Fuzzy Sets

If X is a collection of objects denoted generically by x, then the fuzzy set A in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), |x \in X\}$is called the membership function of x in A that maps X to the membership space M (When M contains only the two points 0 and 1,$\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$is identical to the characteristic function of a nonfuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite.

2.2 Normal Fuzzy Set

A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ε X such that μA(x) = 1.

2.3 α – cut

The α Cut of a α level set of a fuzzy set $\tilde{A}$is a set consisting of those elements of the universe X whose membership values exceed the threshold level α

$$\tilde{A}_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

2.4 Intuitionistic Fuzzy set

An Intuitionistic fuzzy set (IFS) A assigns to each element x of the universeXa membership degree $\mu_A(x)\in[0,1]$andanon-membership degree$\gamma_A(x)\in[0,1]$such that$\mu_A(x) + \gamma_A(x) \leq 1$. IFS is mathematically represented as $\{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in X\}$.

2.5 Intuitionistic Fuzzy number

1. Let A∈ F(R)and A is normal that is there exists x∈ Rsuch that A(x) =1then A is called a fuzzy number.
2. Whenever λ ∈ [0,1] then $A_\lambda = \{x, A(x) \geq \lambda\}$is a closed interval denoted by$\lambda[A_\lambda^-, A_\lambda^+]$

An Intuitionistic fuzzy subset $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle / x \in R\}$of the real lineis called an intuitionistic fuzzy number if $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \mu_A(x_1) = 1$

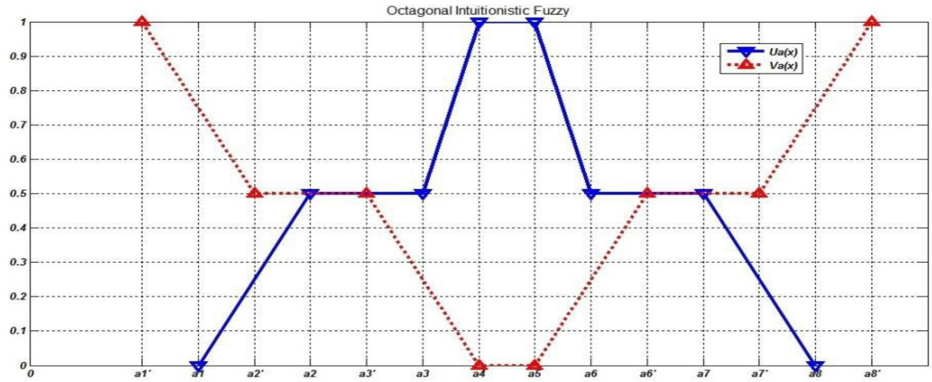
A is convex and$\mu_A$is upper semi continuous and $\gamma_A$is lower continuous.

2.6 Octagonal Intuitionistic Fuzzy Number

An Octagonal Intuitionistic Fuzzy number with the following membership function$\mu_A(x)$ and $\vartheta_A(x)$

$$\mu_A(x) = \begin{cases} 0, & for\ x < a_1 \\ k(\frac{x-a_1}{a_2-a_1}), & for\ a_1 \leq x \leq a_2 \\ k & for\ a_2 \leq x \leq a_3 \\ k + (1-k)\left(\frac{x-a_3}{a_4-a_3}\right) & for\ a_3 \leq x \leq a_4 \\ 1 & for\ a_4 \leq x \leq a_5 \\ k + (1-k)\left(\frac{a_6-x}{a_6-a_5}\right) & for\ a_5 \leq x \leq a_6 \\ k & for\ a_6 \leq x \leq a_7 \\ k(\frac{a_8-x}{a_8-a_7}), & for\ a_7 \leq x \leq a_8 \\ 0 & x > a_8 \end{cases}$$

$$\vartheta_A(x) = \begin{cases} 0, & for\ a_1' \\ k + (1-k)\left(\frac{a_2'-x}{a_4-a_3}\right) & for\ a_1' \leq x \leq a_2' \\ k & for\ a_2' \leq x \leq a_3' \\ k(\frac{a_4-x}{a_4-a_3'}), & for\ a_3' \leq x \leq a_4 \\ 0 & for\ a_4 \leq x \leq a_5 \\ k(\frac{x-a_5}{a_6'-a_5}), & for\ a_5 \leq x \leq a_6' \\ k & for\ a_6' \leq x \leq a_7' \\ k + (1-k)\left(\frac{x-a_7'}{a_8'-a_7'}\right) & for\ a_7' \leq x \leq a_8' \\ 0 & x > a_8' \end{cases}$$



2.7 Ranking function

The ranking function of octagonal intuitionistic fuzzy number is givenas

$$Mag_{\mu}(A) = \frac{2a_1 + 3a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8}{28}$$

$$Mag_{\vartheta}(A) = \frac{2a_1' + 3a_2' + 4a_3' + 5a_4 + 5a_5 + 4a_6' + 3a_7' + 2a_8'}{28}$$

Problem Formulation

2.8 Fuzzy Assignment Problem

Let $\widetilde{c_{ij}}$ be the octagonal intuitionistic fuzzy numbers cost (payment) if jth job is assigned to pth person (see table). The problem is to find an assignment xij so that the total cost for performing all the jobs is minimum.

Table 1. Fuzzy Assignment cost

Job → *Person ↓*	*Job 1*	*Job 2*	*Job k*	*Job n*
Person 1	$\widetilde{C_{11}}$	$\widetilde{C_{12}}$	$\widetilde{C_{1k}}$	$\widetilde{C_{1n}}$
Person k	$\widetilde{C_{k1}}$	$\widetilde{C_{k2}}$	$\widetilde{C_{kk}}$	$\widetilde{C_{kn}}$
Person n	$\widetilde{C_{n1}}$	$\widetilde{C_{n2}}$	$\widetilde{C_{nk}}$	$\widetilde{C_{nn}}$

The chosenFuzzy Assignment Problem (FAP) may be formulated into the following fuzzy linear programming problem:

$$Min\ Z = \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \widetilde{C_{ij}} x_{ij}$$

Subject to :

$$(AP) = \begin{cases} \sum_{i=1}^{i=n} x_{ij} = 1, & j = 1,2, \dots \dots \dots \dots \dots n \\ \sum_{j=1}^{j=n} x_{ij} = 1 & i = 1,2, \dots \dots \dots \dots \dots n \end{cases}$$

where $x_{ij} = \begin{cases} 1 & , if\ the\ i^{th}\ person\ assign\ the\ j^{th}\ job \\ 0 & , otherwise \end{cases}$

2.9 Algorithm for Assignment Problem using Modified Best Candidate Method

Our method is based on determination of the best candidates then elimination the unwanted one in order to minimize the number of solution combinations to decide the optimal solution [Hlayel (2012)]. However, we can notice that the solution approach using this method as one of LAP methods is divide into two phases. We will describe each phase and clarify the new modifications. The first Phase, is to elect the best candidates through choosing the prime candidate and its alternative in each row depending on the

objective function (maximum or minimum value) then elect one candidate for the columns that have no candidate. There are no new modifications in this phase and the solution findings steps are as follows:
Step1: Prepare the matrix. If the matrix is unbalanced, we balance it and we would not use the added row or column candidates in our solution process.
Step2: Determination of the best candidate, it is used for minimization problems (minimum cost) or maximization problem (maximum profit): Elect the best two candidates in each row, if the candidate repeated more than one times elect it also. Check the columns that not have candidates and elect one candidate from them, if the candidate repeated more than one time elect it also.
The second phase, will introduce the following steps:
a. At the end of phase one an index matrix is produced that shows the position for each candidate.
b. Find the direct combinations and calculating the cost for each.
c. Check the unused candidates, by finding the possible candidates for them then calculate the cost for each.
d. Find the optimal solution according to the objective function.

2.10 Algorithm to solve Octagonal Intuitionistic fuzzy assignment problem with Modified BCM
Step 1: First test whether the given fuzzy cost matrix of an fuzzy assignment problem is a balanced/unbalanced. If not change this unbalanced assignment problem by adding the dummy row (s) / column(s) and the values for the entries are zero. If it is a balanced onethen go to step 2. If it is an unbalanced one then convert it into a balanced one and then go to step2.
Step 2: Replace the cost matrix Octagonal Intuitionistic fuzzy number.
Step 3: Defuzzify the fuzzy cost by using ranking method
Step 4: Replace Octagonal numbers by their respective ranking indices.
Step 5: Apply Modified BCM to determine the best combination to produce the lowest total weight of the costs, where elect the best two candidates in each row, if the candidate repeated more than one times elect it also. Check the columns that not have candidates and elect one candidate from them, if repeated more than once elect them.
Step 6: Construct an index matrix and find the direct combination. Calculate cost for each combination. Check for unused candidates, find the possible candidates for them and calculate cost for them also. Now find optimal solution from all the combinations.

3.Numerical Example
An Octagonal Intuitionistic Assignment problem with rows representing 3 machines M1,M2,M3 with 3 jobs J1,J2,J3 is considered.In this Fuzzy Assignment Problem the cost matrix elements are Symmetric Octagonal Intuitionistic Fuzzy Numbers.

Machines	**Jobs**		
	J1	**J2**	**J3**
M1	(4,6,8,10,12,14,16,18; 1,4,7,10,12,15,18,21)	(6,8,10,12,14,16,18,20;3,6,9,12,14,17,20,23)	(9,11,13,15,17,19,21,23;6,9,12,15,17,20,23,26)
M2	(10,12,14,16,18,20,22,24;9,10,13,16,18,21,24,27)	(3,5,7,9,11,13,15,17;0,3,6,9,11,14,17,20)	(4,6,8,10,12,14,16,18;1,4,7,10,12,15,18,21)
M3	(3,5,7,9,11,13,15,17;0,3,6,9,11,14,17,20)	(8,10,12,14,18,20,22;5,8,11,14,16,19,22,25)	(7,9,11,16,15,17,19,21;4,7,10,13,15,18,21,24)

Solution
The given problem is a balanced problem. Now we have to obtain $R(\tilde{C}_{ij})$ of each $(\tilde{C}_{ij})$ using ranking function as follows,

$$Mag_{\mu}(A) = \frac{2a_1 + 3a_2 + 4a_3 + 5a_4 + 5a_5 + 4a_6 + 3a_7 + 2a_8}{28}$$

$$Mag_{\vartheta}(A) = \frac{2a_1^{'} + 3a_2^{'} + 4a_3^{'} + 5a_4 + 5a_5 + 4a_6^{'} + 3a_7^{'} + 2a_8^{'}}{28}$$

For (4,6,8,10,12,14,16,18;1,4,7,10,12,15,18,21)

$$Mag(A) = \frac{8 + 18 + 32 + 50 + 60 + 56 + 48 + 36}{28}$$

$$Mag(A) = \frac{308}{28}$$

$$Mag_{\mu}(A) = 11$$

Similarly, the other values are determined and they are as follows,

Machines	**Jobs**		
	J1	**J2**	**J3**
M1	11	13	16
M2	17	10	11
M3	10	15	14

The formulated fuzzy assignment problem is given as:

$$\text{Min}\{\text{R}(11)x_{11} + R(13)x_{12} + R(16)x_{13} + R(17)x_{21} + R(10)x_{22} + R(11)x_{23} + R(10)x_{31} + R(15)x_{32} + R(14)x_{33}$$

Subject to

$$x_{11} + x_{12} + x_{13} = 1 \qquad x_{11} + x_{21} + x_{31} = 1$$
$$x_{21} + x_{22} + x_{23} = 1 \qquad x_{12} + x_{22} + x_{32} = 1$$
$$x_{31} + x_{32} + x_{33} = 1 \qquad x_{13} + x_{23} + x_{33} = 1$$

where $x_{ij} \in [0,1]$

The above Octagonal Intuitionistic Assignment problem is solved using Modified Best Candidate Method as follows,

Phase 1 : Election of Candidates

Step 1: The matrix is Balanced, where the number of rows is equal to the number of columns as shown in table 1.

Table 1: Profit matrix

Machines	**Jobs**		
	J1	**J2**	**J3**
M1	11	13	16
M2	17	10	11
M3	10	15	14

Step 2 : Elect the best Candidates as shown in table 2

Table 2: Best Candidates Determination Matrix

Machines	**Jobs**		
	J1	**J2**	**J3**
M1	11	13 ◯	16 ◯
M2	17 ◯	10	11 ◯
M3	10	15 ◯	14 ◯

Phase 2 : Obtain the BCM Combinations.

a.Draw the following index matrix as in Table 3

Table 3 : Best Candidate Combination Position Matrix.

Machines	**Jobs**		
	J1	**J2**	**J3**
M1	-	A2	A3
M2	B1	-	B3
M3	-	C2	C3

From the above table we obtain the solution set {A2, A3,B1,B3,C2,C3}.

b. The direct combinations for all the candidates to calculate the cost for each:

Combination 1 :{A2,B1,C3} = 13+17+14 = 44
Combination 2 :{A3,B1,C2} = 16+17+15 = 48
c. Check for unused candidates{B3}, then find the possible combinations:
Combination 3 : {A2,B3} then we add to them C1 and become {A2,B3,C1}= 13+11+10 =**34**
Combination 4 : {B3,C2} then we add to them A1 and become {A1,B3,C2}= 11+11+15 = 37
d. Find the optimal solution according to the objective function (maximum of minimum cost):
In our case it is combination number 3 (modified-BCM solution).

Machine M1 → Job J2
Machine M2 → Job J3
Machine M3 → Job J1

4. CONCLUSION

The fuzzy assignment costs are taken as Octagonal Intuitionistic Fuzzy Number. The membership and non-membership functions are defuzzified using the ranking function. Further the assignment problem is solved by using Modified Best Candidate Method for its time reducibility and optimal solution.

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