Placement of Staff in a company using Fuzzy Assignment Problem

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ABSTRACT
The decision to place the right person for the right job is difficult because of uncertainty and imprecise information. However, fuzzy assignment problems can certainly solve the purpose of placing four candidates for four different designations. In this paper, we solve the dodecagonal fuzzy assignment problem, which is converted into a crisp model by robust ranking technique and then solved using Modified Best candidate Method.

Keywords: Modified Best candidate method, Dodecagonal fuzzy number, Defuzzification technique.

1. INTRODUCTION
The fuzzy set theory was put forward by Zadeh [8] in the year 1965. For the past six decades, researchers have given more attention to fuzzy set theory. It may be applied in fields like operations research, control theory, neural networks, management science, finance, etc. In industry, assignment problems (AP) play a vital role. Determining the optimal assignment that minimizes the assigning cost is the main goal of AP. The assumptions made are:
- Each person is assigned to exactly one job
- Each person can do at most one job.

As a special case, this article discusses the algorithm to solve using fuzzy parameters with Dodecagonal fuzzy numbers. Real-life data is being collected from a European-based company for the placement of candidates for the following four jobs: Job1 – Logistics Manager; Job 2 – HR; Job 3 – Procurement Supervisor; and Job 4 – Sales Supervisor. The scores and rankings of the candidates are formed as a fuzzy assignment problem, and the best candidate is chosen using the modified best candidate methodology. The entries in the assignment matrix are taken as a dodecagonal fuzzy number along with the alpha cut operations. Robust's ordering technique is used for ordering the Dodecagonal fuzzy numbers, which are converted to a crisp model for further usage of the methodology.

2. Preliminaries
Literature Review
Fuzzy assignment problems have been studied by many researchers. [6] The author introduced the Dodecagonal fuzzy number and its properties. [2] The article uses Robust's ranking technique to defuzzify the FAP, and the optimal solution is found by using the Ones Assignment problem. [3] In 2012, Hlayel introduced a new method, the Best Candidate Method (BCM), of electing the candidates among the others and finding the solution for the optimization problems. [1] Furthermore, with slight modifications, Hlayel extended the method and proposed the Modified Best Candidate method.

Basic Definitions
2.1 Fuzzy Sets:
If X is a collection of objects denoted generically by x, then the fuzzy set A in X is a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), |x \in X\}$is called the membership function of x in A.

2.2 Normal Fuzzy Set:
A fuzzy set A of the universe of discourse X is called a normal fuzzy set implying that there exist at least one x ε X such that$\mu_A(x)$ = 1.

2.3 α – cut :

The α Cut of a α level set of a fuzzy set $\tilde{A}$is a set consisting of those elements of the universe X whose membership values exceed the threshold level α

$$\tilde{A}_{\alpha} = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}$$

2.4 Triangular Fuzzy Number:

For a triangular fuzzy number A(x), it can be represented by A(a,b,c;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ \frac{(c-x)}{(c-b)}, & b \leq x \leq c \\ 0 & otherwise \end{cases} \qquad \text{where } a \leq b \leq c$$

2.5 Trapezoidal Fuzzy Number:

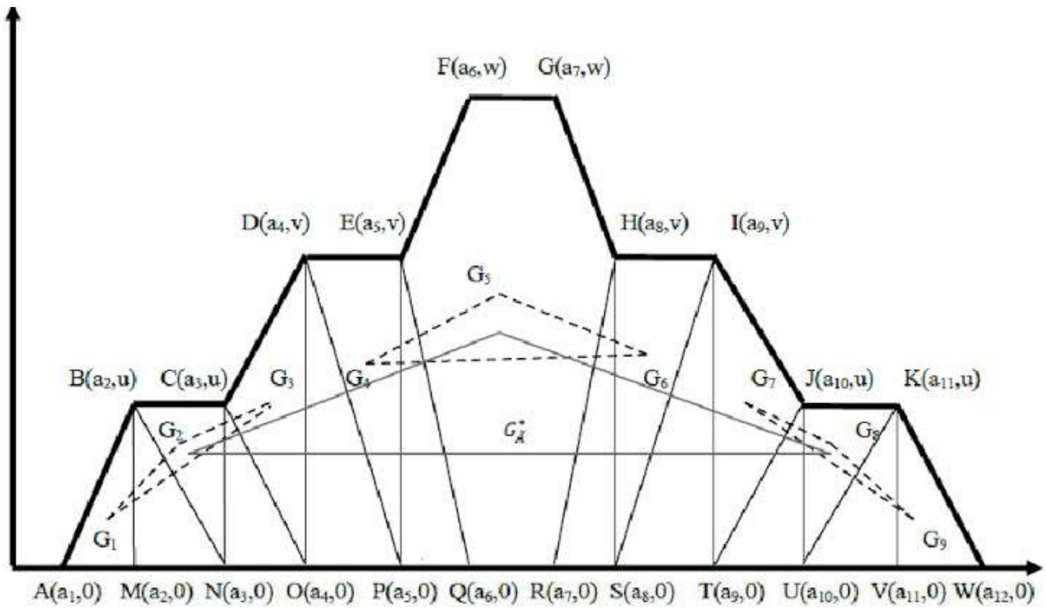
For a triangular fuzzy number A(x), it can be represented by A(a,b,c,d;1) with membership function μ(x) given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{(d-x)}{(d-c)}, & c \leq x \leq d \\ 0 & otherwise \end{cases} \quad \text{where } a \leq b \leq c \leq d$$

2.6 Dodecagonal Fuzzy Number:

A fuzzy number $\tilde{A}_H$is a dodecagonal fuzzy number denoted by $\tilde{A}_H\,(a,b,c,d,e,f,g,h,i,j,k,l)$ wherea,b,c,d,e,f,g,h,i,j,k,l are real numbers and its membership function$\mu_{\tilde{A}}(x)$ is given below

$$\mu(x) = \begin{cases} 0 & x < a \\ \dfrac{x-a}{b-a}, & a \leq x \leq b \\ \dfrac{c-x}{c-b}, & b \leq x \leq c \\ \dfrac{x-d}{e-d}, & d \leq x \leq e \\ \dfrac{f-x}{f-e}, & e \leq x \leq f \\ 1 & x \leq f \\ \dfrac{x-g}{g-f}, & f \leq x \leq g \\ \dfrac{h-x}{h-g}, & g \leq x \leq h \\ \dfrac{x-i}{i-h}, & h \leq x \leq i \\ \dfrac{j-x}{j-i}, & j \leq x \leq k \\ \dfrac{l-x}{l-k}, & k \leq x \leq l \\ 0 & x > l \end{cases}$$



2.7 Alpha Cut of Dodecagonal Fuzzy Number:
The classical set $\tilde{A}_\alpha$ called alpha cut set is the set of elements whose degree of membership is the set of elements whose degree of membership in $\tilde{A}_H$ $(a,b,c,d,e,f,g,\text{?},i,j,k,l)$is no less than α, B it is defined as
$A_\alpha = \{x \in X / \mu_{\tilde{A}_H}(x) \geq \alpha\}$

$$= \begin{cases} [P_1(\alpha), P_2(\alpha), P_3(\alpha), P_4(\alpha), P_5(\alpha)] & for\ \alpha \in [0,0.5) \\ [Q_1(\alpha), Q_2(\alpha), Q_3(\alpha), Q_4(\alpha), Q_5(\alpha)] & for\ \alpha \in [0.5,1] \end{cases}$$

2.8 Alpha cut Operations:
If we get crisp interval by α cut operations Interval A_α shall be obtained as follows for all α∈ [0,1]
Consider $P_1(x) = \alpha$

$$\frac{x-a}{b-a} = \alpha$$
$$x = \alpha(b-a) + a$$
$$P_1(\alpha) = \alpha(b-a) + a$$

Similarly evaluating the other alpha cuts we obtain,

$$A_\alpha = [[(b-a)\alpha + a, c-(c-b)\alpha] + [(e-d)\alpha + d, f-(f-e)\alpha] + [(g-f)\alpha + g, \text{?} - (\text{?} - g)\alpha] + [(i-\text{?})\alpha + i, j-(j-i)\alpha] + [(k-j)\alpha + k, l-(l-k)\alpha]$$

2.9 Defuzzification:
Defuzzification is the process of finding singleton value (crisp value) which represents the average value of the Dodecagonal Fuzzy numbers. Here Robust's Ranking technique is used to defuzzify the Dodecagonal Fuzzy numbers because of its simplicity and accuracy.

2.10 Robust Ranking Technique:
Robust's ranking technique which satisfy compensation, linearity, and additively properties and provides results which are consist human intuition. If ã is a fuzzy number then the Robust Ranking is defined by

$$R(\tilde{a}) = \int_0^1 0.5(a_L^\alpha, a_R^\alpha)\,d\alpha$$

where (a_L^α, a_R^α) is the α – level cut of fuzzy number $\tilde{a}$
The Robust's Ranking technique for dodecagonal fuzzy number is :

$$R(\tilde{a}) = \int_0^1 0.5\{[P_1(\alpha), P_2(\alpha)P_1(\alpha), P_2(\alpha), P_3(\alpha), P_4(\alpha), P_5(\alpha)], [Q_1(\alpha), Q_2(\alpha), Q_3(\alpha), Q_4(\alpha), Q_5(\alpha)]\}d\alpha$$

$$R(\tilde{a}) = \int_0^1 0.5\{[(b-a)\alpha + a, c-(c-b)\alpha] + [(e-d)\alpha + d, f-(f-e)\alpha] + [(g-f)\alpha + g, \text{?} - (\text{?} - g)\alpha] + [(i-\text{?})\alpha + i, j-(j-i)\alpha] + [(k-j)\alpha + k, l - (l-k)\alpha]\}d\alpha$$

This method is for Ranking the objective values. The Robust ranking index R(ã) gives the representative value of fuzzy number ã.

Problem Formulation

2.11 Fuzzy Assignment Problem:

Let $\tilde{C}\ ij$ be the dodecagonal fuzzy numbers cost (payment) if jth job is assigned to pth person (see table). The problem is to find an assignment xij so that the total cost for performing all the jobs is minimum.

Table 2. Fuzzy Assignment cost

Job → *Person ↓*	*Job 1*	*Job 2*	*Job k*	*Job n*
Person 1	$\widetilde{C_{11}}$	$\widetilde{C_{12}}$	$\widetilde{C_{1k}}$	$\widetilde{C_{1n}}$
Person k	$\widetilde{C_{k1}}$	$\widetilde{C_{k2}}$	$\widetilde{C_{kk}}$	$\widetilde{C_{kn}}$
Person n	$\widetilde{C_{n1}}$	$\widetilde{C_{n2}}$	$\widetilde{C_{nk}}$	$\widetilde{C_{nn}}$

The chosen Fuzzy Assignment Problem (FAP) may be formulated into the following fuzzy linear programming problem:

$$Min\ Z = \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \widetilde{C_{ij}}\, x_{ij}$$

Subject to:

$$(AP) = \begin{cases} \displaystyle\sum_{i=1}^{i=n} x_{ij} = 1, & j = 1,2, \ldots\ldots\ldots\ldots n \\ \displaystyle\sum_{j=1}^{j=n} x_{ij} = 1 & i = 1,2, \ldots\ldots\ldots\ldots n \end{cases}$$

where $x_{ij} = \begin{cases} 1 & , if\ the\ i^{th}\ person\ assign\ the\ j^{th}\ job \\ 0 & , otherwise \end{cases}$

2.12 Algorithm for Assignment Problem using Modified Best Candidate Method:

Our method is based on determination of the best candidates then elimination the unwanted one in order to minimize the number of solution combinations to decide the optimal solution [Hlayel (2012)]. However, we can notice that the solution approach using this method as one of LAP methods is divide into two phases. We will describe each phase and clarify the new modifications. The first Phase, is to elect the best candidates through choosing the prime candidate and its alternative in each row depending on the objective function (maximum or minimum value) then elect one candidate for the columns that have no candidate. There are no new modifications in this phase and the solution findings steps are as follows:

Step1: Prepare the matrix. If the matrix is unbalanced, we balance it and we would not use the added row or column candidates in our solution process.

Step2: Determination of the best candidate, it is used for minimization problems (minimum cost) or maximization problem (maximum profit): Elect the best two candidates in each row, if the candidate repeated more than one times elect it also. Check the columns that not have candidates and elect one candidate from them, if the candidate repeated more than one time elect it also.

The second phase, will introduce the following steps:

a. At the end of phase one an index matrix is produced that shows the position for each candidate.

b. Find the direct combinations and calculating the cost for each.

c. Check the unused candidates, by finding the possible candidates for them then calculate the cost for each.

d. Find the optimal solution according to the objective function.

2.13 Algorithm to solve Dodecagonal fuzzy assignment problem with Modified Best Candidate Method:

Step 1: First test whether the given fuzzy cost matrix of fuzzy assignment problem is a balanced/unbalanced. If not change this unbalanced assignment problem by adding the dummy row (s) /

column(s) and the values for the entries are zero. If it is a balanced onethen go to step 2. If it is an unbalanced one then convert it into a balanced one and then go to step2.
Step 2: Replace the cost matrix Dodecagonal fuzzy number.
Step 3: Defuzzify the fuzzy cost by using Robust's ranking method
Step 4: Replace Dodecagonal numbers by their respective ranking indices.
Step 5: Apply Modified BCM to determine the best combination to produce the lowest total weight of the costs, where elect the best two candidates in each row, if the candidate repeated more than one times elect it also. Check the columns that not have candidates and elect one candidate from them, if repeated more than once elect them.
Step 6: Construct an index matrix and find the direct combination. Calculate cost for each combination. Check for unused candidates, find the possible candidates for them and calculate cost for them also. Now find optimal solution from all the combinations.

3. Numerical Illustration

In this paper four candidates namely P1,P2,P3,P4 and four jobs namely Job1 – Logistics Manager, Job 2 – HR, Job 3 – Procurement Supervisor, Job 4 – Sales Supervisor are taken for placement in an Fuzzy Assignment problem. As every person has varying capability and skills, the task here is to place a right person for a right job in order to improve the performance of the organization based on different criteria like qualification, experience and performance in tests. So here we have dodecagonal fuzzy assignment problem where each entry in the matrix is score gained by the employee for different jobs and our objective is to find optimum placement of right candidate at right place.
The Fuzzy Assignment problem is converted to crisp model using Robust Ranking technique and it is then solved using Modified Best Candidate method.

Person/ Jobs	**J1**	**J2**	**J3**	**J4**
P1	(10,13,16,19, 22,25,28,31,34, 37,40,43)	(7,10,13,16, 19,22,25,28, 31,34,37,40)	(2,4,6,8,10, 12,14,16,18, 20,22,24)	(1,2,3,4,5, 7,9,11,13, 17,21,25)
P2	(3,7,11,13,17, 21,22,25,29, 32,40,43)	(2,4,6,8,9, 13,15,16,18, 20,21,25)	(2,3,7,8,9,11, 13,15,16,21, 33,37)	(1,3,5,7,9,12, 15,18,21,25, 29,33)
P3	(1,2,3,4,7,10, 13,15,16,17, 22,26)	(5,8,10,13, 16,21,23,28, 31,32,37,39)	(4,6,7,9,10, 11,18,23,24, 26,27,30)	(2,3,5,7,10 ,13,17,21,25, 29,34,39)
P4	(8,11,14,17, 20,23,26,29, 32,35,38,41)	(6,9,12,15, 18,21,24,27, 30,33,36,39)	(1,2,3,4,7, 10,13,16,20, 24,28,32)	(1,4,7,10,13, 16,19,22,25, 28,31,34)

The FAP can be formulated as:

$$\begin{aligned}\text{Min}\{&R(10,13,16,19,22,25,28,31,34,37,40,43)x_{11} + R(7,10,13,16,19,22,25,28,31,34,37,40)x_{12}\\ &+ R(2,4,6,8,10,12,14,16,18,20,22,24)x_{13} + R(1,2,3,4,5,7,9,11,13,17,21,25)x_{14}\\ &+ R(3,7,11,13,17,21,22,25,29,32,40,43)x_{21} + R(2,4,6,8,9,13,15,16,18,20,21,25)x_{22}\\ &+ R(2,3,7,8,9,11,13,15,16,21,33,37)x_{23} + R(1,3,5,7,9,12,15,18,21,25,29,33)x_{24}\\ &+ R(1,2,3,4,7,10,13,15,16,17,22,26)x_{31} + R(5,8,10,13,16,21,23,28,31,32,37,39)x_{32}\\ &+ R(4,6,7,9,10,11,18,23,24,26,27,30)x_{33} + R(2,3,5,7,10,13,17,21,25,29,34,39)x_{34}\\ &+ R(8,11,14,17,20,23,26,29,32,35,38,41)x_{41} + R(6,9,12,15,18,21,24,27,30,33,36,39)x_{42}\\ &+ R(1,2,3,4,7,10,13,16,20,24,28,32)x_{43} + R(1,4,7,10,13,16,19,22,25,28,31,34)x_{44}\end{aligned}$$

Subject to

$$\begin{gathered} x_{11} + x_{12} + x_{13} + x_{14} = 1 \\ x_{21} + x_{22} + x_{23} + x_{24} = 1 \\ x_{31} + x_{32} + x_{33} + x_{34} = 1 \\ x_{41} + x_{42} + x_{43} + x_{44} = 1 \end{gathered}$$

and

$$\begin{gathered} x_{11} + x_{21} + x_{31} + x_{41} = 1 \\ x_{12} + x_{22} + x_{32} + x_{42} = 1 \\ x_{13} + x_{23} + x_{33} + x_{43} = 1 \\ x_{14} + x_{24} + x_{34} + x_{44} = 1 \end{gathered}$$

where $x_{ij} \in [0,1]$

Now we calculate the crisp values using Robust Ranking formula for R(10,13,16,19,22,25,28,31,34,37,40,43). The alpha cut of dodecagonal fuzzy number is given by,

$$(a_\alpha^L, a_\alpha^U) = [(b-a)\alpha + a, c-(c-b)\alpha] + [(e-d)\alpha + d, f-(f-e)\alpha] + [(g-f)\alpha + g, h-(h-g)\alpha] + [(i-h)\alpha + i, j-(j-i)\alpha] + [(k-j)\alpha + k, l-(l-k)\alpha]$$

R(10,13,16,19,22,25,28,31,34,37,40,43)

$$= 0.5\int_0^1 [3\alpha + 10 + 16 - 3\alpha + 3\alpha + 19 + 25 - 3\alpha + 3\alpha + 28 + 30 - 3\alpha + 3\alpha + 34 + 37 - 3\alpha + 3\alpha + 40 + 43 - 3\alpha]d\alpha$$

$$\mathrm{R}(10,13,16,19,22,25,28,31,34,37,40,43) = 141.5$$

Similarly the Robust's Ranking indices is calculated as follows:

$$R(\tilde{a}_{12}) = 126.5 \;\; R(\tilde{a}_{13}) = 70 \;\; R(\tilde{a}_{14}) = 55 \;\; R(\tilde{a}_{21}) = 121.25 \;\; R(\tilde{a}_{22}) = 70.75$$
$$R(\tilde{a}_{23}) = 83.5 \;\; R(\tilde{a}_{24}) = 82 \;\; R(\tilde{a}_{31}) = 64 \;\; R(\tilde{a}_{32}) = 119.75 \;\; R(\tilde{a}_{33}) = 92$$
$$R(\tilde{a}_{34}) = 95.25 \;\; R(\tilde{a}_{41}) = 131.5 \;\; R(\tilde{a}_{42}) = 121.5 \;\; R(\tilde{a}_{43}) = 75.5 \;\; R(\tilde{a}_{44}) = 96.5$$

We replace these values for their corresponding a_{ij} in which result in a convenient assignment problem in the linear programming problem.

	J1	**J2**	**J3**	**J4**
P1	141.5	126.5	70	55
P2	121.25	70.75	83.5	82
P3	64	119.75	92	95.25
P4	131.5	121.5	75.5	96.5

We use modified best candidate method to get the optimal solution.

Phase 1 : Elect Candidates

Step 1: The matrix is Balanced, where the number of rows is equal to the number of columns as shown in table 1.

Table 1: Person-Job assignment Profit matrix after balance

	J1	**J2**	**J3**	**J4**
P1	141.5	126.5	70	55
P2	121.25	70.75	83.5	82
P3	64	119.75	92	95.25
P4	131.5	121.5	75.5	96.5

Step 2 : Election of Best Candidates

Table 2: Best Candidates Determination Matrix

	J1	**J2**	**J3**	**J4**
P1	141.5	126.5	70	55
P2	121.25	70.75	83.5	82
P3	64	119.75	92	95.25
P4	131.5	121.5	75.5	96.5

Phase 2 : Obtain the BCM Combinations.

a. Draw the index matrix (Table 3) showing the position of each candidate as follows

Table 3 : Best Candidate Combination Position Matrix.

	J1	**J2**	**J3**	**J4**
P1	A1	A2	-	-
P2	B1	-	B3	-
P3	-	C2	-	C4
P4	D1	D2	-	-

From the above table we obtain the solution set {A1,A2,B1,B3,C2,C4,D1,D2}.
b. The cost is calculated from direct combinations for all the candidates from the solution set as:
Combination 1 :{A1,B3,C4,D2} = 141.5+83.5+95.25+121.5 = 441.75
Combination 2 : {A2,B3,C4,D1} = 126.5+83.5+95.25+131.5 = 436.75
c. Check for unused candidates in the solution set{B1,C2}, then find the possible combinations and calculate cost for each:
Combination 3 : {A2,B1,C4} then we add to them D3 and become {A2,B1,C4,D3}= 126.5+121.25+95.25+75.5 = **418.5**
Combination 4 : {A1,B3,C2} then we add to them D4 and become {A1,B3,C2,D4}= 141.5+83.5+119.75+96.5 = 441.25
d. Find the optimal solution according to the objective function (maximum of minimum cost):
In our case it is combination number 3 (modified-BCM solution).

Person 1	Job 2
Person 2	Job 1
Person 3	Job 4
Person 4	Job 3

4. CONCLUSION

This study gives the reader a clear idea of the solvation technique of a dodecagonal fuzzy assignment problem, where the solution is obtained for the real-life problem. The result says that the modified BCM is the most efficient and accurate compared to other currently used methods, and it can be easily used in different areas and applications of optimization problems.

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