An Inventory Model for Seasonal products with varying demand function and price discount

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ABSTRACT
Typically, demand is modelled as either an increasing or decreasing function. However, when dealing with seasonal products, the demand tends to follow a variable pattern. In this paper, we adopt a sin function to represent the fluctuating demand. Additionally, we introduce a discount strategy for the product, activated after a specific duration. In this context, we are focusing on optimizing $t_0$to enhance profit maximization.

Keywords: Sine function, Seasonal products and Discount.

INTRODUCTION

In our everyday lives, we encounter numerous demands, and a significant portion of them lacks formal representation. Consider seasonal products as an illustration; typically, these items experience heightened demand during specific periods, followed by a decline. Examples include raincoats, sweaters, umbrellas, and fashionable attire during festive seasons.

In [1] Lee C. and Ma C. discussed the optimal inventory strategy for deteriorating items, deteriorating items is crucial. Overordering could result in products languishing in the warehouse, leading to spoilage and diminished value. Therefore, it is imperative to optimize the ordering quantity to ensure efficient inventory management. In this context, we embrace the optimization of ordering quantity as a necessary step. In initial paperthe product under consideration is also perishable, butin [2] Lee C.C. and Hsu S. L., the distinction lies in the fact that the demand is time-dependent. This temporal dependency aids in the adoption and application of our specific demand function.[3] Manna S. K. and Chaudhuri K. S. discusses a product characterized by a ramp-type demand within the framework of a purchasing model.Providing valuable insights for selecting an appropriate.In [4]Maragatham M and Gnanavel G gives the purchasing model for a deteriorating item that incorporates a delayed payment structure and offers a price discount for the product. In this context, the adoption of a price discount for retailers is integrated into the model.[7] bySeth B. K. Sarkar B. Goswami A. This paper presents the concept of stimulating demand, facilitating both rising and falling demand scenarios.

Assumptions and Notations

Assumptions

- Demand is a sine function over (0-T) where (T=180) days.
- One product is considered over the period of time.
- All products are taken as good.
- Shortages are allowed with lost sales.
- The retailer offers a price discount after the time t_1.

Notations

- P – Purchasing rate per unit time.
- Q - Inventory level at $t = 0$
- D(t) – demand rate, $D(t) = a + b \sin t$ where a and b are positive constant.
- T- The length of total time we consider (T=180 days).
- t_0 – At the time the inventory gets empty.

- t_1 – At the time the retailer gives a discount the selling price between the time$(0-t_1)$ is s_{p1} and for the time $(t_1 - t_0)$ is s_{p2}, where $s_{p1} > s_{p2}$.
- h – holding cost per unit.
- s – shortage cost per unit.
- TC – Total Cost.
- TR – Total Revenue.

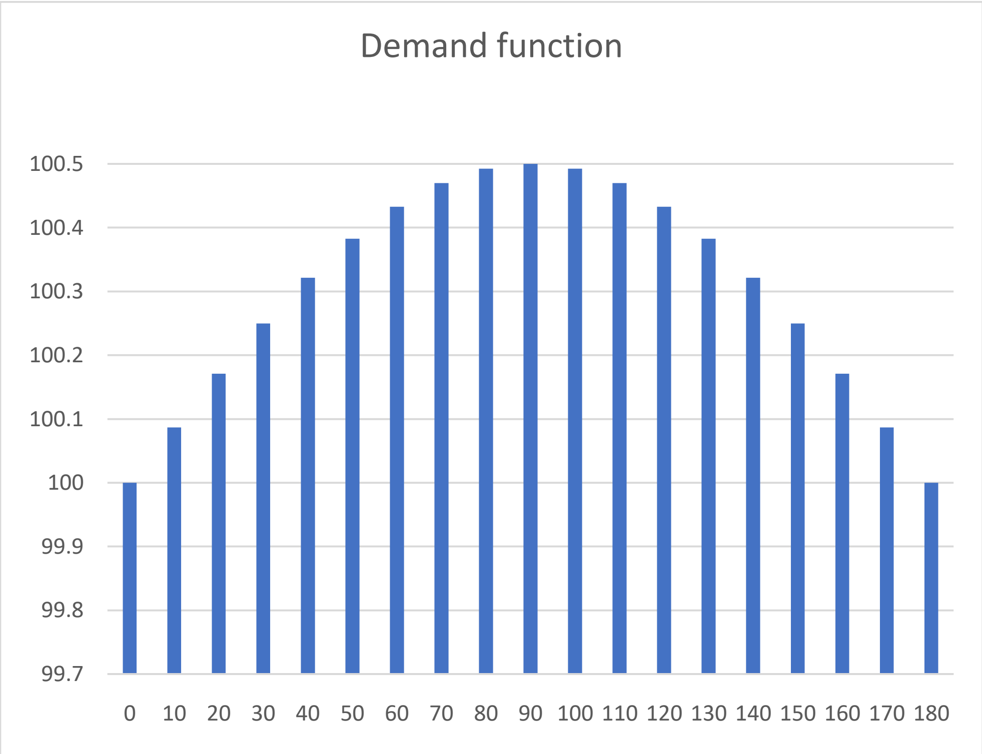
Mathematical Modelling

$$\frac{dI(t)}{dt} = -(a + b\sin t) \qquad 0 \le t \le t_0 \quad \ldots\ldots (1)$$

$$\frac{dI(t)}{dt} = -(a + b\sin t) \qquad t_0 \le t \le T \quad \ldots\ldots (2)$$

With the boundary conditions we have

$I(t_0) = 0$ and $I(0) = Q$



From equation (1)

$$I(t) = -at + b\cos t + at_0 - b\cos t_0 \qquad 0 \le t \le t_0 \quad \ldots\ldots (3)$$

using the boundary condition that $I(0) = Q$ when $t = 0$

$$Q = at_0 + b(1 - \cos t_0) \quad \ldots\ldots (4)$$

From equation (2)

$$I(t) = a(t_0 - t) + b(\cos t - \cos t_0) \qquad t_0 \le t \le T \quad \ldots\ldots (5)$$

Our aim is to Maximize the profit

Where, Total Profit= Total Revenue - Total Cost

Since, Total cost = Purchasing cost + Holding cost +Shortage cost

$$TC = PQ + h\int_0^{t_0} I(t) + s\int_{t_0}^{T} I(t) \quad \ldots\ldots (6)$$

Purchasing cost

$$PQ = p(at_0 + b(1 - \cos t_0))$$

$$PQ = pat_0 + pb - pb\cos t_0 \quad \ldots\ldots (7)$$

Holding Cost

$$Hc = h\int_0^{t_0} I(t)$$

$$HC = h\left[\frac{1}{2}at_0^2 + b\sin t_0 - b.t_0.\cos t_0\right] \quad \ldots\ldots (8)$$

Shortage Cost

$$SC = s\int_{t_0}^{T} I(t)$$

$$SC = s\left[\, at_0.T - \frac{aT^2}{2} + b\sin T - bT\cos t_0 - at_0^2 + \frac{at_0^2}{2} - b\sin t_0 + b.t_0.\cos t_0\right] \text{ ....(9)}$$

Total Cost

$$TC = p(at_0 + b - b\cos t_0) + h\left[\frac{1}{2}at_0^2 + b\sin t_0 - b.t_0.\cos t_0\right] + s\left[\, at_0.T - \frac{aT^2}{2} + b\sin T - bT\cos t_0 - at_0^2 + \frac{at_0^2}{2} - b\sin t_0 + b.t_0.\cos t_0\right] \text{ ....(10)}$$

Differentiating With Respect to "t_0"

$$\frac{\partial TC}{\partial t_0} = saT + pa + (pb + bt_0h + sbT - sbt_0)\sin t_0 - sat_0 + hat_0$$

Again, differentiating with respect to "t_0"

$$\frac{\partial^2 TC}{\partial t_0^2} = pb\cos t_0 + ha + bt_0h\cos t_0 + sbT\cos t_0 - sa - sbt_0\cos t_0$$

Where

$\frac{\partial^2 TC}{\partial t_0^2} > 0$and now $\frac{\partial TC}{\partial t_0} = 0$

Total Cost:

$$= PQ + h\left[\frac{1}{2}at_0^2 + b\sin t_0 - bt_0\cos t_0\right] + s\left[a\,t_0T - \frac{aT^2}{2} + b\sin T - bT\cos t_0 - \frac{at_0^2}{2} - b\sin t_0\, bt_0\cos t_0\right]$$

$$TC = P[at_0 = b(1-\cos t_0)] + h\left[\frac{1}{2}at_0^2 + b\sin t_0 - bt_0\cos t_0\right] + s\left[a\,t_0T - \frac{aT^2}{2} + b\sin T - bT\cos t_0 - \frac{at_0^2}{2} - b\sin t_0\, bt_0\cos t_0\right]$$

During the period from $0 - t_1$the selling price is denoted asS_{p1}. Subsequently, from $t_1 - t_0$, the selling price will be S_{p2}.

Total Revenue $= S_{p1}\int_0^{t_1} I(t)\,dt + S_{p2}\int_{t_1}^{t_0} I(t)\,dt$ ....(11)

$$= S_{p1}\int_0^{t_1}[-at + b\cos t + at_0 - b\cos t_0]\,dt + S_{p2}\int_{t_1}^{t_0}[-at + b\cos t + at_0 - b\cos t_0]\,dt$$

$$= TR = S_{p1}\left[-\frac{at_1^2}{2} + b\sin t_1 + at_0t_1 - b\cos t_0\, t_1\right] + S_{p2}\left[-\frac{at_0^2}{2} + b\sin t_0 + at_0^2 - bt_0\cos t_0\right] - S_{p2}\left[-\frac{at_1^2}{2} + b\sin t_1 + a\,t_0t_1 - bt_1\cos t_0\right] \text{ ....(12)}$$

Our Ultimate aim is to Maximize the profit, where profit is $TR - TC$

$$\text{Profit} = (S_{p1} - S_{p2})\left[-\frac{at_1^2}{2} + b\sin t_1 + at_0t_1 - b\cos t_0\, t_1\right] + S_{p2}\left[-\frac{at_0^2}{2} + b\sin t_0 + at_0^2 - bt_0\cos t_0\right] - \left\{P[at_0 = b(1-\cos t_0)] + h\left[\frac{1}{2}at_0^2 + b\sin t_0 - bt_0\cos t_0\right] + s\left[a\,t_0T - \frac{aT^2}{2} + b\sin T - bT\cos t_0 - \frac{at_0^2}{2} - b\sin t_0\, bt_0\cos t_0\right]\right\}$$

.... (13)

We can verify this with a numerical example.

Example:

A	**B**	**p**	**H**	**S**	**t0**	**Q**	**TC**	**TR**	**Profit**
100	0.5	80	10	60	170.1377	17014.75	15543910	168662389.3	153118480
100	0.5	50	5	10	131.2582	13126.8	3776185	103126156	99349971
100	0.5	50	2	10	165.4863	16549.55	3460956	160184964.9	156724009
100	0.5	50	10	30	147.9428	14795.28	10142983	129768907.9	119625925
100	0.5	50	5	27	167.3909	16740.07	7627861	163635276.6	156007415

CONCLUSION

Normally we discuss a business strategy related to seasonal products and the goal of maximizing profit over a short period of time. The context mentions the consideration of not necessarily ordering the maximum quantity, as doing so may not result in the maximum profit.

The example provided, suggests that the third option, despite not having the maximum quantity (Q) yields the maximum profit. This decision-making process takes into account factors beyond just quantity, such as cost, demand, and potentially other variables.

The statement also hints at the possibility of future expansions in the paper, possibly involving considerations of shortages or managing inventory across two warehouses.

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