MULTIPLE SUMMATION FORMULAE FOR THE MODIFIED MULTIVARIABLE I-FUNCTION

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ABSTRACT. The importance of I-function, H-function and many more special functions has a wide range of applications in applied mathematics and applied physics. Some of the multiple summations for the modified multivariable I-function(MMIF) has been discussed in the present article. Some of the summation formulae are concluded at the end of the paper as special cases of our primary results. Also these summation formulae leads to develop the solution of a boundary value problem.

1. INTRODUCTION

Recent advancements of special functions and their applications in mathematical modelling attracting researchers. The motivation of this work is by the applications of special functions like G, H and I-functions by several authors([1], [2], [3]). The generalization of H-function, namely I-function has great importance in Physics and Applied Mathematics. Prasad [15] generalized the I-function and studied many results. In the literature of the special functions like H, G, Meijer etc., many authors established integral results and solved boundary value problems also([7], [11], [5]). Recently, I-function has found its applications in wireless communication.

Srivastava and Panda [8, 9] studied multivariable H-function. The extension of the same as two functions H and I studied by Prasad and Singh [14, 15]. Here we establish four different summation formulae for the MMIF defined by Prasad [15] and a number of summation formulae derived as particular cases.

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Assume $\mathbb{C}, \mathbb{R}$ and $\mathbb{N}$ as set of complex, real and positive integers respectively and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. We define MMIF as :

(1.1) $$I(Z_1, ..., Z_r) =$$

$$I^{0,n_2;0,n_3;....;0,n_r:\left|R^1:m^1,n^1;....;m^{(r)},n^{(r)}\right.}_{p_2,q_2;p_3,q_3;....;p_r,n_r:\left|R:p^1,q^1;....;p^{(r)},q^{(r)}\right.} \left[\begin{array}{c} Z_1 \\ \vdots \\ Z_r \end{array}\middle| \begin{array}{c} \left(a_{2j};\alpha^1_{2j},\alpha^{11}_{2j}\right)_{1,p_2};\left(\alpha_{3j};\alpha^1_{3j},\alpha^{11}_{3j},\alpha^{111}_{3j}\right)_{1,p_3}; \\ \\ \left(b_{2j};\beta^1_{2j},\beta^{11}_{2j}\right)_{1,q_2};\left(\beta_{3j};\beta^1_{3j},\beta^{11}_{3j},\beta^{111}_{3j}\right)_{1,q_3}; \end{array}\right.$$

$$...;(a_{rj};\alpha'_{rj},...,\alpha^{(r)}_{rj})_{1,p_r} : (e_j;u'_j g'_j,...,u^{(r)}_j g^{(r)}_j)_{1,R'} :$$

$$\cdot$$

$$\cdot$$

$$...;(b_{rj};\beta'_{rj},...,\beta^{(r)}_{rj})_{1,q_r} : (l_j;U'_j f'_j,...,U^{(r)}_j f^{(r)}_j)_{1,R} :$$

$$\left.\begin{array}{c} (a'_j;\alpha'_j)_{1,p^{(1)}},(a^{(r)}_j;\alpha^{(r)}_j)_{1,p^{(r)}} \\ \cdot \\ \cdot \\ (b'_j;\beta'_j)_{1,q^{(1)}},(b^{(r)}_j;\beta^{(r)}_j)_{1,q^{(r)}} \end{array}\right)$$

$$= \frac{1}{(2\pi\, w)^r}\int_{L_1}....\int_{L_r}\xi(s_1,....,s_r)\prod_{i=1}^{r}\phi(s_i)\,{z_i}^{s_i}ds_1\,....ds_r$$

where $\xi(s_1, ..., s_r)$ and $\phi(s_i)$ clearly mentioned in [6]. The MMIF is analytic if

(1.2)
$$\sum_{k=1}^{p_2}\alpha^{(i)}_{2k}+\sum_{k=1}^{p_3}\alpha^{(i)}_{3k}+...+\sum_{k=1}^{p_s}\alpha^{(i)}_{sk}-\sum_{k=1}^{q_2}\beta^{(i)}_{2k}-\sum_{k=1}^{q_3}\beta^{(i)}_{3k}-...-\sum_{k=1}^{q_s}\beta^{(i)}_{sk}-\sum_{j=1}^{R}f^{(i)}_j \le 0$$

The contour integral in (1.1) converges absolutely if $|\arg z_i| < \frac{1}{2}\Omega_i\pi$, where

(1.3) $$\Omega_i=\sum_{k=1}^{n^{(i)}}\alpha^{(i)}_k-\sum_{k=n^{(i)}+1}^{p^{(i)}}\alpha^{(i)}_k+\sum_{k=1}^{m^{(i)}}\beta^{(i)}_k-\sum_{k=m^{(i)}+1}^{q^{(i)}}\beta^{(i)}_k+\sum_{k=1}^{n_2}\alpha^{(i)}_{2k}-\sum_{k=n_2+1}^{p_2}\alpha^{(i)}_{2k}$$

$$+\sum_{k=1}^{n_3}\alpha^{(i)}_{3k}-\sum_{k=n_3+1}^{p_3}\alpha^{(i)}_{3k}+....+\sum_{k=1}^{n_r}\alpha^{(i)}_{rk}-\sum_{k=n_r+1}^{p_r}\alpha^{(i)}_{rk}$$

$$-\sum_{k=1}^{q_2}\beta^{(i)}_{2k}-\sum_{k=1}^{q_3}\beta^{(i)}_{3k}....-\sum_{k=1}^{q_r}\beta^{(i)}_{rk}+\sum_{j=1}^{R'}g^{(i)}_j-\sum_{j=1}^{R}f^{(i)}_j>0 \text{ (i=1,...,r)}.$$

We note

$$\text{(1.4)} \qquad \text{A}=(a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};....;\left(a_{(r-1)j};\alpha'_{(r-1)j},....,\alpha^{r-1}_{(r-1)j}\right)_{1,p_{r-1}}$$

$$\text{(1.5)} \qquad \text{B}=\left(b_{2j};\beta'_{2j},\beta''_{2j}\right)_{1,q_2};....;\left(b_{(r-1)j};\beta'_{(r-1)j},....,\beta^{r-1}_{(r-1)j}\right)_{1,q_{r-1}}$$

$$\text{(1.6)} \qquad \mathbf{A}=\left(a_{rj};\alpha'_{rj},....,\alpha^{(r)}_{rj}\right)_{1,p_r};\Im=(a'_j,\alpha'_j)_{1,p'};....;\left(a^{(r)}_j,\alpha^{(r)}_j\right)_{1,p^{(r)}}$$

$$\text{(1.7)} \qquad \mathbf{B}=\left(b_{rj};\beta'_{rj},....,\beta^{(r)}_{rj}\right)_{1,q_r};\Re=\left(b'_j,\beta'_j\right)_{1,q'};....;\left(b^{(r)}_j,\beta^{(r)}_j\right)_{1,q^{(r)}}$$

$$\mathbf{E}=\left(e_j;u'_jg'_j,....,u^{(r)}_jg^{(r)}_j\right)_{1,R'};L=\left(l_j;U'_jf'_j,....,U^{(r)}_jf^{(r)}_j\right)_{1,R}$$

$$\text{(1.8)} \qquad U=p_2,q_2;p_3,q_3;...;p_{r-1},q_{r-1}\,;\;V=0,n_2;0,n_3;...;0,n_{r-1}$$

$$\text{(1.9)} \qquad Y=(p',q')\,;....\left(p^{(r)},q^{(r)}\right);X=(m',n')\,:....;\left(m^{(r)},n^{(r)}\right)$$

2. Main results

In this section, we establish the summation formulae for the MMIF as follows:

Theorem 2.1.

(2.1)

$$\sum_{u_1,...u_m=0}^{\infty}\prod_{J=1}^{m}\frac{((w_j))_{u_j}}{u_j!}\;I^{V;0,n_r+1:|R':X}_{U;p_r+1,q_r+1:|R:Y}\left(\begin{array}{c|c} z_1 & A; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\text{B}, \end{array}\right.$$

$$\left.\begin{array}{c}(1-g-\sum_{j=1}^{m}t_j;a_1,...,a_r),\text{A}:E:\Im \\ \cdot \\ \cdot \\ (1-h-\sum_{j=1}^{m}t_j;b_1,...,b_r):L:\Re\end{array}\right)$$

$$=I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+2:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-g;a_1,...a_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\text{B}, \end{array}\right.$$

$$\left.\begin{matrix} (1+g-h+\sum_{j=1}^{m} w_j; b_1-a_1,...,b_r-a_r), \mathrm{A} : E : \Im \\ . \\ . \\ (1-h+\sum_{j=1}^{m} w_j; b_1,...,b_r), (1+g-h; b_1-a_1,...,b_r-a_r) : L : \Re \end{matrix}\right)$$

Following the lines of Braaksma([4], p.278), we may establish the asymptotic expansion in the following convenient way :

$$a_i, b_i, b_i - a_i > 0 (i = 1, ..., r),\ \mathrm{Re}(h - g - \sum_{j=1}^{m} w_j) > 0 \textit{ and } |arg(z_i)| < \tfrac{1}{2}(\Omega_i - 2b_i)\pi$$

Proof. To establish the Theorem (2.1), expressing the MMIF by Prasad [15] in the Mellin-Barnes multiple integrals contour using (1.1) and interchanging the order of summation and integration, we obtain

$$I = \frac{1}{(2\pi w)^r} \int_{L_1} ... \int_{L_r} \phi(s_1,...,s_r) \prod_{k=1}^{r} \theta_k(s_k) z_k^{s_k} \frac{\Gamma(g+\sum_{j=1}^{m} a_i s_i)}{\Gamma(h+\sum_{j=1}^{m} b_i s_i)}$$

$$\times \sum_{u_1,...u_m=0}^{\infty} \prod_{J=1}^{m} \frac{((w_j))_{u_j}}{u_j!} \frac{\left(g+\sum_{j=1}^{m} a_i s_i\right)_{\sum_{j=1}^{m} t_j}}{\left(h+\sum_{j=1}^{m} b_i s_i\right)_{\sum_{j=1}^{m} t_j}} ds_1...ds_2$$

Now applying result of Panda([12], p.108, Eq.2) and Gauss's theorem ([10], p.28, Eq.1.7.6) in the above equation and interpreting the resulting expression with the help of (1.1), we arrive at Theorem (2.1).

□

Theorem 2.2.

(2.2)

$$\sum_{u_1,...u_m=0}^{\infty} \prod_{j=1}^{m} \frac{((w_j))_{u_j}}{u_j!} I_{U;p_r+2,q_r+2:|R:Y}^{V;0,n_r+2:|R':X} \left(\begin{matrix} z_1 \\ . \\ . \\ z_r \end{matrix} \middle| \begin{matrix} A; (1-g-\sum_{j=1}^{m} t_j; a_1,...,a_r), \\ . \\ . \\ B; \mathrm{B}, \end{matrix} \right.$$

$$\left.\begin{matrix} (1-g'-\sum_{j=1}^{m} t_j; a'_1,...,a'_r), \mathrm{A} : E : \Im \\ . \\ . \\ (g'-g-\sum_{j=1}^{m} t_j; a_1-a'_1,...,a_r-a'_r), (\sum_{j=1}^{m} w_j - g - \sum_{j=1}^{m} t_j; a_1,...,a_r) : L : \Re \end{matrix}\right)$$

$$= I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+3:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-\frac{g}{2};\frac{a_1}{2},...,\frac{a_r}{2}),(1-g';a'_1,...,a'_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B},(g'-\frac{g}{2};\frac{a_1}{2}-a'_1,...,\frac{a_r}{2}-a'_r), \end{array}\right.$$

$$\left.\begin{array}{c} (g'+\sum_{j=1}^m w_j-\frac{a}{2};\frac{a_1}{2}-a'_1,...,\frac{a_r}{2}-a'_r),\mathrm{A}:E:\Im \\ \cdot \\ \cdot \\ (\sum_{j=1}^m w_j+g'-g;a_1-a'_1,...,a_r-a'_r):L:\Re \end{array}\right)$$

provided

$a_i, a'_i, a_i - 2a_i > 0 (i = 1, ..., r)$, $\mathrm{Re}(g' - \frac{g}{2} - \sum\limits_{j=1}^{m} w_j) > 0$ *and* $|arg(z_i)| < \frac{1}{2}(\Omega_i - 2a_i)\pi$

Theorem 2.3.

(2.3)

$$\sum_{u_1,...u_m=0}^{\infty}\prod_{j=1}^{m}\frac{((w_j))_{u_j}}{u_j!}\ I^{V;0,n_r+4:|R':X}_{U;p_r+4,q_r+4:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-g-\sum_{j=1}^m t_j;a_1,...,a_r), \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B}, \end{array}\right.$$

$$\begin{array}{c} (1-g'-\sum_{j=1}^m t_j;a'_1,...,a'_r),(1-g''-\sum_{j=1}^m t_j;a''_1,...,a''_r), \\ \cdot \\ \cdot \\ (g'-g-\sum_{j=1}^m t_j;a_1-a'_1,...,a_r-a'_r),(\sum_{j=1}^m w_j-g-\sum_{j=1}^m t_j;a_1,...,a_r) \end{array}$$

$$\left.\begin{array}{c} (-\frac{g}{2}-\sum_{j=1}^m t_j;\frac{a_1}{2},...,\frac{a_r}{2}),\mathrm{A}:E:\Im \\ \cdot \\ \cdot \\ (1-\frac{g}{2}-\sum_{j=1}^m t_j;\frac{a_1}{2},...,\frac{a_r}{2}),(g''-g-\sum_{j=1}^m t_j;a_1-a''_1,...,a_r-a''_r):L:\Re \end{array}\right)$$

$$= I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+3:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-g';a'_1,...,a'_r) \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B},(g'-g+\sum_{j=1}^m w_j;a_1-a'_1,...,a_r-a'_r), \end{array}\right.$$

$$\begin{array}{c} (1-g'';a''_1,...,a''_r),(g'+g''-g+\sum_{j=1}^m w_j;a_1-a''_1,...,a_r-a''_r), \\ \cdot \\ \cdot \\ (g''-g+\sum_{j=1}^m w_j;a_1-a''_1,...,a_r-a''_r), \end{array}$$

$$\left.\begin{matrix} \mathrm{A} : E : \Im \\ \cdot \\ \cdot \\ (g' + g'' - g + \sum_{j=1}^{m} w_j; a_1 - a_1' - a_1'', ..., a_r - a_r' - a_r'') : L : \Re \end{matrix}\right)$$

provided $a_i, a_i', a_i'', a_i - a_i' - a_i'' > 0 (i = 1, ..., r)$, $\mathrm{Re}(g' + g'' - g - \sum_{j=1}^{m} w_j) < 1$ *and* $|arg(z_i)| < \frac{1}{2}(\Omega_i - \frac{7}{2}a_i)\pi$.

Theorem 2.4.

(2.4)

$$\sum_{u_1,...u_m=0}^{\infty} \prod_{j=1}^{m} \frac{((w_j))_{u_j}}{u_j!} I_{U;p_r+3,q_r+3:|R:Y}^{V;0,n_r+3:|R':X} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix} \middle| \begin{matrix} A; (-\frac{g}{2} - \sum_{j=1}^{m} t_j; \frac{a_1}{2}, ..., \frac{a_r}{2}), \\ \cdot \\ \cdot \\ B; \mathrm{B}, (1 - \frac{g}{2} - \sum_{j=1}^{m} t_j; \frac{a_1}{2}, ..., \frac{a_r}{2}), \end{matrix} \right.$$

$$\begin{matrix} (-\frac{g}{2} - \sum_{j=1}^{m} t_j; \frac{a_1}{2}, ..., \frac{a_r}{2}), (1 - g - \sum_{j=1}^{m} t_j; a_1, ..., a_r), \\ \cdot \\ \cdot \\ (1 - \frac{g}{2} - \sum_{j=1}^{m} t_j; \frac{a_1}{2}, ..., \frac{a_r}{2}), (-g - \sum_{j=1}^{m} t_j + \sum_{j=1}^{m} w_j; a_1, ..., a_r), \end{matrix}$$

$$\left.\begin{matrix} (g'' - \sum_{j=1}^{m} t_j; a_1'', ..., a_r''), \mathrm{A} : E : \Im \\ \cdot \\ \cdot \\ (g' - g - \sum_{j=1}^{m} t_j; a_1 - a_1'', ..., a_r - a_r'') : L : \Re \end{matrix}\right)$$

$$= I_{U;p_r+3,q_r+3:|R:Y}^{V;0,n_r+3:|R':X} \left(\begin{matrix} z_1 \\ \cdot \\ \cdot \\ z_r \end{matrix} \middle| \begin{matrix} A; \\ \cdot \\ \cdot \\ B; \mathrm{B}, \end{matrix} \right.$$

$$\begin{matrix} (\frac{1-g}{2}; \frac{a_1}{2}, ..., \frac{a_r}{2}), (1 - g''; a_1'', ..., a_r''), \\ \cdot \\ \cdot \\ (\frac{1-g}{2} + g'; \frac{a_1}{2} - a_1', ..., \frac{a_r}{2} - a_r'), (\frac{1-g}{2} + \sum_{j=1}^{m} w_j; \frac{a_1}{2}, ..., a_r), \end{matrix}$$

$$\left.\begin{matrix} (\frac{1-g}{2} + \sum_{j=1}^{m} w_j; \frac{a_1}{2}, ..., \frac{a_r}{2}, \mathrm{A} : E : \Im \\ \cdot \\ \cdot \\ (g' - g + \sum_{j=1}^{m} w_j; a_1 - a_1', ..., a_r - a_r') : L : \Re \end{matrix}\right)$$

provided
$a_i, a'_i, a_i - a'_i - 2a''_i > 0 (i = 1, ..., r)$, $\mathrm{Re}(g' - \frac{g}{2} - \sum_{j=1}^{m} w_j) < \frac{1}{2}$ *and* $|arg(z_i)| < \frac{1}{2}(\Omega_i - \frac{5}{2}a_i)\pi$

To prove Theorems (2.2, 2.3 and 2.4), we follow the similar lines with the help of ([10], p.52, Eq.(2.3.3.5)), ([10], p.56, Eq.(2.3.4.5)) and ([10], p.245, Eq.(III.22)) respectively, instead of Gauss's theorem.

3. Particular Cases

In this section, we observe several particular cases. If we take $a'_i = 0 (i = 1, ..., r)$and assume $g' \to \infty$ in Theorem (2.2) and Theorem (2.4), also using the following properties of confluence,

$$\lim_{\lambda \to \infty} \left[(\lambda)_m \left(\frac{z}{\lambda} \right)^m \right] = z^m \tag{3.1}$$

and

$$\lim_{\rho \to \infty} \left[\frac{(\rho w)^m}{(\rho)_m} \right] = w^m,\ m = 0, 1, ... \tag{3.2}$$

After algebraic simplification, we obtain the following corollaries :

Corollary 3.1.

(3.3)

$$\sum_{u_1, ... u_m = 0}^{\infty} \prod_{j=1}^{m} \frac{(-1)^{t_j} ((w_j))_{u_j}}{u_j!} I_{U;p_r+1,q_r+1:|R:Y}^{V;0,n_r+1:|R':X} \left(\begin{array}{c|c} z_1 & A; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B; \mathrm{B}, \end{array} \right. \left. \begin{array}{c} (1 - g - \sum_{j=1}^{m} t_j; a_1, ..., a_r), \mathrm{A} : E : \Im \\ \cdot \\ \cdot \\ (\sum_{j=1}^{m} w_j - g - \sum_{j=1}^{m} t_j; a_1, ..., a_r) : L : \Re \end{array} \right)$$

$$= I_{U;p_r+1,q_r+1:|R:Y}^{V;0,n_r+1:|R':X} \left(\begin{array}{c|c} z_1 & A; (1 - \frac{g}{2}; \frac{a_1}{2}, ..., \frac{a_r}{2}), \mathrm{A} : E : \Im \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B; \mathrm{B}, (\sum_{j=1}^{m} w_j - \frac{g}{2}; \frac{a_1}{2}, ..., \frac{a_r}{2}) : L : \Re \end{array} \right)$$

provided $a_i > 0 (i = 1, ..., r)$, $\mathrm{Re}(\sum_{j=1}^{m} w_j) > 0$ *and* $|arg(z_i)| < \frac{1}{2}(\Omega_i - a_i)\pi$.

Corollary 3.2.

(3.4)

$$\sum_{u_1,\dots u_m=0}^{\infty}\prod_{j=1}^{m}\frac{(-1)^{t_j}(w_j)_{u_j}}{u_j!}\; I^{V;0,n_r+2:|R':X}_{U;p_r+2,q_r+2:|R:Y}\left(\begin{array}{c|c} z_1 & A; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B}, \end{array}\right.$$

$$\left.\begin{array}{c} (-\frac{g}{2}-\sum_{j=1}^{m}t_j;\frac{a_1}{2},\dots,\frac{a_r}{2}),(1-g-\sum_{j=1}^{m}t_j;a_1,\dots,a_r),\mathrm{A}:E:\Im \\ \cdot \\ \cdot \\ (1-\frac{g}{2}-\sum_{j=1}^{m}t_j;\frac{a_1}{2},\dots,\frac{a_r}{2}),(-g-\sum_{j=1}^{m}t_j+\sum_{j=1}^{m}w_j;a_1,\dots,a_r):L:\Re \end{array}\right)$$

$$=I^{V;0,n_r+1:|R':X}_{U;p_r+1,q_r+1:|R:Y}\left(\begin{array}{c|c} z_1 & A;(\frac{1-g}{2};\frac{a_1}{2},\dots,\frac{a_r}{2}),\mathrm{A}:E:\Im \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B},(\frac{1-g}{2}+\sum_{j=1}^{m}w_j;\frac{a_1}{2},\dots,\frac{a_r}{2}):L:\Re \end{array}\right)$$

provided $a_i>0(i=1,\dots,r)$, $\mathrm{Re}(\sum_{j=1}^{m}w_j)<\frac{1}{2}$ *and* $|arg(z_i)|<\frac{1}{2}(\Omega_i-\frac{3}{2}a_i)\pi$.

Taking $a_i=0(i=1,\dots,r)$ and assume $g''\to\infty$ in Theorem (2.3). Also using the equations (2.4),(3.1) and after algebraic manipulations, we obtain the following corollary.

Corollary 3.3.

(3.5)

$$\sum_{u_1,\dots u_m=0}^{\infty}\prod_{j=1}^{m}\frac{(-1)^{t_j}(w_j)_{u_j}}{u_j!}\; I^{V;0,n_r+3:|R':X}_{U;p_r+3,q_r+3:|R:Y}\left(\begin{array}{c|c} z_1 & A; \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B}, \end{array}\right.$$

$$\begin{array}{c} (1-g-\sum_{j=1}^{m}t_j;a_1,\dots,a_r),(1-g'-\sum_{j=1}^{m}t_j;a'_1,\dots,a'_r), \\ \cdot \\ \cdot \\ (g'-g-\sum_{j=1}^{m}t_j;a_1-a'_1,\dots,a_r-a'_r),(\sum_{j=1}^{m}w_j-g-\sum_{j=1}^{m}t_j;a_1,\dots,a_r) \end{array}$$

$$\left.\begin{array}{c} (-\frac{g}{2}-\sum_{j=1}^{m}t_j;\frac{a_1}{2},\dots,\frac{a_r}{2}),\mathrm{A}:E:\Im \\ \cdot \\ \cdot \\ (1-\frac{g}{2}-\sum_{j=1}^{m}t_j;\frac{a_1}{2},\dots,\frac{a_r}{2}):L:\Re \end{array}\right)$$

$$= I^{V;0,n_r+1:|R':X}_{U;p_r+1,q_r+1:|R:Y}\left(\begin{array}{c|c} z_1 & A;(1-g';a'_1,...,a'_r),\mathrm{A}:E:\Im \\ \cdot & \cdot \\ \cdot & \cdot \\ z_r & B;\mathrm{B},(g'-g+\sum_{j=1}^{m} w_j;a_1-a'_1,...,a_r-a'_r):L:\Re \end{array}\right)$$

provided $a_i > 0(i=1,...,r)$, $\mathrm{Re}(\sum\limits_{j=1}^{m} w_j) < \frac{1}{2}$
and $|arg(z_i)| < \frac{1}{2}(\Omega_i - \frac{3}{2}a_i)\pi$.

We can give a number of corollaries by specializing the parameters. The multiple summation formulae involved in this article are general in nature in their manifold.

4. Concluding Remarks

If I-function defined by Prasad [15] reduces in generalized form of H-function defined by Prasad and Singh [14], we obtain the similar relations using analogue techniques. Also by modifying the functions defined by Srivastava and Panda([8], [9]) and Goyal and Garg [13], we can obtain similar type of relations.

The importance of all these results are common in nature. We can obtain single, double or multiple summation formulae by making use of general multiple summation formulae used here. By specializing various parameters and variables in the MMIF, we get several useful product of such functions like E, F, G, H and I of one and several variables. These formulae are useful in many interesting cases of Applied Mathematics and Mathematical Physics. In the next extension of this work, we are going to apply these summation formulae to obtain the solutions of Boundary value problems.

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