Valuation of Hybrid Stochastic Pricing Models for Equity Warrants: A Comparative Analysis

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ABSTRACT
The valuation of warrant pricing models is vital for improving the methodologies employed in the stock market, providing investors with deeper insights for making informed decisions. Traditional methods, such as the Black-Scholes model, often fall short due to assumptions of constant volatility and constant interest rates, leading to inaccuracies in equity warrant valuation. Additionally, existing alternative models have primarily focused on pricing techniques rather than empirical validation. Therefore, there is a pressing need for a mathematical model that incorporates both stochastic volatility and stochastic interest rate to enhance the accuracy of warrant pricing. Recent studies by Roslan et al. (2020) and Roslan et al. (2022) have investigated the pricing of hybrid equity warrants under these stochastic conditions. At present, a comparative analysis to assess the relative effectiveness of these methods has yet to be conducted.This study aims to empirically assess the performance of hybrid equity warrants pricing models in the Malaysian market, considering the influence ofCox-Ingersoll-Ross (CIR) model for stochastic interest rates and the Heston model for stochastic volatility. The algorithm is created for both pricing models, followed by model calibration andstatistical error measurements.Overall, the results indicate that the pricing model developed by Roslan et al. (2022) outperforms Roslan et al. (2020) model in terms of performance and accuracy. However, in terms of execution time, Roslan et al. (2022)'s pricing model recorded 38.12 seconds to 62.62 seconds, whereas the pricing model of Roslan et al. (2020) took 7.25 seconds to 8.19 seconds. The findings of this study will be highly beneficial in quest of a model commonly used by local investment analysts for evaluating equity warrants.

Keywords: Equity Warrants, Heston-CIR Model, Hybrid Models, Stochastic Volatility, Stochastic Interest Rate

INTRODUCTION

Derivatives are one type of financial contract whose value is based on the expected future price movements of the underlying asset, and used as a tool for hedging, speculating, and arbitraging. Warrants that are issued by a third-party issuer, such as an authorized broker or financial institution, will only grant rights but not obligations to purchase or sell the underlying instrument at the predetermined price in the future. It is just like making a reservation earlier to purchase or sell the underlying instrument at a certain price in the future. In other words, a warrant is a type of financial derivative that confers the privilege, without imposing the duty, to purchase or sell securities at a certain price before their expiration or on the expiration day.

Basically, there are two types of warrants: American and European. American-style warrants can be exercised at any time before the expiration date, while European-style warrants can only be exercised on the day of expiration. American warrant styles are being practiced in Malaysia. The security listed on Bursa Malaysia, along with the renowned stock exchange, plays a crucial role in the financial industry.There are two types of warrants traded in Bursa Malaysia; company warrants and structured warrants. Company warrants, or stock warrants, are issued by the company itself to raise funds for the company. This type of warrant gives the right to buy stock in a company within the contract period and at a specific price. Once the warrant holder chooses to exercise their rights, the company will issue new shares or additional shares for the transaction. Structured warrants, also known as exchange-traded options, do not involve shares; the settlement is only in cash. In these types of warrants, the goal is to create capital liquidity in the market. In other words, a stock warrant applies when the corporation grants holders the choice, without imposing a duty, to purchase new common shares at a predetermined price within a specified period. Additionally, the corporation issues structured warrants.

In the literature, the Black-Scholes model has beencommonly usedfor warrant valuation. A modified version of the original Black-Scholes model by Merton accounts for dividend on the underlying asset, where it eliminates the absence of dividend payments from the underlying share (Merton, 1978). These models do not take into account the effects of changes in a company's overall value or changes in its capital structure, and may underestimate oroverestimate equity warrants. In fact, according to Ukhov (2004) and Xiao et al. (2014), a number of difficulties arose, such as inversely correlated stock volatilities, steady interest rates, and consistent volatility assumptions. Besides, data related to the company's worth and fluctuationare not directly observable, leading to unreliability in terms of accuracy and precision. One of the factors that may influence the pricing of financial derivatives such as warrants is the short life of the interest rate, which varies over time. These short-term interest rates could be problematic for an equity warrant's lengthy lifespan. Volatility is also important in determining warrants price. The limitation of the Black-Scholes model is its assumption of constant volatility, which fails to capture the dynamic nature of market volatility.In reality, volatility fluctuates from time to time, particularly during times of market turmoil or unexpected events.

Besides that, the Black Scholes model did not reflect the real world based on several assumptions, which may cause mispricing of these financial derivatives. To overcome these problems in determining the warrant price, it is important to consider stochastic interest rate and stochastic volatility to allow fluctuations in the price of underlying asset. A new equity warrant pricing model needs to be developed in order to enhance market characterization and minimize errors brought on by fixed interest rates and fixed volatility,by combining hybridizations of stochastic volatility and stochastic interest rates.

In this paper, we presenta comparative analysisconsistinghybridization of the Heston stochastic volatility model and the Cox-Ingersoll-Ross (CIR) stochastic interest rate model for the pricing formulation of equity warrants in Malaysia. The relative benefits of one over the other have not yet been discussed empirically, despite the fact that Roslan et al. (2020) and Roslan et al. (2022) had performed studies for pricing hybrid equity warrants with these stochastic properties. The outcomes of this study may be used as a benchmark pricing tool for equity warrants.The next section outlines the related literature, while the subsequent section details the methods and phases involved. The results and analysis section evaluates bothpricingmodelsagainst the real market to validate and assesstheir performance, accuracy, and efficiency.Finally, the last section concludes this study.

LITERATURE REVIEW

A warrant model evaluator is critical for improving Malaysia's stock market and economy. It provides investors with valuable data for making better choices and improves their understanding of warrant pricing dynamics. Equity warrants are financial instruments that grant the holder the right to purchase (call warrant) or sell (put warrant) an underlying asset at a certain price before the expiration date (Sawal, Ibrahim, and Roslan, 2022). Warrants, unlike options, are issued by firms rather than exchanges. They frequently have longer expiration dates, which can lead to dilution when exercised. The differences between warrants and call options of debt-free companies are discussed by Bertrand (2024), who noted that there are differences between structural and reduced form pricing models. It is important to have structural models to determine the value of a new warrant. There is complexity in valuing equity warrants compared to valuing options because, in valuing warrants, it takes into consideration policy changes and market conditions that influence volatility (Tian et al., 2019). Ibrahim et al. (2020) offered an in-depth review of stochastic approaches used in pricing equity warrants that specifically focus on stochastic volatility and interest rate models.

The Black-Scholes model, which assumed that the price of the underlying asset follows geometric Brownian motion, has limitations in terms of volatility (Sawal et al., 2022). According to Glória, Dias, and Cruz (2024), many studies on warrant valuation assumed that warrant volatility is constant. Meanwhile Tian et al. (2019) highlighted the Black-Scholes model has many deviations from market conditions from its underlying assumptions. In addition, Bertrand (2024) mentioned that the Black-Scholes formula for valuing warrant pricing is less accurate at low interest rates, and its responses to fluctuation estimation increase. Therefore, it is important to enhance the Black-Scholes model by incorporating other characteristics to more accurately represent the intricacies of the real world (Tian et al., 2019).

Today, there exists many new pricing formulas using stochastic analysis and certain theories, along with the advancement of pricing techniques for financial products (Cheng, 2024). One example is hybrid models for pricing equity warrants using CIRmodel for interest rates and Heston model for volatility (Roslan et al., 2020; Roslan et al., 2022). Roslan et al. (2022) applied their hybrid model, the Black Scholes model, and the Noreen Wolfson model to real market data, and the hybrid model shows superior performance whereby 96.875% of the warrants have strike prices lower than the current market price, which can bring profit to the investors.Gu and Wei (2023) found that the Fourier-Cosine method for pricing equity-indexed annuities under the Heston model is as accurate as those produced by the Monte Carlo simulation technique. Peng (2023) stated that the outcomes of simulating stock prices using mixed bi-fractional Brownian motion are more accurate compared to the traditional Black-Scholes model. Yoon (2022) mentioned that subscription warrant prices typically amount to 69.3% of their adjusted Black-Scholesmodel pricing when initially listed, and after listing, prices tend to rise, accompanied by substantial trading volumes, short sales, and long-term underperformance, which indicated the possibility of arbitrage opportunities.

Therefore, it is important to have profitable warrants for gain more returns in the future. According to Roslan et al. (2022), it is imperative to have a mathematical model that integrates both stochastic volatility and interest rates to achieve precise warrant pricing. Meanwhile, Sawal et al. (2022) have developed more recent models that incorporate jump diffusion, stochastic volatility, and stochastic interest rates. Accurate pricing necessitates the use of advanced models that take into account volatility surfaces, jumps in asset prices, and fat-tailed distributions. Optimal portfolio selection requires balancing risk and return preferences, considering factors such as asset correlation and investor risk tolerance. On the other hand, Tian et al. (2019) calculated the price of equity warrants using a mean-reverting stock model and uncertainty theorywhich attains more effective results.

Meanwhile, in valuing equity warrants, Shokrollahi (2022) adopts uncertainty theory and uses a formula for an uncertain stock model. Still, Wang et al. (2022) showed a way to price equity warrants that mixes fractional Brownian motion with interest rates based on the Merton short rate model. They used delta hedging to create partial differential equations for equity warrants that include clear pricing formulas and numerical data. Ibrahim et al. (2022)on the other hand, used a pricing method for call warrants that was based on mixed-fractional Brownian motion with Merton jump-diffusion. This method used risk-neutral valuation and quasi-conditional anticipation.

In valuing warrants, Carrion, Imerman, and Zhang (2024) adopted Monte Carlo simulations by using path-dependent cumulative Parisian redemption features, where the existence of these features makes complex warrants have a high impact on the price. Lvand Jiang (2024) adopted a 4/2 stochastic volatility model using CIR interest rates in valuing foreign exchange options by using market data. This model was considered precise and better than the Monte Carlo simulation method, and proved to have better performance compared to the Heston model with CIR interest rates.Recently, He and Lin (2024) introduced a three-factor Heston-Hull White (HW)foreign exchange model for valuing foreign currency options, which is directly comparable to the one-factor Heston-HW model. The three-factor model combines the Heston model for volatility with the CIR process for interest rates and volatility.

In addition, Chen and Jiang (2022) adopted fractional Brownian motion models for stock warrant pricing and claimed that the models perform better and the result is consistent with the real market value. This model can be used to value equity warrant pricing by using a variety of complex models beyond fractional Brownian motion. Glória et al. (2024) have developed an enhanced version of the stock warrant pricing model that incorporates jumps and stochastic components. This model introduced leaps in asset price dynamics to increase its accuracy, as well as a pricing formula that takes into account jumps, stochastic volatility, and interest rates. On the other hand, IndrawatiandKohardinata (2022) found that both ownership concentration and the inclusion of warrants do not affect stock under-pricing, and COVID-19 had a negative influence on stock under-pricing. Variations in the number of derivative warrants purchased and sold could provide significant insights into the related stock, potentially leading to price correction the following day.

RESEARCH METHODOLOGY

This study's primary goal is to conduct comparative analysis for Malaysian equity warrants pricing formulation using two stochastic hybrid models of Roslan et al. (2020) and Roslan et al. (2022). Altogether, there are four phases involved in this research's method. The first phase is data gathering and literature search which involves reviewing the relevant equity warrants pricing models. In the second phase, model implementation will be conducted to apply the models into the practical sense. In the next stage, computer test and calibration are performed for validation purpose. Ultimately, the outcomes will be examined in the final stage. The following is a description of the detailed procedures:

Phase 1: Data gathering and model review

Data for this study are gathered in the first phase from two sources: Bursa Malaysia in terms of open data and purchased ones, as well as from external parties. Due to lengthier lengths until expiration, only 160 European and American style warrants are chosen for this research, in the period of December 2015 to December 2019. The emphasis is on equity warrants that are actively traded.

The Heston-CIR model used as the benchmark in this study, can be specified as

$dS(t) = r(t)S(t)dt + \sqrt{v(t)}S(t)d\widetilde{w}_1(t),$

$dv(t) = k^*(\theta^* - v(t))dt + \sigma\sqrt{v(t)}d\widetilde{w}_2(t),$

$dr(t) = \alpha^*(\beta^* - r(t))dt + \eta\sqrt{r(t)}d\widetilde{w}_3(t).$

The underlying price, volatility, and interest rate are represented respectively by $S(t)$, $v(t)$, and $r(t)$. In particular, $v(t)$ indicates the asset's volatility, $S(t)$ reflects the price of the asset driven by the drift $r(t)$.k^* denotes the mean-reversion process, θ^* is the long term-mean, and σ is the volatility of the instantaneous variance process of $v(t)$. Furthermore, $r(t)$ is defined as the instantaneous interest rate, where α^* denotes the rate's mean-reversion speed, β^* is the rate's long-term mean, and η tracks the rate's volatility. The model's correlation between processes are defined by $(d\widetilde{w}_1(t), d\widetilde{w}_2(t)) = \rho dt, (d\widetilde{w}_1(t), d\widetilde{w}_3(t)) = 0$, and $(d\widetilde{w}_2(t), d\widetilde{w}_3(t)) = 0$, with $-1 < \rho < 1$ and $0 \leq t \leq T$.To ensure that the square root processes are always positive, the requirements$2k^*\theta^* \geq \sigma^2$ and $2\alpha^*\beta^* \geq \eta^2$must be met.

Roslan et al. (2022) pricing model

Roslan et al. (2022)appraises the price of the equity warrant $W(t)$at time $t \in [0, T]$ as

$$W(t)(S(t), T, t, G, \sigma, v, r, k, N, M) = \frac{1}{N+Mk}\left[kS(t)\phi(d_1) - NGe^{-r(T-t)}\phi(d_2)\right]$$

where

$$d_1 = \frac{\ln\frac{kS}{NG} - \ln P(r,t,T) + \frac{1}{2}L(T-t) + \frac{1}{2}Q}{\sqrt{L(T-t) + Q}},$$

$$Q = \eta^2 r\int_t^T \left(\frac{2(e^{2R} - 2e^R + 1)}{2((\alpha^*)^2 + 2\eta^2) + (e^R - 1)(C)}\right) ds,$$

$$R = (T-s)\sqrt{(\alpha^*)^2 + 2\eta^2},$$

$$C = \left(\alpha^*\sqrt{(\alpha^*)^2 + 2\eta^2} + (\alpha^*)^2 + 3\eta^2 + (e^R)\left((\alpha^*)^2 + \alpha^*\sqrt{(\alpha^*)^2 + 2\eta^2} + \eta^2\right)\right),$$

$$d_2 = d_1 - \sqrt{L(T-t) + \eta^2 r\int_t^T B^2(s,T)ds},$$

$$P(r,t,T) = A(t,T)e^{-B(t,T)r(t)},$$

$$A(t,T) = \left(\frac{2\left(e^{\left(\alpha^* + \sqrt{(\alpha^*)^2 + 2\eta^2}\right)\frac{(T-t)}{2}}\right)\left(\sqrt{(\alpha^*)^2 + 2\eta^2}\right)}{\left(2\sqrt{(\alpha^*)^2 + 2\eta^2}\right) + \left(\alpha^* + \sqrt{(\alpha^*)^2 + 2\eta^2}\right)\left(e^{(T-t)\sqrt{(\alpha^*)^2 + 2\eta^2}} - 1\right)}\right)^{\frac{2\alpha^*\beta^*}{\eta^2}}$$

$$B(t,T) = \frac{2\left(e^{(T-t)\sqrt{(\alpha^*)^2 + 2\eta^2}} - 1\right)}{\left(2\sqrt{(\alpha^*)^2 + 2\eta^2}\right) + \left(\alpha^* + \sqrt{(\alpha^*)^2 + 2\eta^2}\right)\left(e^{(T-t)\sqrt{(\alpha^*)^2 + 2\eta^2}} - 1\right)}.$$

and

$\Phi(\cdot)$represents the cumulative Gaussian distribution function.

Roslan et al. (2020) pricing model

According to Roslan et al. (2020), the payoff function of a warrant $W(S, v, r, T)$can be written in semi-closed formula

$$W(S, v, r, \tau) = NG\left(\frac{k}{NG}e^{\widetilde{C}(\tau) + \widetilde{D}(\tau)v + \widetilde{E}(\tau)r} - 1\right)$$

where $\widetilde{C}(\tau)$, $\widetilde{D}(\tau)$, and $\widetilde{E}(\tau)$ are representatives for $\widetilde{C}(-i,\tau)$, $\widetilde{D}(-i,\tau)$ and $\widetilde{E}(-i,\tau)$ respectively.

The equation of $\widetilde{C}(\tau)$, $\widetilde{D}(\tau)$, and $\widetilde{E}(\tau)$ are given as follows:

$$\widetilde{D}(\tau) = \frac{\tilde{a} + \tilde{b}}{\sigma^2}\left(\frac{1 - e^{\tilde{b}\tau}}{1 - \tilde{g}e^{\tilde{b}\tau}}\right),$$

$\tilde{a} = k^* - \rho\sigma, \tilde{g} = \frac{\tilde{a}+\tilde{b}}{\tilde{a}-\tilde{b}}, \tilde{b} = \sqrt{\tilde{a}^2},$
$\frac{d\tilde{E}}{d\tau} = \frac{1}{2}\eta^2\tilde{E}^2 - (\alpha^* + B(T-\tau,T)\eta^2)\tilde{E} + 1,$
$\frac{d\tilde{C}}{d\tau} = k^*\theta^*\tilde{D} + \alpha^*\beta^*\tilde{E}.$

Phase 2: Model implementation
Following the above specified formulas, the algorithm is constructed for both pricing models. In this phase, computer coding using MATLAB is conducted. The version of Matlab Software used is R2020a.

Phase 3: Computer tests and calibration
The aim of calibration is to determine the parameter set that minimize the distance between model prediction and observed market price. In other words, it relates to the error between the market and model prices or implied volatilities (Chang et al., 2021). Thus, calibration has been carried out by minimizing the loss function through the nonlinear least squares algorithm lsqnonlin provided in the Matlab Optimization Toolbox, which belongs to the class of local optimizers. The function lsqnonlin(costfun,x_0,lb,ub) starts at the point x_0 and finds a minimum of the sum of squares of the cost functions described in costfun. A vector of lower and upper bounds is defined respectively, so that the solution which means that Heston-CIR calibrated initial parameters will be between those bounds.
After the development of computer codes, pilot test is performed to examine both models' performance. Following this, the validation procedure will compare both models' prices with the actual warrant prices from Bursa Malaysia. Enhancing the models' usability and functionality is the goal.

Phase 4: Analysis of findings
At this point, additional examination is done to evaluate how well the involved models perform in terms of pricing errors. From literature, Mrázek & Pospíšil (2017) utilized three methods to calculate errors in pricing European call options such asMaximum Absolute Relative Error (MARE), Average Absolute Relative Error (AARE), and Root Mean Square Error (RMSE). Meanwhile, Bauer (2012) used Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), Mean Percentage Error (MPE), and RMSE to measure the option prices model. In contrast, Chang et al. (2021) only chosed two methods- MSE and RMSE to calculate errors.
Three methods are chosen to calculate errors in this study namely MAE, MAPE, and RMSE. These metrics are utilised to indicate the average difference to know the most accurate method. The MSE is a reliable tool to assess the performance of the methods or models (Fisher, 1920). RMSE and MAE are standard metrics used for method assessment, and the RMSE is widely used as an appropriate measurement for model errors in geosciences (McKeen et al., 2005).
The most appropriate measure of average error is MAE according to the perspective aligned with previous analyses (Mielke & Berry, 2007). The RMSE is the square root of the MSE. Applying the square root operation does not change the relative rankings of the method, but it results in a metric that shares similar measures. This metric efficiently obtains the standard error for normal distribution errors. Moreover, the MAE is a statistical measure derived from the L2 norm and L1 norm respectively. The RMSE is a frequent statistical measure used to evaluate model performance in meteorology, air quality, and climate research. MSE is another reliable tool to assess the performance of the method or model (Chai & Draxler, 2014; Hodson, 2022) and measure the volatility of errors (Baur, 2012). Meanwhile, MAPE is a statistical technique for computing the margin error resulting from applying the predicted Least Square Method (Khair et al., 2017), and quantifies the degree of prediction error in relation to the actual value (Khair et al., 2017).
The respective formulas for the error calculation in this study are as follows:
MAE $= \frac{1}{n}\left(\sum_{i=1}^{n} |y_i(x) - \bar{y}_i(x)|\right),$
MAPE $= \frac{1}{n}\left(\sum_{i=1}^{n} \frac{|y_i(x) - \bar{y}_i(x)|}{y_i(x)}\right),$
RMSE $= \sqrt{\frac{1}{n}\left(\sum_{i=1}^{n} (y_i(x) - \bar{y}_i(x))^2\right)},$
where $y_i(x)$ represent the approximate warrant price from a model and $\bar{y}_i(x)$ refer to the actual value of warrant price. The model with the lowest error is the best.

Results and Analysis

Warrant pricing models' performance

Roslan et al. (2022) conferred that their pricing strategy outperformed other warrants pricing models such as the Black Scholes model and the Noreen Wolfson model with remarkable results. Consequently, this subsection appraises the pricing models of Roslan et al. (2020) and Roslan et al. (2022) numerically in quest of a pricing model which can be regarded as an equity warrants benchmark price tool. The respective predictions are recorded in Table 1-5 along the years of 2015 until 2019 to explore both model's efficiency.

Table 1. Roslan et al. (2020) and Roslan et al. (2022) warrant prices along the market price for year 2015.

Warrants	Roslan et al. (2020)	Roslan et al. (2022)	Market price
APPASIA-WA	-0.5862744460343730	0.0800000005747	0.080
AZRB-WA	-0.2431964358763730	0.2380000017049	0.238
BIMB-WA	6.6564107642286200	0.3430000028249	0.343
BTM-WB	-0.5977598359593730	0.1550000040117	0.155
DIGISTA-WB	-0.5042500360393730	0.0750000006771	0.075
DNONCE-WA	0.0450774391348180	0.0830000000000	0.083
DOMINAN-WA	0.2145284393348180	0.2830000000000	0.283
DPS-WB	-0.5640434360563730	0.0580000001760	0.058
ECOWLD-WA	4.9162874579936700	0.4180000000000	0.418
EG-WC	0.1057584397868180	0.7350000000000	0.735
GPA-WA	-0.5247716360813730	0.0330000000873	0.033
GUNUNG-WB	-0.5283796359863730	0.1280000031744	0.128
INIX-WA	0.0208398390798180	0.0280000000000	0.028
JIANKUN-WA	0.0471532176636730	0.0880000000015	0.088
KIMLUN-WA	-0.1178930757013730	0.4130000080938	0.413
LBS-WB	1.5445964393118100	0.2600000000000	0.260
LEWEKO-WB	0.0643550391248180	0.0730000000000	0.073
LUSTER-WB	-0.4496295360763730	0.0380000000381	0.038
MAGNA-WB	0.2995009393948180	0.3430000000000	0.343
MBL-WA	-0.4759308358993730	0.2150000086723	0.215
MCLEAN-WB	0.0446709391248180	0.0730000000000	0.073
OCK-WA	0.5624675492348180	0.1830000000000	0.183
PENSONI-WB	-0.5450196359263730	0.1880000076934	0.188
POHUAT-WB	0.2134234400768180	1.0250000000000	1.025
REACH-WA	0.9477619802289200	0.0530000000037	0.053
SERSOL-WA	-0.5840576159863730	0.1280000009369	0.128
SOLID-WA	0.1924764399748180	0.9230000000000	0.923
SRIDGE-WA	-0.6010404360563730	0.0580000017593	0.058
SYMLIFE-WB	0.3660987577986730	0.2230000000044	0.223
THRIVEN-WB	0.3182119591648180	0.1130000000000	0.113
WCT-WE	2.5726348392398100	0.1880000000000	0.188
WZSATU-WA	-0.3557106353643730	0.7500000028764	0.750

Table 2. Roslan et al. (2020) and Roslan et al. (2022) warrant prices along the market price for year 2016

Warrants	Roslan et al. (2020)	Roslan et al. (2022)	Market Price
APPASIA-WA	-0.5958685306008210	0.2480000007096	0.248
AZRB-WA	-0.2531285206058210	0.2430000012612	0.243
BIMB-WA	6.8658170794241700	0.2730000024541	0.273

BTM-WB	-0.6076919206908210	0.1580000023105	0.158
DIGISTA-WB	-0.5670536407838210	0.0650000004715	0.065
DNONCE-WA	0.0450773325820280	0.0730000000000	0.073
DOMINAN-WA	0.2145283327070280	0.1980000000000	0.198
DPS-WB	-0.5739755208108210	0.0380000001112	0.038
ECOWLD-WA	5.8157230638633800	0.3700000000000	0.370
EG-WC	0.1056088330420280	0.5330000000000	0.533
GPA-WA	-0.5347037208208210	0.0280000000695	0.028
GUNUNG-WB	-0.5383117206758210	0.1730000026270	0.173
INIX-WA	0.0231397325320280	0.0230000000000	0.023
JIANKUN-WA	0.0471406635733880	0.0800000000016	0.080
KIMLUN-WA	-0.1114989201758210	0.6730000089736	0.673
LBS-WB	1.8164513329240200	0.4150000000000	0.415
LEWEKO-WB	0.0643549325640280	0.0550000000000	0.055
LUSTER-WB	-0.4595616208208210	0.0280000000152	0.028
MAGNA-WB	0.2982219330390280	0.5300000000000	0.530
MBL-WA	-0.4864141204438210	0.4050000073185	0.405
MCLEAN-WB	0.0446708325540280	0.0450000000000	0.045
OCK-WA	0.6187164827170280	0.2080000000000	0.208
PENSONI-WB	-0.5549517206408210	0.2080000061985	0.208
POHUAT-WB	0.2134233332820280	0.7730000000000	0.773
REACH-WA	0.8115847764705490	0.1130000000023	0.113
SERSOL-WA	-0.5939897007438210	0.1050000006385	0.105
SOLID-WA	0.1941508332570280	0.7480000000000	0.748
SRIDGE-WA	-0.6109725207958210	0.0530000011855	0.053
SYMLIFE-WB	0.3660862036263880	0.1330000000006	0.133
THRIVEN-WB	0.3182118525790280	0.0700000000000	0.070
WCT-WE	2.7020648126920200	0.1830000000000	0.183
WZSATU-WA	-0.4002010202458210	0.6030000018333	0.603

Table 3. Roslan et al. (2020) and Roslan et al. (2022) warrant prices along the market price for year 2017

Warrants	Roslan et al. (2020)	Roslan et al. (2022)	Market Price
APPASIA-WA	-0.6168942515554860	0.25800000073246	0.258
AZRB-WA	-0.2443399513034860	0.51000000195972	0.510
BIMB-WA	7.0683672684795100	0.29300000332200	0.293
BTM-WB	-0.6360712516504860	0.16300000308989	0.163
DIGISTA-WB	-0.4925190917484860	0.06500000023260	0.065
DNONCE-WA	0.0460550225475530	0.19300000000000	0.193
DOMINAN-WA	0.2145280225845530	0.23000000000000	0.230
DPS-WB	-0.6029932517704860	0.04300000015784	0.043
ECOWLD-WA	6.1228527471335500	0.35300000000001	0.353
EG-WC	0.1117132427475530	0.39300000000000	0.393
GPA-WA	-0.5637214517754860	0.03800000008324	0.038
GUNUNG-WB	-0.5672982516854860	0.12800000281672	0.128
INIX-WA	0.0252914224525530	0.09800000000000	0.098
JIANKUN-WA	0.0519577068555570	0.07500000000262	0.075
KIMLUN-WA	-0.1232882512084860	0.60500001252732	0.605

LBS-WB	1.9398460234295500	1.07500000000000	1.075
LEWEKO-WB	0.0643546225345530	0.18000000000000	0.180
LUSTER-WB	-0.4641667517654860	0.04800000004393	0.048
MAGNA-WB	0.2993394227645530	0.41000000000000	0.410
MBL-WA	-0.5154318512584860	0.55500001054816	0.555
MCLEAN-WB	0.0446705224175530	0.06300000000000	0.063
OCK-WA	0.6187218526125530	0.25800000000000	0.258
PENSONI-WB	-0.5839694515984860	0.21500000768327	0.215
POHUAT-WB	0.2196650231495530	0.79500000000000	0.795
REACH-WA	0.8112349088476040	0.11300000000126	0.113
SERSOL-WA	-0.6230074317184860	0.09500000103309	0.095
SOLID-WA	0.0820547324925530	0.13800000000000	0.138
SRIDGE-WA	-0.6399902517504860	0.06300000236438	0.063
SYMLIFE-WB	0.3660256069505570	0.17000000000098	0.170
THRIVEN-WB	0.3182115423995530	0.04500000000000	0.045
WCT-WE	3.0146219426175500	0.26300000000000	0.263
WZSATU-WA	-0.4291877512704860	0.54300000230610	0.543

Table 4. Roslan et al. (2020) and Roslan et al. (2022) warrant prices along the market price for year 2018

Warrants	Roslan et al. (2020)	Roslan et al. (2022)	Market Price
APPASIA-WA	-0.6337042675737930	0.040000000797450	0.040
AZRB-WA	-0.3028327175107920	0.103000001527921	0.103
BIMB-WA	7.3150447525242000	0.138000005665921	0.138
BTM-WB	-0.6503179675737920	0.040000002394644	0.040
DIGISTA-WB	-0.5093356075837920	0.030000000184336	0.030
DNONCE-WA	0.0604629800197350	0.128000000000000	0.128
DOMINAN-WA	0.2147879799547350	0.063000000000000	0.063
DPS-WB	-0.6198097675907920	0.023000000267042	0.023
ECOWLD-WA	6.1228367919814100	0.178000000000008	0.178
EG-WC	0.1115494000567350	0.165000000000000	0.165
GPA-WA	-0.5805379675957920	0.018000000196653	0.018
GUNUNG-WB	-0.5841147675687920	0.045000005245761	0.045
INIX-WA	0.0258900799247350	0.033000000000000	0.033
JIANKUN-WA	0.0519417518634090	0.060000000010420	0.060
KIMLUN-WA	-0.1210434873537920	0.260000012593268	0.260
LBS-WB	0.8991875801117350	0.220000000000000	0.220
LEWEKO-WB	0.0643545799347350	0.043000000000000	0.043
LUSTER-WB	-0.4809832675857920	0.028000000073885	0.028
MAGNA-WB	0.2993393800717350	0.180000000000000	0.180
MBL-WA	-0.5149931672907920	0.323000019842525	0.323
MCLEAN-WB	0.0446704799197350	0.028000000000000	0.028
OCK-WA	0.6187218099747350	0.083000000000000	0.083
PENSONI-WB	-0.6007859674807920	0.133000012916104	0.133
POHUAT-WB	0.2198809803867350	0.495000000000000	0.495
REACH-WA	0.8110591345852650	0.068000000001872	0.068
SERSOL-WA	-0.6398239475787920	0.035000001296996	0.035
SOLID-WA	0.0823047999847350	0.093000000000000	0.093

SRIDGE-WA	-0.6540363875307930	0.083000006366859	0.083
SYMLIFE-WB	0.4648741318314090	0.028000000000214	0.028
THRIVEN-WB	0.2386526199467350	0.055000000000000	0.055
WCT-WE	2.9583308599447300	0.053000000000000	0.053
WZSATU-WA	-0.4429027875387920	0.075000000849504	0.075

Table 5. Roslan et al. (2020) and Roslan et al. (2022) warrant prices along the market price for year 2019

Warrants	Roslan et al. (2020)	Roslan et al. (2022)	Market Price
APPASIA-WA	-0.6558798587758340	0.088000000148371	0.088
AZRB-WA	-0.3245530486788340	0.185000000252882	0.185
BIMB-WA	7.6271086613641600	0.228000000898383	0.228
BTM-WB	-0.6720382987458340	0.118000000811604	0.118
DIGISTA-WB	-0.5310559388458340	0.018000000015144	0.018
DNONCE-WA	0.0652998193445530	0.143000000000000	0.143
DOMINAN-WA	0.2147878192365530	0.035000000000000	0.035
DPS-WB	-0.6415300988258340	0.038000000046692	0.038
ECOWLD-WA	6.1228125848728500	0.238000000000001	0.238
EG-WC	0.1080934792745530	0.073000000000000	0.073
GPA-WA	-0.6022582988258340	0.038000000026699	0.038
GUNUNG-WB	-0.6058350987358340	0.128000001404027	0.128
INIX-WA	0.0298013192095530	0.008000000000000	0.008
JIANKUN-WA	0.0519175447028580	0.068000000001654	0.068
KIMLUN-WA	-0.1294414185958340	0.268000002196129	0.268
LBS-WB	0.8918200592545530	0.053000000000000	0.053
LEWEKO-WB	0.0643544192395530	0.038000000000000	0.038
LUSTER-WB	-0.4927035988308340	0.033000000010146	0.033
MAGNA-WB	0.2993392193495530	0.148000000000000	0.148
MBL-WA	-0.5387542982888340	0.575000003707543	0.575
MCLEAN-WB	0.0446703192565530	0.055000000000000	0.055
OCK-WA	0.6805626492845530	0.083000000000000	0.083
PENSONI-WB	-0.6225062988038340	0.060000001570295	0.060
POHUAT-WB	-0.4703280983608340	0.503000004130376	0.503
REACH-WA	0.8222855692395530	0.038000000000000	0.038
SERSOL-WA	0.0272671361129360	0.040000000003767	0.040
SOLID-WA	-0.6177199787908340	0.073000000304881	0.073
SRIDGE-WA	0.0252395393095530	0.108000000000000	0.108
SYMLIFE-WB	-0.2262004488458340	0.018000000306296	0.018
THRIVEN-WB	0.2610597846528580	0.018000000000017	0.018
WCT-WE	2.9620310192395500	0.038000000000000	0.038
WZSATU-WA	-0.5058743987608340	0.103000000161639	0.103

Tables 1-5 display the contrast between both pricing models with the market prices which acts as the baseline from year 2015 to year 2019. Roslan et al. (2022)'s pricing model obtained warrant prices with a good fit to the market prices for the designated years. The differences between market prices and this pricing model are observed quite negligible. On the contrary, the pricing model of Roslan et al. (2020) is perceived to underestimate most of the warrants' prices throughout the years, from 62.5% of warrants under-pricing in year 2018 and 2019, up to 75% of warrants under-pricing in year 2015. Additionally, this model also exhibited consistent negative values along those years for certain warrants, such as

DIGISTA-WB, WZSATU-WA and APPASIA-WA. It is important to note that such inconsistencies may result in significant mispricing issues. Understanding the causes of inconsistent pricing of warrants is crucial for investors and financial analysts. Market volatility, interest rate changes, dividend announcements, liquidity issues, model assumptions, stochastic elements, market sentiment, and corporate actions all play significant roles in driving pricing inconsistencies. By considering these factors, market participants can better anticipate and respond to price variations, improving their strategies and decision-making processes in the warrant market.

Further analysis on the efficiency of both pricing models is conducted on the aspect of model's execution time. Along the specified years, Roslan et al. (2022)'s pricing model recorded 38.12 seconds to 62.62 seconds of execution time, whereas the pricing model of Roslan et al. (2020) took 7.25 seconds to 8.19 seconds. Even though the number of parameters in both pricing models is the same, the distinction in execution time might be due to the different techniques utilized in both pricing models. This matter might be interesting for future research.

Warrant pricing models' accuracy

The comparative criterion for the accuracy of the pricing models in this study is also known as statistical error measurement. The root mean squared error (RMSE), mean absolute percentage error (MAPE), and mean absolute error (MAE) are the error metrics that are utilized for this purpose. The smallest value, which is the centre of the judgment scale utilizing MAE, MAPE, and RMSE values, is considered to be the best. Tables 6-8 display the investigation on the accuracy of Roslan et al. (2020) and Roslan et al. (2022) models from year 2015 until 2019.

Table 6. Pricing errors of Roslan et al. (2020) and Roslan et al. (2022) models for year 2015 and 2016

Year	2015		2016	
Pricing Model / Error Measurements	Roslan et al. (2020)	Roslan et al. (2022)	Roslan et al. (2020)	Roslan et al. (2022)
MAE	0.86702	1.35348E-09	0.916428	1.13069E-09
MAPE	5.250423	7.23231E-09	5.589136	4.77478E-09
RMSE	1.545571326	2.79532E-09	1.680484414	2.48921E-09

Table 7. Pricing errors of Roslan et al. (2020) and Roslan et al. (2022) models for year 2017 and 2018

Year	2017		2018	
Pricing Model / Error Measurements	Roslan et al. (2020)	Roslan et al. (2022)	Roslan et al. (2020)	Roslan et al. (2022)
MAE	0.923035	1.5283E-09	0.872127	2.19473E-09
MAPE	5.097786	6.17149E-09	11.86534	1.92459E-08
RMSE	1.73918242	3.41041E-09	1.788883885	5.0946E-09

Table 8. Pricing errors of Roslan et al. (2020) and Roslan et al. (2022) models for year 2019

Year	2019	
Pricing Model / Error Measurements	Roslan et al. (2020)	Roslan et al. (2022)
MAE	0.911511	4.99892E-10
MAPE	11.05866	3.11961E-09
RMSE	1.827549176	1.14338E-09

Overall, the results from the above tables reveal that the Roslan et al. (2022) pricing model produced the smallest valuesfor RMSE, MAPE, and MAE respectively, and discovered in line with those shown in Tables 1-5. In fact, the values of MAPE reported by the Roslan et al. (2020) model along the designated years are observed being very large, ranging around 5 up to 11 times compared to the real market price. This phenomenon clearly resulted in major pricing inaccuracies and found improper for pricing warrants. In this sense, the Roslan et al. (2022) pricing model can be regarded as the most accurate method for estimating equity warrants, and may be applied in a real-world financial setting.

CONCLUSION

This study evaluates the performance of two hybrid equity warrants pricing models in the Malaysian market by comparing the methods developed by Roslan et al. (2020) and Roslan et al. (2022). Incorporating the CIRmodel for stochastic interest rates and the Heston model for stochastic volatility, the analysis reveals that the Roslan et al. (2022) model significantly outperforms the Roslan et al. (2020) model in aspects of accuracy and market price alignment, with minimal pricing errors. In contrast, the Roslan et al. (2020) model consistently underestimates warrant prices, leading to substantial inaccuracies and potential mispricing. Although the Roslan et al. (2022) model has a longer execution time, it proves to be a more reliable and accurate method for pricing equity warrants, making it more suitable for real-world financial applications.

Investors, financial advisors, remisiers, financial institutions, and other market participants may use the Roslan et al. (2022) warrant pricing model to forecast the market price of warrants in the future. This will allow them to decide whether to purchase or sell warrants, which could help them to earn profit and avoid them from losing money. For instance, an investor should hold a warrant and sell it in the future if the warrant's anticipated price increases in order to earn profit. Meanwhile, investors should sell the warrant immediately if the warrant's anticipated price decreases in the future in order to prevent a big loss. Furthermore, financial advisors and remisiers can easily advise potential investors in making investment decisions using the predicted warrant price. Additionally, this pricing model can be used for other investment securities pricing such as foreign exchange, share, bond and other derivatives.

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