Evenvertex Oblong Mean Labeling of Subdivision of Some Connected Graphs

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ABSTRACT
The n^{th} oblong number is denoted by$O_n$and is defined by $O_n = n(n+1)$. Let G = (V, E) be a loop-free, not having multiple edges, finite, and non-directed graphwith |V(G)| = p and |E(G)| = q. An even vertex oblong mean labeling is an injective function $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \dots, 2O_q\}$ where O_q is the q^{th} oblong number that induces a bijective edge labeling $\gamma^*_{on} : E(G) \rightarrow \{O_1, O_2, \dots, O_q\}$ defined by $\gamma^*_{on}(uv) = \frac{\gamma_{evom}(u)+\gamma_{evom}(v)}{2}$ for all e = uv ∈E(G). Every graph that accommodates evenvertex oblong mean labeling is called an evenvertex oblong mean graph. In the context of this paper, evenvertex oblong mean labeling of subdivision of a few path embraced graphs and some rooted trees are studied.

Keywords: oblong number, even vertex oblong mean labeling, even vertex oblong mean graph.

1. INTRODUCTION

In the context of a graph G = (V, E) with p vertices and q edges, we refer to a loop-free, not having multiple edges, finite, and non-directed graph. The expressions prescribed here are employed in accordance with Harary's conventions [3]. For number-theoretic terminology, we adhere to the definitions outlined in [1].

A graph labeling involves assigning integers to vertices(edges/both), dependent on particular criteria. When the input space encompasses the collection of vertices (edges/both), the labeling is termed an vertex(edge/total)labeling.

The survey in [2] comprehensively explores various hierarchical approaches to graph labeling. The notion of mean labeling was innovated and explored by Somasundaram and Ponraj[4].

Building on the aforementioned articles, M. P. Syed Ali Nisaya and K. Somasundari introduced evenvertex oblong mean labeling in [5]. The exploration of even vertex oblong mean labeling for further graphs is detailed in [6] and [7].

2. Initial Descriptions

Definition 2.1: A path $P_n$is obtained by joining u_s to the consecutive vertices $u_{s+1}$for $1 \leq s \leq n-1$.

Definition 2.2: A graph in which every vertex is connected and no cycles exist is precisely termed a tree.

Definition 2.3: The Comb graph $CB(P_n)$ is acquired by appending an individual hanging edge to all vertices in a path.

Definition 2.4: The Generalized butane graph is attained from a path P_n by annexing precisely one hanging vertex namely u_s and v_s in the vicinity of the both sides on P_n except for the maiden and the ultimate vertices.

Definition 2.5: A Y – structured graph $Y(P_n)$, is derived from the path P_n by adhering a hanging vertex to the penultimate vertex of P_n.

Definition 2.6: A F – Structured graph $F(P_n)$, acquired from a path $P_n$by affixing two hanging vertices to the penultimate vertex and the ultimate vertex of P_n.

Definition 2.7: A Coco palm $Ct(P_n, m)$ is sourced from the path P_n by adhering m contemporary hanging edges at the maiden vertex of P_n.

Definition 2.8: The graph $S_{k,m}$ is the single vertex union of k counterparts of P_m.

Definition 2.9: A graph in which every vertex is connected and no cycles exist is precisely termed a tree.

Definition 2.10: A tree in which one vertex (called a root) is distinguished from all the others is called a rooted tree. In a rooted tree, vertex v_s is said to be at level l_s, if v_s is at a distance l_s from the root. The root is at level 0.
Definition 2.11: An olive tree is a rooted tree consisting of n branches where the s^{th} branch is path of length s. It is denoted by OT_n.
Definition 2.12: A graph obtained by adhering the apex vertex of m-counterparts of stars of equal order to a hanging vertex by m distinct edges is called a m – Balanced Balloon Tree. It is denoted by $m - BBT_n$.
Definition 2.13: A graph obtained by adhering m number of stars of distinct order to a hanging vertex by m distinct edges is called $n_1, n_2, \dots, n_m$ –Unbalanced Balloon tree. It is denoted by $n_1, n_2, \dots, n_m - UBT_n$.
Definition 2.14: A graph obtained from a given graph by breaking up every edge into two segments by inserting an immediate vertex is called a Subdivision graph S(G).
Definition 2.15: An oblong number is the product of a number with its successor, algebraically it has the form $n(n + 1)$. The oblong numbers are 2, 6, 12, 20, 30, 42, ….
Definition 2.16: An even vertex oblong mean labeling is an injective function $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \dots, 2O_q\}$ where O_q is the q^{th} oblong number that induces a bijective edge labeling $\gamma^*_{on} : E(G) \rightarrow \{O_1, O_2, \dots, O_q\}$ defined by $\gamma^*_{on}(uv) = \frac{\gamma_{evom}(u) + \gamma_{evom}(v)}{2}$ for every single e = uv $\in E(G)$. The graph that accommodates even vertex oblong mean labeling is called an even vertex oblong mean graph.
Demonstration.2.12: Even vertex oblong mean graph is given in the figure below.

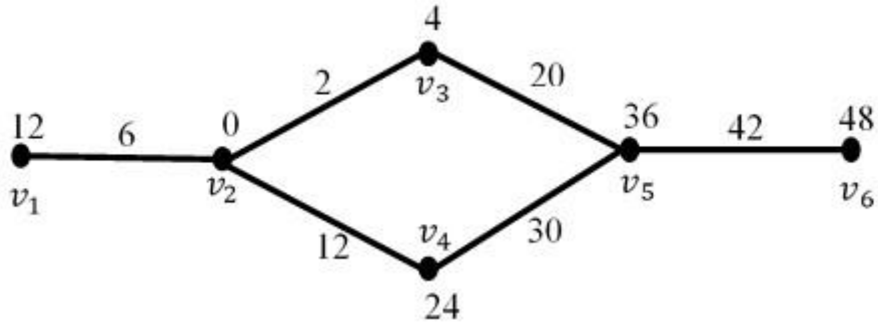


Fig.2.13

3. Principal Outcomes

3.1: Even vertex Oblong Mean Labeling of Path Embraced Graphs:

This section discusses the evenvertex oblong mean labeling of subdivision of some path embraced graphs: comb graph, generalized butane graph, coco palm tree, F – structured tree, Y – structured tree and one point union of path.

Theorem 3.1.1: Every Subdivision of comb $S(CB(P_n))$, for all $n \geq 3$ is an evenvertex oblong mean graph.
Proof: Hypothesize G be a $S[CB(P_n)]$ graph for $n \geq 3$ having vertex set $V(G) = \{u_s, u_s', v_s : 1 \leq s \leq n\} \cup \{v_s' : 1 \leq s \leq n-1\}$ edge set $E(G) = \{v_s v_s' : 1 \leq s \leq n-1\} \cup \{v_s' v_{s+1} : 1 \leq s \leq n-1\} \cup \{v_s u_s' : 1 \leq s \leq n\} \cup \{v_s' v_s : 1 \leq s \leq n\}$ with 4n vertices and 4n - 1 edges.
Describe $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \dots, 2O_{4n-1}\}$ in the following manner.
$\gamma_{evom}(u_1) = 0$
$\gamma_{evom}(u_1') = 4$
$\gamma_{evom}(v_1) = 8$
$\gamma_{evom}(v_1') = 16$
$\gamma_{evom}(v_2) = 24$
$\gamma_{evom}(u_2') = 36$
$\gamma_{evom}(u_2) = 48$
For $3 \leq s \leq n$, $\gamma_{evom}(v_{s-1}') = 2(4s-1)(4s) - \gamma_{evom}(v_{s-1})$
For $3 \leq s \leq n$, $\gamma_{evom}(v_s) = 2(4s-4)(4s-3) - \gamma_{evom}(v_{s-1}')$
For $3 \leq s \leq n$, $\gamma_{evom}(u_s') = 2(4s-3)(4s-2) - \gamma_{evom}(v_s)$
For $3 \leq s \leq n$, $\gamma_{evom}(u_s) = 2(4s-2)(4s-1) - \gamma_{evom}(u_s')$

Conspicuously γ_{evom} is an injective function which induces a bijective edge labeling$\gamma^*_{on}: E(G) \rightarrow \{O_1, O_2, \dots, O_{4n-1}\}$ defined as follows.
$\gamma^*_{on}(u_1v_1) = O_1$
$\gamma^*_{on}(v_sv_{s+1}) = O_{s+1}$, for $1 \leq s \leq n-1$
$\gamma^*_{on}(u_sv_s) = O_{n+s}$, for $2 \leq s \leq n-1$,
$\gamma^*_{on}(u_nv_n) = O_{n+2}$
Clearly, all the edge labels are $O_1, O_2, \dots, O_{2(m+n+1)}$.. Hence, $CB(P_n)$, for all $n \geq 3$ admits evenvertex oblong mean labeling. Thus $CB(P_n)$, for all $n \geq 3$ is an evenvertex oblong mean graph.

Example.3.1.2: The Evenvertex oblong mean labeling of $S(CB(P_5))$ is illustrated in Fig. 3.1.3

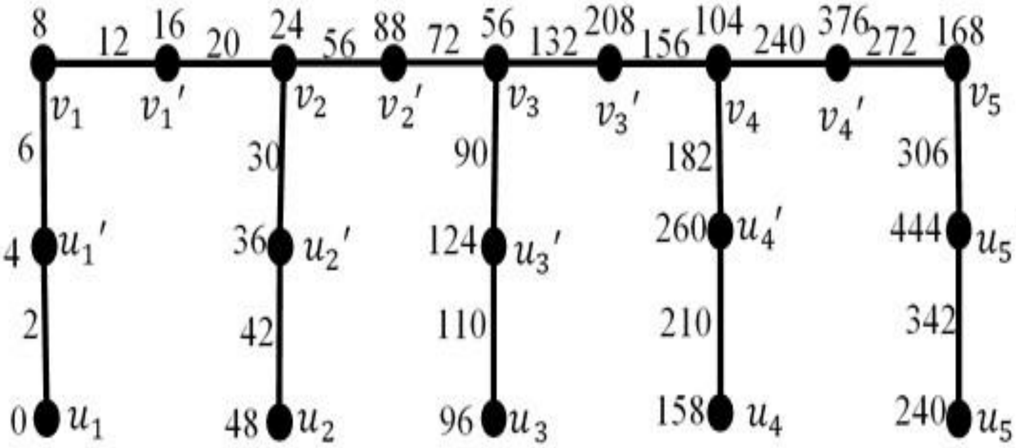


Fig. 3.1.3: The Evenvertex oblong mean labeling of $S(CB(P_3))$

Theorem 3.1.4: Every subdivision of generalized butane graph $S[GB(P_n)]$ for $n \geq 3$ is an evenvertex oblong mean graph.
Proof: Hypothesize G as a subdivision graph of generalized butane graph having vertex set $V(G) = \{w_s: 1 \leq s \leq n, w_s': 1 \leq s \leq n-1, u_s, v_s, u_s', v_s': 2 \leq s \leq n-1\}$and edge set $E(G) = \{w_sw_s': 1 \leq s \leq n-1\} \cup \{w_s'w_{s+1}: 1 \leq s \leq n-1\} \cup \{w_su_s', w_sv_s': 2 \leq s \leq n-1\} \cup \{u_su_s', v_sv_s': 2 \leq s \leq n-1\}$ with $6n+3$ vertices and $6n+2$ edges.
Describe $\gamma_{evom}: V(G) \rightarrow \{0, 2, 4, \dots, 2O_{3n+1}\}$ as follows.
For$1 \leq s \leq n, \gamma_{evom}(w_s) = (2s-1)^2 - 1$
For$1 \leq s \leq n-1, \gamma_{evom}(w_s) = 2s^2$
For $2 \leq s \leq n-1$, $\gamma_{evom}(u_s') = 2(2n+s-2)(2n+s-1) - \gamma_{evom}(w_s)$
For $2 \leq s \leq n-1$, $\gamma_{evom}(u_s) = 2(4n+s-6)(4n+s-5) - \gamma_{evom}(u_s')$
For $2 \leq s \leq n-1, \gamma_{evom}(v_s') = 2(3n+s-2)(3n+s-3) - \gamma_{evom}(w_s)$
For $2 \leq s \leq n-1$, $\gamma_{evom}(v_s) = 2(5n+s-8)(5n+s-7) - \gamma_{evom}(v_s')$
Transparently γ_{evom} is an injective function which induces a bijective edge labeling$\gamma^*_{on}: E(G) \rightarrow \{O_1, O_2, \dots, O_{6n+2}\}$ defined as follows.
$\gamma^*_{on}(w_1w_1') = O_1$
For $1 \leq s \leq n, \gamma^*_{on}(w_sw_{s+1}') = O_{2s-1}$
For $1 \leq s \leq n-1, \gamma^*_{on}(w_{s-1}'w_s) = O_{2s}$
For $2 \leq s \leq n-1, \gamma^*_{on}(w_su_s') = O_{2n+s+2}$
For $2 \leq s \leq n-1, \gamma^*_{on}(u_s'u_s) = O_{3n+s+2}$
For $2 \leq s \leq n-1, \gamma^*_{on}(w_sv_s') = O_{4n+s+2}$
For $2 \leq s \leq n-1, \gamma^*_{on}(v_s'v_s) = O_{5n+s+2}$
Apparently, all the edge labels are $O_1, O_2, \dots, O_{2(m+n+1)}$. Hence, The subdivision of generalized butane graph $S(GB(P_n))$ for $n \geq 3$ admits evenvertex oblong mean labeling. Thus the subdivision of generalized butane graph $S(GB(P_n))$ for $n \geq 3$is an evenvertex oblong mean graph.

Theorem 3.1.5: The subdivision of every $Y-$structured graph $Y(P_n)$ for $n \geq 3$ is an evenvertex oblong mean graph.
Proof: Consider a $Y-$structured graph $Y(P_n)$ for $n \geq 3$. Let $G = S(Y(P_n))$ having vertex set $V(G) = \{v, v', v_s: 1 \leq s \leq n, v_s': 1 \leq s \leq n\}$ and edge set $E(G) = \{v_{n-1}v', v'v, v_s' v_{s+1}, v_s v_s' : 1 \leq s \leq n-1\}$. Then G has $2n+1$ vertices and $2n$ edges.
Describe $\gamma_{evom} : V(G) \to \{0, 2, 4, \dots, 2O_{2n}\}$ in following way.
For $1 \leq s \leq n$, $\gamma_{evom}(v_s) = (2s+1)^2 - 1$
For $1 \leq s \leq n-1$, $\gamma_{evom}(v_s') = 4s^2$
$\gamma_{evom}(v') = 4(n^2 + 2n - 2)$
$\gamma_{evom}(v) = 4(n^2 - n + 2)$
Indisputably γ_{evom} is an injective function which induces a bijective edge labeling $\gamma^*_{on}: E(G) \to \{O_1, O_2, \dots, O_{2n}\}$ defined as follows.
$\gamma^*_{on}(v_s v_s') = O_{2s-1}$ for $1 \leq s \leq n-1$
$\gamma^*_{on}(v_s' v_{s+1}) = O_{2s}$ for $1 \leq s \leq n-1$
$\gamma^*_{on}(v_{n-1}v') = O_{2n-1}$
$\gamma^*_{on}(v'v) = O_{2n}$
Evidently, all the edge label are $O_1, O_2, \dots, O_{2n}$. Hence, $S(Y(P_n))$ for $n \geq 3$ admits evenvertex oblong mean labeling. Thus $S(Y(P_n))$ for $n \geq 3$ is an evenvertex oblong mean graph.

Theorem 3.1.6: The subdivision of every $F-$structured graph $S(F(P_n))$ for $n \geq 3$ is an evenvertex oblong mean graph.
Proof : Consider a $F-$structured graph $F(P_n)$ for $n \geq 3$Let $G = S(FP_n)$ having vertex set $V(G) = \{u, u', v, v', v_s: 1 \leq s \leq n, v_s': 1 \leq s \leq n\}$ and edge set $E(G) = \{u'v, v'v, v_n u', v_{n-1}v', v_s' v_{s+1}, v_s v_s': 1 \leq s \leq n\}$. Then G has $2n+3$ vertices and $2n+2$ edges. Describe $\gamma_{evom} : V(G) \to \{0, 2, 4, \dots, 2O_{2n+2}\}$ as follows.
$\gamma_{evom}(v_s) = (2s-1)^2 - 1$, for $1 \leq s \leq n$
$\gamma_{evom}(v_s') = 4s^2$, for $1 \leq s \leq n-1$
$\gamma_{evom}(v') = 4(n^2 + 2n - 2)$
$\gamma_{evom}(v) = 4(n^2 - n + 2)$
$\gamma_{evom}(u') = 4(n^2 + 2n + 1)$
$\gamma_{evom}(u) = 4(n^2 + n + 2)$
Evidently $\gamma_{evo\,m}$ is an injective function which induces a bijective edge labeling $\gamma^*_{on}: E(G) \to \{O_1, O_2, \dots, O_{2n+2}\}$ defined as follows.
$\gamma^*_{on}(v_s' v_{s+1}) = O_{2s}$, for $1 \leq s \leq n-1$
$\gamma^*_{on}(v_s' v_s) = O_{2s-1}$, for $1 \leq s \leq n$
$\gamma^*_{on}(v_{n-1}v') = O_{2n-1}$
$\gamma^*_{on}(v'v) = O_{2n}$
$\gamma^*_{on}(v_n u') = O_{2n+1}$
$\gamma^*_{on}(u'u) = O_{2n+2}$
Evidently, all the edge labels are $O_1, O_2, \dots, O_{2n+2}$. Hence, $S(F(P_n))$ for $n \geq 3$ admits evenvertex oblong mean labeling. Thus $S(F(P_n))$ for $n \geq 3$ is an evevertex oblong mean graph.

Theorem 3.1.7: The subdivision graph of every coco palm $S(Ct(P_n, m))$ for $n \geq 3$ and $m \geq 2$ is an evenvertex oblong mean graph.
Proof : Consider a coco palm $Ct(P_n, m)$ for $n \geq 3$ and $m \geq 2$. Let $G = S(Ct(P_n, m))$ for $n \geq 3$ and $m \geq 2$ having vertex set $V(G) = \{v_s, v_s': 1 \leq s \leq n, u_h u_h': 1 \leq h \leq m\}$ and edge set $E(G) = \{v_s v_s': 1 \leq s \leq n\} \cup \{v_s' v_{s+1}: 1 \leq s \leq n-1\} \cup \{v_1 u_h': 1 \leq h \leq m\} \cup \{u_h' u_h: 1 \leq h \leq m\}$. Then, G has $2n+2m+1$ vertices and $2n+2m$ edges. Describe $\gamma_{evom} : V(G) \to \{0, 2, 4, \dots, 2O_{2n+2m}\}$ as follows.
$\gamma_{evom}(v_1) = 0$
$\gamma_{evom}(u_h') = 2s(s+1)$, for $1 \leq h \leq m$
$\gamma_{evom}(u_h) = 2(n+s)(n+s+1) - \gamma_{evom}(u_h')$, for $1 \leq h \leq m$
$\gamma_{evom}(v_s') = 2(2m+2s-1)(2m+2s) - \gamma_{evom}(v_s)$, for $1 \leq s \leq n$
$\gamma_{evom}(v_s) = 2(2m+2s)(2m+2s+1) - \gamma_{evom}(v_{s-1}')$, for $2 \leq s \leq n$

Clearly γ_{evom} is an injective function which induces a bijective edge labeling$\gamma_{on}^{*}: E(G) \to \{O_1, O_2, \ldots, O_{2n+2m}\}$ defined as follows.
$\gamma_{on}^{*}(v_1 u_s{}') = O_s$, for $1 \le h \le m$
$\gamma_{on}^{*}(u_h{}' u_h) = O_{s+m}$, for$1 \le h \le m$
$\gamma_{on}^{*}(v_s{}' v_s) = O_{2s+2m}$, for$1 \le s \le n$
Noticeably, all the edge labels are $O_1, O_2, \ldots, O_{2n+2m}$. Hence, $S(Ct(P_n, m))$ for $n \ge 3$ and $m \ge 2$ admits evenvertex oblong mean labeling. Thus Coco palm $S(Ct(P_n, m))$ for $n \ge 3$ and $m \ge 2$ is an evenvertex oblong mean graph.

Theorem 3.1.8: The subdivision graph of $S_{k,m}$ for $k, m \ge 3$ is an evenvertex oblong mean graph.
Proof: Hypothesize P_m be a path on m vertices. Consider k counterparts of P_m. Let us join any of the ultimate vertex of each counterpart of P_m with the contemporary vertex w. The resultant graph is $S_{k,m}$. Let us subdivide each edge of $S_{k,m}$.
Then $G = S(S_{k,m})$ for $k, m \ge 3$ having the vertex set $V(G) = \{w,\ u_{sh}{}', u_{sh} : 1 \le s \le k \text{ and } 1 \le h \le m\}$ and edge set $E(G) = \{w\, u_{s1}{}' : 1 \le s \le k\} \cup \{u_{sh}{}' u_{sh} : 1 \le s \le k \text{ and } 1 \le h \le m-1\} \cup \{u_{sh}\, u_{s(h+1)}{}' : 1 \le s \le k \text{ and } 1 \le h \le m-1\}$. Hence, G has $2km + 1$ vertices and $2km$ edges.
Describe $\gamma_{evom} : V(G) \to \{0, 2, 4, \ldots, 2O_{2km}\}$ as follows.
$\gamma_{evom}(w) = 0$
$\gamma_{evom}(u_{s1}{}') = 2s(s+1)$, for $1 \le s \le k$
For $1 \le s \le k$ andfor $2 \le h \le m$,
$\gamma_{evom}(u_{s\hbar}{}') = (\hbar - 1)k(\hbar k + 2s + 1) + 2s(s+1)$
$For\ 1 \le s \le k\ andfor\ 1 \le \hbar \le m,$
$\gamma_{evom}(u_{s\hbar}) = (\hbar - 1)k(\hbar k + 2s + 1)$
Apprehensively γ_{evom} is an injective function which induces a bijective edge labeling$\gamma_{on}^{*}: E(G) \to \{O_1, O_2, \ldots, O_{2km}\}$ defined as follows.
$\gamma_{on}^{*}(wu_{1s}) = O_s for 1 \le s \le k$
$\gamma_{o_n}^{*}(u_{1s}{}' u_{1s}) = O_{k+s} for\ 1 \le s \le k$
$\gamma_{o_n}^{*}(u_{\hbar s}{}' u_{\hbar s}) = O_{\hbar k+s} for\ 1 \le s \le k\ and\ 2 \le \hbar \le m$
$\gamma_{o_n}^{*}(u_{\hbar s} u_{(\hbar+1)s}{}') = O_{(\hbar-1)k+s} for\ 1 \le s \le k\ and\ 1 \le \hbar \le m$
Effectively, all the edge labels are $O_1, O_2, \ldots, O_{2km}$. Hence, $S(S_{k,m})\ for\ k, m \ge 3$ admits evenvertex oblong mean labeling. Thus the graph $S(S_{k,m})\ for\ k, m \ge 3$ is an evenvertex oblong mean graph.

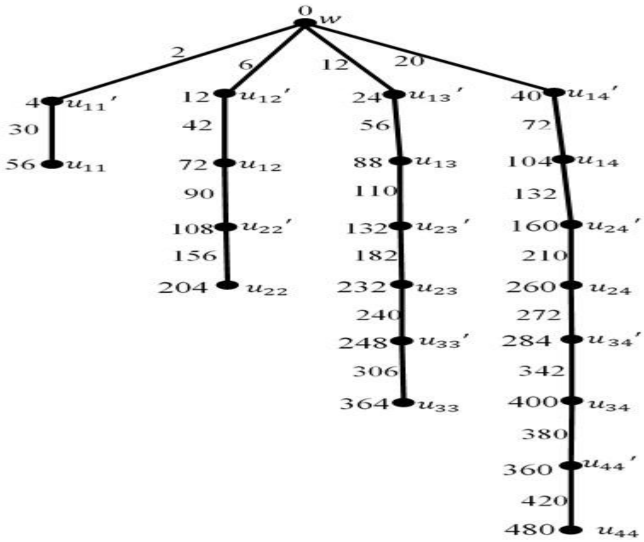
3.2 Evenvertex oblong mean labeling of subdivision of rooted trees

This section discusses the evenvertex oblong mean labeling of subdivision of some rooted trees: olive tree,2 – balanced balloon tree, 2 – unbalanced Balloon tree.

Theorem 3.2.1: The subdivision graph of any olive tree $S(OT_n) for\ n \ge 3$ is an evenvertex oblong mean graph.
Proof: Consider an olive tree $OT_n for\ n \ge 3$. Let $G = S(OT_n) for\ n \ge 3$ having the vertex set $V(G) = \{w, u_{s\hbar} : 1 \le s \le n\ and\ s \le \hbar \le n, u_{s\hbar}{}' : 1 \le s \le n\ and\ s \le \hbar \le n\}$ and edge set $E(G) = \{wu_{s1} : 1 \le s \le n\} \cup \{u_{s\hbar} u_{s\hbar}{}', u_{s\hbar} u_{s(\hbar+1)} : 1 \le s \le n\ and\ s \le \hbar \le n\}$. Then G has$n(n+1) + 1$ vertices and $n(n+1)$ edges.
Describe $\gamma_{evom} : V(G) \to \{0, 2, 4, \ldots, 2O_{n(n+1)}\}$ as follows.
$\gamma_{evom}(w) = 0$
$\gamma_{evom}(u_{s1}{}') = 2s(s+1)for1 \le s \le n$
$\gamma_{evom}(u_{s1}) = 2(n+s)(n+s+1) - \gamma_{evom}(u_{s1}{}')for1 \le s \le n$
$\gamma_{evom}(u_{s\hbar}{}') = 2(ns + \hbar - 1)(ns + \hbar) - \gamma_{evom}(u_{(s-1)\hbar})for2 \le s \le n\ and\ s \le \hbar \le n$
$For\ 2 \le s \le n\ and\ for\ s \le \hbar \le n,$
$\gamma_{evom}(u_{s\hbar}) = 2[sn + (s-1)(n-1) + \hbar - 1][sn + (n-1)(s-1) + \hbar] - \gamma_{evom}(u_{s\hbar}{}')$
Apprehensively γ_{evom} is an injective function which induces a bijective edge labeling$\gamma_{on}^{*}: E(G) \to \{O_1, O_2, \ldots, O_{2n}\}$ defined as follows.
$\gamma_{on}^{*}(wu_{s1}{}') = O_s for 1 \le s \le n$
$\gamma_{on}^{*}(u_{s1}{}' u_{s1}) = O_{2s} for\ 1 \le s \le n$
$\gamma_{on}^{*}(u_{s\hbar} u_{(s+1)\hbar}{}') = O_{s\hbar+s} for\ 2 \le s \le n\ and\ s \le \hbar \le n,$
$\gamma_{on}^{*}(u_{s\hbar} u_{s\hbar}{}') = O_{(s-1)\hbar+sn+\hbar} for\ 2 \le s \le n\ and\ s \le \hbar \le n,$

Effectively, all the edge labels are $O_1, O_2, \dots, O_{2n}$. Hence, $S(OT_n), n \geq 3$ admits evenvertex oblong mean labeling. Thus the graph $S(OT_n), n \geq 3$ is an evenvertex oblong mean graph.

Demonstration 3.2.2: The evenvertex oblong mean labeling of $S(OT_4)$ is given in Fig.3.2.3



Theorem 3.2.4: The subdivision ofevery 2-balanced balloon tree $S(2-BBT_n), n \geq 2$ is an evenvertex oblong mean graph.

Proof: Consider $2-BBT_n$ for $n \geq 2$. Let $G = S(2-BBT_n), n \geq 2$ having vertex set $V(G) = \{u, u_{\hbar s}, u_{\hbar s}{}' : \hbar = 1,2 \; and \; 0 \leq s \leq n\}$and edge set $E(G) = \{uu_{10}{}', uu_{20}{}', u_{10}{}'u_{10}, u_{20}{}'u_{20}, u_{\hbar 0}u_{\hbar s} : \hbar = 1,2 \; and 1 \leq s \leq n\}$. Then, G has with $4n+5$ vertices and $4n+4$ edges.

Describe $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \dots, 2O_{4n+4}\}$ as follows.

$\gamma_{evom}(u) = 0$

$\gamma_{evom}(u_{10}) = 4$

$\gamma_{evom}(u_{20}) = 12$

$\gamma_{evom}(u_{10}{}') = 20$

$\gamma_{evom}(u_{20}{}') = 28$

$\gamma_{evom}(u_{1s}) = 2(s+2)(s+3) - 4$, for $1 \leq s \leq n$

$\gamma_{evom}(u_{2s}) = 2(n+s+2)(n+s+3) - 12$, for $1 \leq s \leq n$

$\gamma_{evom}(u_{s\hbar}{}') = 2(ns+\hbar-1)(ns+\hbar) - \gamma_{evom}(u_{(s-1)\hbar})for2 \leq s \leq n$

Transparently γ_{evom} is an injective function which induces a bijective edge labeling$\gamma_{on}^* : E(G) \rightarrow \{O_1, O_2, \dots, O_{4n+4}\}$ defined as follows.

$\gamma_{on}^*(uu_{10}{}') = O_1$

$\gamma_{on}^*(uu_{20}{}') = O_2$

$\gamma_{on}^*(u_{10}{}'u_{10}) = O_3$

$\gamma_{on}^*(u_{20}{}'u_{20}) = O_4$

$\gamma_{on}^*(u_{10}u_{1s}) = O_{4+s}$, for $1 \leq s \leq n$

$\gamma_{on}^*(u_{20}u_{2s}) = O_{4+n+s}$, for $1 \leq s \leq n$

$\gamma_{on}^*(u_{1s}{}'u_{1s}) = O_{4+2n+s}$, for $1 \leq s \leq n$

$\gamma_{on}^*(u_{2s}{}'u_{2s}) = O_{4+3n+s}$, for $1 \leq s \leq n$

Apparently, all the edge labels are $O_1, O_2, \dots, O_{4n+4}$. Hence, the subdivision of every 2 – balanced balloon tree$S(2-BBT_n), n \geq 2$ admits evenvertex oblong mean labeling. Thus the subdivision of every 2 – balanced balloon tree$S(2-BBT_n), n \geq 2$is an evenvertex oblong mean graph.

Theorem 3.2.5: The Subdivision ofevery 2-unbalanced balloon tree $S(2-UBT_{n_1,n_2}), n_1, n_2 \geq 2$ is an evenvertex oblong mean graph.

Proof: Consider $2-UBT_{n_1,n_2}$ for $n_1, n_2 \geq 2$. Let $G = S(2-UBT_{n_1 n_2}), n_1 n_2 \geq 2$ having vertex set $V(G) = \{u, u_{h0}, u_{1s}{}', u_{1s} : 1 \leq s \leq n_1, u_{2s}{}', u_{2s} : 1 \leq s \leq n_2\}$and edge set

$E(G) =$
$\{uu_{10}{}', uu_{20}{}', u_{10}{}'u_{10}, u_{20}{}'u_{20}, u_{10}u_{1s}{}' : 1 \leq s \leq n_1, u_{20}u_{2s}{}' : 1 \leq s \leq n_2, u_{1s}{}'u_{1s} : 1 \leq s \leq n_1, u_{2s}{}'u_{2s} : 1 \leq s \leq n_2\}$. Then, G has $2(n_1 + n_2) + 5$ vertices and $2(n_1 + n_2) + 4$ edges.
Describe $\gamma_{evom} : V(G) \rightarrow \{0, 2, 4, \dots, 2O_{2(n_1+n_2)+4}\}$ as follows.
$\gamma_{evom}(u) = 0$
$\gamma_{evom}(u_{10}) = 4$
$\gamma_{evom}(u_{20}) = 12$
$\gamma_{evom}(u_{10}{}') = 20$
$\gamma_{evom}(u_{20}{}') = 28$
$\gamma_{evom}(u_{1s}{}') = 2(s + 4)(s + 5) - 20$, for $1 \leq s \leq n$,
$\gamma_{evom}(u_{2s}{}') = 2(n_1 + s + 4)(n_1 + s + 5) - 28$, For $1 \leq s \leq n$
$\gamma_{evom}(u_{1s}) = 2(n_1 + n_2 + s + 4)(n_1 + n_2 + s + 5) - \gamma_{evom}(u_{1s}{}')for1 \leq s \leq n_1$
$\gamma_{evom}(u_{2s}) = 2(2n_1 + n_2 + s + 4)(2n_1 + n_2 + s + 5) - \gamma_{evom}(u_{2s}{}')for1 \leq s \leq n_2$
Transparently γ_{evom} is an injective function which induces a bijective edge labeling$\gamma_{on}^{*} : E(G) \rightarrow \{O_1, O_2, \dots, O_{2(n_1+n_2)+4}\}$ defined as follows.
$\gamma_{on}^{*}(uu_{10}{}') = O_1$
$\gamma_{on}^{*}(uu_{20}{}') = O_2$
$\gamma_{on}^{*}(u_{10}{}'u_{10}) = O_3$
$\gamma_{on}^{*}(u_{20}{}'u_{20}) = O_4$
$\gamma_{on}^{*}(u_{10}u_{1s}) = O_{4+s}$, for $1 \leq s \leq n$
$\gamma_{on}^{*}(u_{20}u_{2s}) = O_{4+n_1+s}$, for $1 \leq s \leq n$
$\gamma_{on}^{*}(u_{1s}{}'u_{1s}) = O_{4+n_1+n_2+s}$, for $1 \leq s \leq n$
$\gamma_{on}^{*}(u_{2s}{}'u_{2s}) = O_{4+2n_1+n_2+s}$, for $1 \leq s \leq n$
Apparently, all the edge labels are $O_1, O_2, \dots, O_{2(n_1+n_2)+4}$. Hence, the subdivision of 2 – unbalanced balloon tree $S(2 - UBT_{n_1n_2})$ $n_1, n_2 \geq 2$ admits evenvertex oblong mean labeling. Thus the subdivision of every 2 – unbalanced balloon tree$S(2 - UBT_{n_1n_2})$, $n_1, n_2 \geq 2$is an evenvertex oblong mean graph.

4. CONCLUSION

In wrapping up our study, we have meticulously examined the evenvertex oblong mean labeling of subdivision several graph structures such as path embraced graphs and some rooted trees. The process of subdivision effectively divides each edge into two, resulting in a new graph structure that retains the original vertices while adding new ones between them. The subdivision graph preserves the overall connectivity of the original graph while refining its structure by increasing the number of vertices. The simplicity of our findings carries profound implications, providing a solid foundation for future exploration.

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