On Weakly Symmetries of δ-Lorentzian Para Trans-Sasakian Manifolds

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ABSTRACT
The purpose of the paper is to introduce the notion of δ -Lorentzian para trans-Sasakian manifolds of type (α, β) and study some of its basic results. Also,weakly symmetries of δ -Lorentzian para trans-Sasakian manifolds have been introduced. An example has been given to show the existence of δ -Lorentzian para trans-Sasakian manifolds.

Keywords: Trans-Sasakian manifolds, δ -Lorentzian manifolds, δ-Lorentziam α Sasakian manifolds, δ-Lorentzian-β-Kenmotsu manifolds, δ- Lorentzian α- Sasakian manifolds.

1. INTRODUCTION

Many authors such as U. C. De and Krishnendu De [25] and Abdul Haseeb, Mobin Ahmad and Mohd. Danish Siddiqi [29],S. M. Bhati [27] have studied Lorentzian α - Sasakian manifolds and Lorentzian β-Kenmotsu manifolds [9]. In 2011, S. S. Pujar and V. J. Khairnar [6] have initiated the study of Lorentzian Trans-Sasakian manifolds and studied the basic results with some of its properties. Earlier to this, ([23],[22],[27]) has initiated the study of δ -Lorentzian α- Sasakian manifolds and Lorentzian β -Kenmotsu manifolds ([7],[9]).

In 2010, S. S. Shukla and D. D. Singh [10] have introduced the notion of ϵ- trans-Sasakian manifolds and studied its basic results. Earlierin 1969 [13] had introduced the notion of almost contact metric manifold equipped with Pseudo Riemannian metric. In particular, he studied the Sasakian manifolds equipped with semi-Riemannian metric g. These indefinite almost contact metric manifolds and indefinite Sasakian manifolds are also known asϵ-almost contact metric manifolds and ϵ- Sasakian manifolds respectively.

M. M.Tripathi [14] has observed that there does not exist a light-like surface in the ϵ- Sasakian manifolds. On the other hand in almost para contact manifold defined by [3], the semi-Riemannian manifold has the index 1 and the structure vector field ξ is always a time-like.This motivated others [14] to introduce ϵ-almost para contact structure where the vector field ξ is space-like or time-like according as $\epsilon = 1$ or $\epsilon = -1$.

In this paper, in Section 2, we have introduced the notion of δ- Lorentzian para trans- Sasakian manifolds and studied its basic results. In fact, we have made an attempt to combine both δ-Lorentzian α-Sasakian manifolds and δ Lorentzian β Kenmotsu manifolds and called the δ-Lorentzian trans-Sasakian manifold of odd dimension and of type (α, β), where α and β are some smooth functions on M.A concrete example to ensure the existence of weakly symmetries of δ- Lorentzian para trans-Sasakian manifold are studied.

In Section 3, we extended the work of ([11],[12]) for weakly symmetric δ- Lorentzian para trans-Sasakian manifolds has been discussed. In this Section, series of Theorems and corresponding Corollaries to indicate the special cases of the Theorems are given. As a special case, in one of the Corollaries, it is proved that there is no weakly symmetries of δ- Lorentzian para Sasakian manifold M (n >1) unless the sum of the associated 1-forms is everywhere zero, that is, A + B + D = 0. Also, it is proved that there is no weakly symmetries of δ- Lorentzian Kenmotsu manifold M (n >1) unless the sum of the associated 1-forms is everywhere zero, that is, A + B + D = 0.

2. δ -Lorentzian Para Trans-Sasakian Manifolds

S. Tanno. [20] classified the connected almost contact metric manifolds whose automorphism groups possesses the maximum dimension. For such a manifold the sectional curvature of the plane section containing ξ is constant, say c. He showed that they can be divided into three classes. First class is homogeneous normal contact Riemannian manifolds with c >0. It is known that the manifolds of class (1) and are characterized by admitting a Sasakian structure. Other two classes can be seen in [20].

In Grey and Harvella [18], the classification of almost Hermitian manifolds, there appears a class, W_4, of Hermitian manifolds which are closely related to the conformal Kaehler manifolds. The class $C_6 \oplus C_5$[19] coincides with the class of the Trans Sasakian structure of type (α, β). In fact, the local nature of the two subclasses, namely C_6 and C_5 of trans-Sasakian structures are characterized completely. An almost contact metric structure on M is called a trans-Sasakian (please see details in ([17], [18]) if (M×R, J, G) belongs to the class W_4, where J is the almost complex structure on M×R defined by

$$J\left(X, f\frac{d}{dt}\right) = \left(\varphi(X) - f\xi, \eta(X)\frac{d}{dt}\right)$$

For all vector fields X on M and smooth function f onM×R and G is the product metric on M×R. This may be expressed by the condition

$(\nabla_X\varphi)(Y) = \alpha\{g(X,Y)\xi - \eta(Y)X\} + \beta\{g(\varphi(X),Y)\xi - \eta(Y)\varphi(X)\}$, (2.1)

for any vector fields X and Y on M,∇ denotes the Levi-Civita connection with respect to g.α and β are smooth functions on M.

A manifold M is said to admit an almost para contact structure [21] (ϕ, ξ, η) if

$\phi^2X = X - \eta(X)\xi, \eta(\xi) = 1$

On the other hand M is said to admit a Lorentzian almost para contact structure [21] (ϕ, ξ, η) if

$\phi^2X = X + \eta(X)\xi, \eta(\xi) = -1$

Here the ξ in this equation is a light-likeunit vector field [22].

K. Matsumoto K [15] has defined the manifold M with the structure (ϕ, ξ, η, g) with usual notions as the Lorentzian para contact([15],[16],[17]) if the following are satisfied.

$\phi^2X = X + \eta(X)\xi, \eta(\xi) = -1$

$g(\xi, \xi) = -1, \eta(X) = g(X, \xi)$

$g(\phi(X), \phi(Y)) = g(X, Y) + \eta(X)\eta(Y)$,

Type equation here.for all vector fields X and Y on M. Further, he also termed a Lorentzian para contact manifold as the Lorentzian α- Sasakian ([16],[17]) if

$$(\nabla_X\varphi)(Y) = \alpha\{g(X,Y)\xi + \eta(Y)X\},$$

for any vector fields X and Y on M. Similarly, a Lorentzian para contact manifold as the Lorentzian β-Kenmotsu if

$$(\nabla_X\varphi)(Y) = \beta\{g(\varphi(X),Y)\xi + \eta(Y)\varphi(X)\},$$

for any vector fields X and Y on M. Considering these, we introduce the following definitions.

Definition 2.1. A (2n+1) dimensional manifold M is said to be the δ- almost para contact metric manifold if it admits a (1,1) tensor field ϕ, a structure tensor field ξ, a 1-form η and an indefinite metric g such that

$\phi^2X= X + \eta(X)\xi, \eta(\xi) = -1$ (2.2)

$g(\xi, \xi) = -\delta, \eta(X) = \delta g(X, \xi)$ (2.3)

$g(\phi(X), \phi(Y) = g(X, Y) + \delta\eta(X)\eta(Y)$(2.4)

for all vector fields X and Y on M, where δ is such that $\delta^2 = 1$ so that $\delta = \pm1$. If δ = 1, thengis the usual Lorentzian metric on M and the vector field ξ is the lightlike [2], that is, M contains a timelike vector field. From the above equations, one can deduce that

$\phi\xi = 0, \eta(\phi(X)) = 0$.

Definition 2.2. A δ - almost contact metric manifold with para contact metric structure (ϕ, ξ, η, g, δ) is said to be δ- Lorentzian para trans -Sasakian manifold M of type (α, β) if

$(\nabla_X\varphi)(Y) = \alpha\{g(X,Y)\xi + \delta\eta(Y)X\} + \beta\{g(\varphi(X),Y)\xi + \delta\eta(Y)\varphi(X)\}$, (2.5)

for any vector fields X and Y on M, ∇ denotes the Levi-Civita connection with respect to g, αandβ are smooth functions on M.

If δ = −1, then the δ- Lorentzian trans-Sasakian manifold is the usual Lorentzian trans -Sasakian manifold of type (α, β) [3]. δ-Lorentzian trans-Sasakian manifold of type (0, 0),(0, β),(α, 0) are the Lorentzian cosympletic,Lorentzian β- Kenmotsu and Lorentzian α- Sasakian manifolds([16],[17]) respectively. In particular if α = 1, β = 0, and α = 0, β = 1, then δ- Lorentzian trans -Sasakian manifold reduces to δ-Lorentzian Sasakian and δ- Lorentzian Kenmotsu manifolds respectively.

Example 2.1: Suppose (ϕ, ξ, η, g, δ) is the δ- Lorentzian almost contact metric structure on M. Put

$$\bar{\varphi} = \varphi, \quad \bar{\xi} = -\xi, \quad \bar{\eta} = -\eta, \quad \bar{g} = -g, \quad \bar{\delta} = -\delta$$

Then $(\bar{\varphi}, \bar{\xi}, \bar{\eta}, \bar{g}, \bar{\delta})$is also δ- Lorentzian almost contact metric structure on M.So, using definition 2.1,

$$\bar{\varphi}^2 X = X + \bar{\eta}(X)\bar{\xi},$$
$$\bar{g}(\bar{\xi}, \bar{\xi}) = -\bar{\delta},$$
$$\bar{\eta}(\bar{\xi}) = -1,$$
$$\bar{\eta}(X) = \overline{\delta \bar{g}}(X, \bar{\xi}),$$
$$\bar{g}(\bar{\varphi}(X), \bar{\varphi}(Y)) = \bar{g}(X, Y)\bar{\xi} + \overline{\delta \bar{\eta}}(X)\bar{\eta}(Y),$$

If (ϕ, ξ, η, g, δ) is the δ- Lorentzian almost normal contact metric structure on M, then $(\bar{\varphi}, \bar{\xi}, \bar{\eta}, \bar{g}, \bar{\delta})$ is also δ- Lorentzian almost normal contact metric structure on M. For, the parallelism with respect to g and$\bar{g}$, are the same so that we have

$$(\bar{\nabla}_X \bar{\varphi})(Y) = (\nabla_X \varphi)(Y) = \alpha\{g(X, Y)\xi + \delta\eta(Y)X\} + \beta\{g(\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)\}$$
$$= \alpha\{\bar{g}(X, Y)\bar{\xi} + \bar{\delta}\bar{\eta}(Y)X\} + \beta\{\bar{g}(\bar{\varphi}(X), Y)\bar{\xi} + \bar{\delta}\bar{\eta}(Y)\varphi(\bar{X})\},$$

for any vector field X,Y on M.

In view of this Example, we may assume without loss of generality δ = 1.

Lemma 2.3: For a δ- Lorentzian para trans-Sasakian manifold, we have

$\nabla_X \xi = \delta\{\alpha\varphi(X) + \beta(X + \eta(X)\xi)\},$ (2.6)

for any vector field X on M.

Proof: From(2.5), we have

$$\nabla_X(\varphi(Y)) - \varphi(\nabla_X Y) = \alpha\{g(X, Y)\xi + \delta\eta(Y)X\} + \beta\{g(\varphi(X), Y)\xi + \delta\eta(Y)\varphi(X)\},$$

for any vector fields X and Y on M. Now taking Y = ξ in the above equation and using (2.2), we get

$$-\varphi(\nabla_X \xi) = \alpha\{g(X, \xi)\xi - \delta X\} - \beta\delta\varphi(X)$$

Applying φon both sides of the above equation and using the fact that $(\nabla_X g)(\xi, \xi) = 0$ which implies $g(\nabla_X \xi, \xi) = 0$ so that $\eta(\nabla_X \xi) = g(\nabla_X \xi, \xi) = 0$, further simplifying, we get (2.6).

Example 2.2: Let us consider the 3-dimensional manifold M = {(x, y, z)ϵR^3}, where x, y, z are the co-ordinates of a point in R^3.Let {e_1, e_2, e_3} be the global frames on M given by

$$e_1 = e^z\left(\frac{\partial}{\partial x} + y\frac{\partial}{\partial z}\right), e_2 = e^z\frac{\partial}{\partial y}, e_3 = e^z\frac{\partial}{\partial z}$$

Let g be the δ- Lorentzian metric on M defined by

$g(e_1, e_2) = g(e_2, e_3) = g(e_1, e_3) = 0$

and

$g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -\delta$

where δ = ±1. Then δ-Lorentzian indefinite metric g on M is in the following form:

$g = \{e^{-2z} - \delta y^2\}(dx)^2 + e^{-2z}(dy)^2 - \delta e^{-2z}(dz)^2 + 2\delta y e^{-z}dxdy$

Let $e_3 = \xi$. Let η be the 1-form defined by

$\eta(U) = \delta g(U, e_3)$,

for any vector field U on Letϕ be (1,1) tensor field defined by

$\phi(e_1) = e_2, \phi(e_2) = e_1, \phi(e_3) = 0$

Then using the linearity of ϕ and g and taking $e_3 = \xi$, one obtains

$\eta(e_3) = -1, \phi^2 U = U + \eta(U)e_3$

and$g(\phi(U), \phi(W)) = g(U, W) + \delta\eta(U)\eta(W)$, (2.7)

for any vector fields X and Y on M. Hence putting W = ξ in (2.6), we have

$\eta(U) = \delta g(U, \xi)$.(2.8)

Putting W = U = ξ in(2.7) and (2.8) respectively, we have $g(\xi, \xi) = -\delta$ and $\eta(\xi) = -1$

Clearly from(2.7), ϕ is symmetric. Thus (ϕ, ξ, η, g, δ) defines a δ- Lorentzian contact metric structure on M.

Let ∇be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g of type (1,3). Then the following results hold.

$[e_1, e_2] = \delta(ye^z e_2 - e^z e_3)$,$[e_1, e_3] = -\delta(ye^z e_3 - e_1)$, $[e_2, e_3] = -\delta e^z e_2$

Taking $e_3 = \xi$ and using the Koszul's formula, that is

$2g(\nabla_X Y, Z) = Xg(Y, Z) + Y g(Z, X) - Zg(X, Y) - g(X, [Y, Z]) + g(Y, [Z, X]) + g(Z, [X, Y])$,

one can easily obtain

$\nabla_{e_1} e_3 = \delta\{-e^z e_1 - \frac{1}{2}e^z e_2\}$,$\nabla_{e_3} e_3 = 0$, $\nabla_{e_2} e_3 = \delta\{-e^z e_2 - \frac{1}{2}e^z e_1\}$

$\nabla_{e_2} e_2 = \delta\{-e_3 + ye^z e_1\}$,$\nabla_{e_1} e_2 = -\frac{\delta}{2}e^z e_3$, $\nabla_{e_3} e_1 = \delta\{\frac{1}{2}e^z e_3 - ye^z e_2\}$

$\nabla_{e_1} e_1 = -\delta e^z e_3$, $\nabla_{e_3} e_2 = \frac{\delta}{2}e^z e_1$, $\nabla_{e_2} e_1 = -\frac{\delta}{2}e^z e_2$.

With these results, M(ϕ, ξ, η, g, δ) defines a δ -Lorentzian para trans-Sasakian manifold of type $(-\frac{\delta}{2}e^z, -\delta e^z)$ and satisfies (2.6) of Lemma 2.3.

Example 2.3: Let us consider the 3-dimensional manifold $M = \{(x, y, z)\epsilon R^3 z \neq 0\}$, where x,y,z are the co-ordinates of a point in R^3.Let $\{e_1, e_2, e_3\}$ be the global frames on M given by

$$e_1 = -e^x z\frac{\partial}{\partial y}, \qquad e_2 = e^x\left(\frac{\partial}{\partial y} - z\frac{\partial}{\partial x}\right), \quad e_3 = \frac{\partial}{\partial z}$$

Let g be the δ- Lorentzian metric on M defined by
$g(e_1, e_2) = g(e_2, e_3) = g(e_1, e_3) = 0$and $g(e_1, e_1) = g(e_2, e_2) = 1, g(e_3, e_3) = -\delta$
where δ = ±1. Let $e_3 = \xi$. Let η be the 1-form defined by
$\eta(U) = \delta g(U, e_3)$,
for any vector field U on M. Let ϕ be (1,1) tensor field defined by
$\phi(e_1) = e_2, \phi(e_2) = e_1, \phi(e_3) = 0$
Then using the linearity of ϕ and g and taking $e_3 = \xi$, one obtains
$\eta(e_3) = -1$, $\phi^2 U = U + \eta(U)e_3$ and $g(\phi(U), \phi(W)) = g(U, W) + \delta\eta(U)\eta(W)$,
for any vector fields X and Y on M. Hence putting W = ξ in in the above equation, we have
$\eta(U) = \delta g(U, \xi)$.
Putting W = U = ξ in the above equations, we have
$g(\xi, \xi) = -\delta$ and $\eta(\xi) = -1$
Clearly,ϕ is symmetric. Thus (ϕ, ξ, η, g, δ) defines a δ- Lorentzian contact metric structure on M.
Let ∇be the Levi-Civita connection with respect to the Lorentzian metric g and R be the curvature tensor of g of type (1,3) Then the following results hold.
$[e_1, e_2] = \delta z e^x e_1, [e_1, e_3] = -\frac{\delta}{2}e_1, [e_2, e_3] = -\frac{\delta}{z^2}e_1 - \frac{\delta}{z}e_2$
Further proceeding as in Example 2.2, one can see that M(ϕ, ξ, η, g, δ) defines a δ -Lorentzian para trans-Sasakian manifold of type $\left(-\frac{\delta}{2z^2}, -\frac{\delta}{z}\right)$and satisfies (2.6) of Lemma 2.3.

Lemma 2.4: For a δ -Lorentzian para trans-Sasakian manifold,we have
$(\nabla_X\eta)(Y) = \alpha g(\phi(X), Y) + \beta\{g(X, Y) + \delta\eta(X)\eta(Y)\}$(2.9)
for all X and Y on M.
Proof: Consider$(\nabla_X\eta)(Y) = \nabla_X(\eta(Y)) - \eta(\nabla_X Y) = \delta\nabla_X(g(Y, \xi)) - \delta g(\nabla_X Y, \xi) = \delta g(Y, \nabla_X\xi)$.
By virtue of (2.6) of Lemma2.3 and noting that $\delta^2 = 1$,we get (2.9).

Lemma 2.5: For a δ -Lorentzian para trans-Sasakian manifold M, we have
$R(X, Y)\xi = (\alpha^2 + \beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi(X) - \eta(X)\phi(Y)\} + \delta\{(X\alpha)\phi(Y) - (Y\alpha)\phi(X) + (X\beta)\phi^2 Y - (Y\beta)\phi^2 X\}$,(2.10)
for all X and Y on M.
Proof: From(2.2)and (2.6) of Lemma 2.3 and using the fact that
$[X, Y] = \nabla_X Y - \nabla_Y X$
we have
$R(X, Y)\xi = \nabla_X\nabla_Y\xi - \nabla_Y\nabla_X\xi - \nabla_{[X,Y]}\xi = \nabla_X[\delta\{\alpha\phi(Y) + \beta\phi^2 Y\}] - \nabla_Y[[\delta\{\alpha\phi(X) + \beta\phi^2 X\}] - \delta[\alpha\phi(\nabla_X Y - \nabla_Y X) + \beta(\nabla_X Y - \nabla_Y X) + \beta\eta(\nabla_X Y - \nabla_Y X)\xi] = \delta[(X\alpha)\phi(Y) - (Y\alpha)\phi(X) + (X\beta)\phi^2 Y - (Y\beta)\phi^2 X] + \alpha\delta[\alpha\{g(X, Y)\xi + \delta\eta(Y)X\} + \beta\{g(\phi(X), Y)\xi + \delta\eta(Y)\phi(X)] - \alpha\delta[\alpha\{g(X, Y)\xi + \delta\eta(X)Y\} + \beta\{g(\phi(Y), X)\xi + \delta\eta(X)\phi(Y)]$
$+ \alpha\delta\phi(\nabla_X Y) - \alpha\delta(\nabla_Y X) + \delta\beta\nabla_X(\phi^2 Y) - \delta\beta\nabla_Y(\phi^2 X) - \delta[\alpha\phi(\nabla_X Y - \nabla_Y X) + \beta(\nabla_X Y - \nabla_Y X) + \beta\eta(\nabla_X Y - \nabla_Y X)\xi]$,
(2.11)
for all vector fields X, Y on M.
Consider,
$\nabla_X(\phi^2 Y) = \nabla_X(Y + \eta(Y)\xi) = \nabla_X Y + (\nabla_X(\eta(Y))\xi + \eta(Y)(\nabla_X\xi) = \nabla_X Y + (\nabla_X\eta)(Y)\xi + \eta(\nabla_X Y)\xi + \eta(Y)\nabla_X\xi = \nabla_X Y + \alpha g(\phi(X), Y) + \beta\{g(X, Y) + \delta\eta(X)\eta(Y)\}\xi + \eta(\nabla_X Y)\xi + \delta\eta(Y)\{\alpha\phi(X) + \beta(X(X)\xi)\}$, (2.12)
Wherein we have used (2.2) and (2.9) of Lemma 2.4. Now substituting for$\delta\beta\nabla_Y(\phi^2 X) - \delta\beta\nabla_X(\phi^2 Y)$
from (2.12) in (2.11), after further simplification, we get (2.10).

Lemma 2.6: For a δ -Lorentzian para trans-Sasakian manifold M, we have
$R(\xi, Y)X = (\alpha^2 + \beta^2)\{\delta g(X, Y)\xi - \eta(X)Y\} + \delta(X\alpha)\phi(Y) - \delta g(\phi(X), Y)(grad\alpha) + \delta(X\beta)(Y + \eta(Y)\xi) - \delta g(\phi(Y), \phi(X))(grad(\beta) + 2\alpha\beta\{\delta g(\phi(X), Y)\xi - \eta(X)\phi(Y)\}$, (2.13)
for any vector fields X,Y on M.
Proof: We have the identity, $g(R(\xi, Y)X, Z) = g(R(X, Y)\xi, Y)$
Now from (2.9) of Lemma 2.5, we have

$g(R(\xi, Y)X, Z) = g(R(X, Z)\xi, Y) = (\alpha^2 + \beta^2)\{\delta g(Z, \xi)g(X, Y) - \eta(X)g(Z, Y)\} + 2\alpha\beta\{\delta g(Z, \xi)g(\phi(X), Y) - \eta(X)g(Z, \phi(Y))\} + \delta\{-(Z\alpha)g(\phi(X), Y) + (X\alpha)g(Z, \phi(Y)) - (Z\beta)g(\phi^2X, Y) + (X\beta))g(Z, \phi^2Y)\}$. (2.14)
After simplification from (2.14), we get (2.13).

Lemma 2.7: For a δ -Lorentzian para trans-Sasakian manifold M, we have$R(\xi, Y)\xi = \{\alpha^2 + \beta^2 + \delta(\xi\beta)\}\phi^2Y + (2\alpha\beta + \delta(\xi\alpha))\phi(Y)$.(2.15)
for any vector fieldY on M.
Proof: Setting$X = \xi$ in (2.13), we get
$R(\xi, Y)\xi = (\alpha^2 + \beta^2)(\eta(Y)\xi + Y) + \delta(\xi\alpha)\phi(Y) + \delta(\xi\beta)(Y + \eta(Y)\xi) - 2\alpha\beta\phi(Y)$which proves (2.15).

Lemma 2.8: For a δ -Lorentzian para trans-Sasakian manifold M, we have
$\eta(R(X, Y)Z) = \delta(\alpha^2 + \beta^2)[\eta(X)g(Y, Z) - \eta(Y)g(Y, Z)] + 2\delta\alpha\beta[\eta(X)g(\phi(Y), Z) - \eta(Y)g(\phi(X), Z)] + [(Y\alpha)(g(\phi(X, Z) - (X\alpha)g(Y, \phi(Z)) + (Y\beta)g(\phi^2X, Z) - (X\beta)g(\phi^2Y, Z)]$, (2.16)
for any vector fields X,Y, Z on M.
Proof: From (2.2), we have $\eta(R(X, Y)Z) = \delta g(R(X, Y)Z, \xi) = -\delta g(R(X, Y)\xi, Z)$.
(2.16) of the above Lemma 2.8 follows from (2.10) of Lemma 2.5.

Lemma 2.9: For a δ -Lorentzian para trans-Sasakian manifold M, we have
$S(X, \xi) = \{2n(\alpha^2 + \beta^2) + \delta(\xi\beta)\}\eta(X) - (2n - 1)\delta(X\beta) + \{2\alpha\beta\eta(X) - \delta(X\alpha)\}f + \delta(\phi(X))\alpha$ (2.17)
$S(\xi, \xi) = -2n(\alpha^2 + \beta^2 + \delta\xi\beta) - (2\alpha\beta + \delta\xi\alpha)f$ (2.18)
where X is any vector field on M and f is given by $f = g(\phi(e_i), e_i)$ (2.19)
repeated indices imply the summation, $\{e_i\}$, for i= 1, 2, ..., 2n + 1 are the orthonormal basis at each point of the tangent space of M.
Proof: From(2.2), we have
$\delta g(R(X, Y)Z, \xi) = \eta(R(X, Y)Z)$.
Now setting $Y = Z = e_i$in the above equation, we get
$\delta g(R(X, e_i)e_i, \xi) = \eta(R(X, e_i)e_i)$.
Further, using the right-hand side expression of (2.16) of Lemma 2.8, the proof follows after multiplying by δ on both sides. Put $X = \xi$ in (2.17) and use (2.2) to get (2.18).
Note2.1: One can choose α and β arbitrarily such that $2\alpha\beta + \delta(\xi\alpha) = 0$ (2.20)
so that the calculations are easier in the next Sections onwards. For instance, take δ = 1 and

$$\alpha = -\frac{e^{-2z}}{2},\ \beta = 1$$

Theorem 2.10: If α is constant and $2\alpha\beta + \delta(\xi\alpha) = 0$, then a δ- Lorentzian trans-Sasakian manifold is always a δ- Lorentzian α-Sasakian manifold.

3. Weakly Symmetries Of Δ-Lorentzian Para Trans-Sasakian Manifolds

In this Section, we define the ξ- sectional curvature of δ- Lorentzian para trans-Sasakian manifold and also weakly symmetries of δ- Lorentzian para trans-Sasakian manifold and study their properties. Here we highlighted the importance of ξ sectional curvature.

Definition 3.1: The ξ-sectional curvature of δ- Lorentzian manifold for a unit vector field X orthogonal to ξ is defined by $K(\xi, X) = R(\xi, X, \xi, X)$ (3.1)
From (2.15) of Lemma 2.7, we have $R(\xi, X, \xi, X) = \{(\alpha^2 + \beta^2) + \delta(\xi\beta)\}g(\phi^2X, X) + (2\alpha\beta + \delta(\xi\alpha))g(\phi(X), X)$.
Using (3.1) of Definition 3.1, choose α and β such that (2.20) of Note 2.1 holds, then the ξ-sectional curvature is given by $K(\xi, X) = \alpha^2 + \beta^2 + \delta(\xi\beta)$(3.2)

Definition 3.2: A non-flat δ -Lorentzian manifold M of dimension 2n+1(n >1) is said to be weakly symmetric if its curvature tensor R of type (0,4) satisfies the condition
$(\nabla_X R)(Y, Z, U, V) = A(X)R(Y, Z, U, V) + B(Y)R(X, Z, U, V) + B(Z)R(Y, X, U, V) + D(U)R(Y, Z, X, V) + D(V)R(Y, Z, U, X)$, (3.3)
for any vector fields X, Y, U, V on M, where A, B and D are associated 1-forms on M (not simultaneously zero).
Let $\{e_i\}$, for i=1,2,.....,2n+1 be the orthonormal basis at each point of the tangent space of M. Setting $Y = U = e_i$in (3.3) of Definition 3.3, we get
$(\nabla_X S)(Z, U) = A(X)S(Z, U) + B(Z)S(X, U) + D(U)S(X, Z) + B((RX, Z)U) + D(R(X, U)Z)$ (3.4)
Next putting $X = Z = U = \xi$ in (3.4) and then using (2.17) of Lemma 2.9, one obtains

$A(\xi)+B(\xi)+D(\xi)=\frac{2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)}{\alpha^2+\beta^2+\delta(\xi\beta)}$, (3.5)

Provided$\alpha^2+\beta^2+\delta(\xi\beta)$ 6= 0. Hence one can state the following Theorem.

Theorem 3.3: In a weakly symmetries of δ -Lorentzian para trans-Sasakian manifold M (n >1) of nonvanishing ξ -sectional curvature if α and β satisfy (2.20), then the relation (3.5) holds.

Putting X = Z = ξ in (3.4) using (2.2), we get

$(\nabla_X S)(\xi, U) = \{A(\xi) + B(\xi)\}S(\xi, U) + \{(-2n + 1)(\alpha^2 + \beta^2 + \delta(\xi\beta)\}D(U)$

$+ (\alpha^2 + \beta^2 + \delta(\xi\beta))\eta(U)D(\xi)$ (3.6)

On the other hand, using (2.17) of Lemma 2.9, we have

$(\nabla_\xi S)(\xi, U) = (\nabla_\xi S)(\xi, U) - S(\nabla_\xi\xi, U) - S(\xi, \nabla_\xi U) = \nabla_\xi S(\xi, U) - S(\xi, \nabla_\xi U) = \{2n(2\alpha(\xi\alpha) + 2\beta(\xi\beta)) + \delta\xi(\xi\beta)\}\eta(U) + \delta(\phi(U))(\xi\alpha) - (2n-1)\delta U(\xi\beta) + \{\xi(2\alpha\beta\eta(U) - \delta U(\xi\alpha)\}f + \{2\alpha\beta\eta(U) - \delta U\alpha\}\xi(f)$ (3.7)

In view of (3.6) and (3.7) equating the right-hand side expressions and further substituting forD(ξ) from (3.5) in (3.6), after lengthy calculations, one obtains

$$D(U) = -\frac{[2n\{2\alpha(\xi\alpha)+2\beta(\xi\beta)\}+\delta\xi(\xi\beta)]\eta(U)+(2\alpha\beta\eta(U)-\delta U\alpha))\xi(f)}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)} - \frac{(-2n+1)\delta U(\xi\beta)+\delta\left(\phi(U)\right)(\xi\alpha)+\left(\xi(2\alpha\beta)\eta(U)-\delta U(\xi\alpha)\right)f}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)} - D(\xi)\left[\frac{(2n-1)(\alpha 2+\beta 2)\eta(U)-\delta U\beta+\delta\phi(U)\alpha}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)}\right]$$

$$-D(\xi)\left[\frac{(2\alpha\beta\eta\,(U)-\delta U\alpha)f}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)}\right]+\frac{2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)^2}[\{2n(\alpha^2+\beta^2)-\delta(\xi\beta)\}\eta(U)+(-2n+1)\delta U\beta+\delta(\phi(U))\alpha+(2\alpha\beta\eta(U)-\delta U\alpha)f] \quad (3.8)$$

for any vector field U on M provided $\alpha^2+\beta^2+\delta(\xi\beta)\neq 0$ and f is given by (2.19). Next puttingX= U = ξ in (3.4) and proceeding as above, one finds

$$B(U) = -\frac{[2n\{2\alpha(\xi\alpha)+2\beta(\xi\beta)\}+\delta\xi(\xi\beta)]\eta(Z)+(2\alpha\beta\eta(Z)-\delta Z\alpha))\xi(f)}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)} - \frac{(-2n+1)\delta Z(\xi\beta)+\delta\left(\phi(Z)\right)(\xi\alpha)+\left(\xi(2\alpha\beta)\eta(Z)-\delta Z(\xi\alpha)\right)f}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)} - B(\xi)\left[\frac{(2n-1)(\alpha^2+\beta^2)\eta(Z)-\delta U\beta+\delta\phi(Z)\alpha}{(2n-1)\left(\alpha^2+\beta^2+\delta(\xi\beta)\right)}\right] - B(\xi)\left[\frac{(2\alpha\beta\eta(Z)-\delta Z(\alpha))f}{(2n-1)(\alpha^2+\beta^2+\delta(\xi\beta))}\right] + \frac{2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)}{(2n-1)(\alpha^2+\beta^2+\delta(\xi\beta))^2}[\{2n(\alpha^2+\beta^2)-\delta(\xi\beta)\}\eta(Z)+(-2n+1)\delta Z\beta+\delta(\varphi(Z))\alpha+(2\alpha\beta\eta(Z)-\delta Z\alpha)f]$$

(3.9)

for any vector field Z on M provided $\alpha^2+\beta^2+\delta(\xi\beta)\neq 0$ and f is given by (2.19). Hence, one can state

Theorem 3.4: In a weakly symmetric δ -Lorentzian trans-Sasakian manifold M (n >1) of nonvanishing ξ -sectional curvature if (2.20) holds, then the associated 1-forms D and B are given by (3.8) and (3.9) respectively.

Setting Z = U = ξ in (3.4), one finds

$(\nabla_X S)(\xi, \xi) = A(X)S(\xi, \xi) + \{B(\xi) + D(\xi)\}S(X, \xi) + B(R(X, \xi)\xi) + D(R(X, \xi)\xi) = -2n(\alpha^2 + \beta^2 + \delta(\xi\beta))A(X) + (B(\xi) + D(\xi))S(X, \xi) - (\alpha^2 + \beta^2 + \delta(\xi\beta))[B(X) + D(X) + (B(\xi) + D(\xi))\eta(X)] = -2n(\alpha^2 + \beta^2 + \delta(\xi\beta))A(X) + (B(\xi) + D(\xi))[(2n - 1)\{(\alpha^2 + \beta^2 + \delta(\xi\beta))\eta(X) - \delta(X\beta)\} + \delta(\phi(X))\alpha + \{2\alpha\beta\eta(X) - \delta(X\alpha)f\}] - (\alpha^2 + \beta^2 + \delta(\xi\beta))(B(X) + D(X))$ (3.10)

On the other hand, we have

$(\nabla_X S)(\xi, \xi) = \nabla_X S(\xi, \xi) - S(\nabla_X\xi, \xi) - S(\xi, \nabla_X\xi) = \nabla_X S(\xi, \xi) - 2S(\nabla_X\xi, \xi)$

which yields by using (2.6), (2.16) of Lemma 2.8, (2.17) and (2.18) Lemma 2.9,

$(\nabla_X S)(\xi, \xi) = -2n[2\alpha(X\alpha) + 2\beta(X\beta) + \delta X(\xi\beta)] - 2\alpha[X\alpha + \eta(X))(\xi\alpha) + (-2n + 1)(\phi(X)\beta) + (\phi(X)\alpha)f - 2\beta[(\phi(X)\alpha + (-2n + 1)\{X\beta + (\xi\beta)\eta(X)\} + (2\alpha\beta\eta(X) - X\alpha))f]$

(3.11)

Equating righthand side expressions of (3.10) and (3.11).one obtains

$-2n(\alpha^2+\beta^2+\delta(\xi\beta))A(X)+(B(\xi)+D(\xi))[(2n-1)\{(\alpha^2+\beta^2)\eta(X)-\delta(X\beta)\}+\delta(\phi(X))\alpha+\{2\alpha\beta\eta(X)-\delta(X\alpha)f\}]-(\alpha^2+\beta^2+\delta(\xi\beta))(B(X)+D(X))=-2n[2\alpha(X\alpha)+2\beta(X\beta)+\delta X(\xi\beta)]X\alpha-2\alpha[X\alpha+\eta(X)(\xi\alpha)+(-2n+1)(\phi X\beta)+(\phi X\alpha)f-2\beta[(\phi(X)\alpha+(-2n+1)\{X\beta+(\xi\beta)\eta(X)\}+(2\alpha\beta\eta(X)-X\alpha)f]$ (3.12)

Adding (3.8) and (3.9) by taking U = Z = X, one obtains

$\{B(\xi)+D(\xi)\}[(2n-1)\{(\alpha^2+\beta^2)\eta(X))-\delta(X\beta)+\delta(\phi(X))\alpha+\{2\alpha\beta\eta(X)-\delta(X\alpha)\}f]$

$=-\{2n(2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)\}\eta(X)+(2\alpha\beta\eta(X)-\delta(X\alpha))\xi(f)$

$-(2n-1)(\alpha^2+\beta^2\delta\xi\beta)\{B(X)+D)X)\}+\frac{2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)}{(2n-1)(\alpha^2+\beta^2+\delta(\xi\beta))}\}[2n\{(\alpha^2+\beta^2)$

$+\delta(\xi\beta)\}\eta(X)+(-2n+1)\delta X\beta+\delta(\phi(X))\alpha+(2\alpha\beta\eta(X)-\delta X\alpha)f]$

Next substituting for the following expression from the above equation in (3.12),

$\{B(\xi)+D(\xi)\}[(2n-1)\{(\alpha^2+\beta^2)\eta(X))-\delta(X\beta\}+\delta(\phi(X))\alpha+\{2\alpha\beta\eta(X)-\delta(X\alpha)\}f]$ after simplification, finally, we get

$$A(X)+B(X)+C(X)=\frac{2\alpha(X\alpha)+2\beta(X\beta)+\delta X(\xi\beta)}{\alpha^2+\beta^2+\delta(\xi\beta)}\frac{\alpha}{n}\left[\frac{X\alpha+\eta(X)(\xi\alpha)+(-2n+1)(\varphi(X)\beta\eta(X))\xi(f)}{\alpha^2+\beta^2+\delta(\xi\beta)}\right]$$
$$+\frac{\beta}{n}\left[\frac{\varphi(X)\alpha+(-2n+1)(X\beta)+(2\alpha\beta\eta(X)-\delta X\alpha)}{\alpha^2+\beta^2+\delta(\xi\beta)}\right]$$
$$-\frac{\{2n\{2\alpha(\xi\alpha)+2\beta(\xi\beta)\}+\delta\xi(\xi\beta)\}\eta(X)-(2\alpha\beta\eta(X)-\delta X\alpha)\xi(f)}{n(\alpha^2+\beta^2+\delta(\xi\beta))}$$
$$-\frac{(-2n+1)\delta X(\xi\beta)+\delta\varphi(X)(\xi\alpha)+\{\xi(2\alpha\beta)\eta(X)-\delta X(\xi\alpha)\}f}{n(\alpha^2+\beta^2+\delta(\xi\beta))}$$
$$+\frac{2\alpha(\xi\alpha)+2\beta(\xi\beta)+\delta\xi(\xi\beta)}{n(\alpha^2+\beta^2+\delta(\xi\beta))^2}\left[\{2n(\alpha^2+2\beta^2)-\delta(\xi\beta)\}\eta(X)\right.$$
$$\left.+(-2n+1)\delta(X\beta)+\delta(\varphi(X))\alpha+(2\alpha\beta\eta(X)-\delta(X\alpha))f\right] \quad (3.13)$$

for any vector field X on M provided $\alpha^2+\beta^2+\delta(\xi\beta)\neq 0$. Hence, we state

Theorem 3.5:In a weakly symmetries of δ -Lorentzian para trans-Sasakian manifold M (n >1) of non-vanishing ξ -sectional curvature if (2.20) holds, then the sum of the associated 1-forms A, B and D are given by (3.13).

Remark 3.6: If we choose β = 0, then from (3.13), one can find an expression for the sum of the associated 1-forms A, B, and D for the weakly symmetries of δ-Lorentzian α -Sasakian manifold M (n >1). Similar arguments follow for the weakly symmetries of δ-Lorentzian β -Kenmotsu manifold M (n >1).
If α = 1 and β = 0, then from (3.13), it is easy to see that A + B + D = 0,Hence we state.

Corollary3.7: Thereare no weakly symmetries of δ-Lorentzian Sasakian manifold M (n >1) unless the sum of associated 1-forms is zero everywhere.
Similarly, if α = 0 and β = 1, then from (3.13), it is easy to see that the sum is zero. Hence we state.

Corollary 3.8: There is no weakly symmetries of δ-Lorentzian Kenmotsu manifold M (n >1) unless the sum of associated 1-forms is zero everywhere.

Corollary 3.9: With the same hypothesis of the Theorem 3.3, if α and β are nonzero constants, then for a δ- Lorentzian para trans-Sasakian manifold of type (α, β), the relation A(ξ) + B(ξ) + D(ξ) = 0holds.
Proof: Follows from Theorem 3.3.

Theorem 3.10: If a weakly symmetries of δ -Lorentzian para trans-Sasakian manifold M (n >1) of non-vanishing ξ -sectional curvature with α and β are such that (2.20) holds, then

$$D(X)-B(X)=(B(\xi)-D(\xi))\left[\frac{(2n-1)\{(\alpha^2+\beta^2)\eta(X)+\delta(X\beta)\}+\delta\varphi(X)\alpha}{(2n-1)(\alpha^2+\beta^2+\delta(\xi\beta))}+\frac{(X\alpha\beta\eta(X)-\delta X(\alpha))f}{(2n-1)(\alpha^2+\beta^2+\delta(\xi\beta))}\right] \quad (3.14)$$

Proof: Follows from (3.7) and (3.8) by forming D(X)−B(X).

Corollary 3.11: In a weakly symmetric δ -Lorentzian β-Kenmotsu manifold M (n >1) with β non-zero constant, the relation

$D\circ\varphi-B\circ\varphi=0$ (3.15) holds

Proof: Putting α = 0 in (3.14) of Theorem 3.10 by taking β constant, we have D(X) − B(X) = (B(ξ) − D(ξ))η(X) (3.16)

Now replacing X by $\phi(X)$ in (3.16), we get (3.15).

Corollary 3.12: In a weakly symmetric δ -Lorentzian β-Kenmotsu manifold M (n >1) if β is nonzero constant and $D(\xi) = B(\xi)$, then D and B are in the same directions.
Proof: Follows from Theorem 3.10 and (3.16).

4. Evaluation Of α&β

In this Section, a concrete way of finding α and β are given.
Lemma 4.1: For a δ -Lorentzian para trans-Sasakian manifold, we have
$(\nabla_X\Phi)(Y, Z) = \delta[\alpha\{g(X, Z)\eta(Y) + g(X, Y)\eta(Z)\}+ \beta\{g(X, \phi(Z))\eta(Y) + g(X, \phi(Y))\eta(Z)\}]$ (4.1)
$(\nabla_X\Phi)(Y, Z) = (\nabla_X\Phi)(Z, Y)$ (4.2)
$\Phi(X, Y) = \Phi(Y, X)$ (4.3)
where Φ is the fundamental 2-form of the structure given by $\Phi(X, Y) = g(X, \phi(Y)$
Proof: Consider,
$(\nabla_X\Phi)(Y, Z) = \nabla_X\Phi(Y, Z) - \Phi(\nabla_XY, Z) - \Phi(Y, \nabla_XZ) = \nabla_X g(Y, \phi(Z)) - g(\nabla_XY, \phi(Z)) - g(Y, \phi(\nabla_XZ))$
$= g(\nabla_XY, \phi(Z) + g(Y, \nabla_X\phi(Z)) - g((\nabla_XY, \phi(Z)) - g(Y, \phi(\nabla_XZ)) = g(\nabla_XY, \phi(Z)) + g(Y,(\nabla_X\phi)Z) + g(Y, \phi(\nabla_XZ))-$
$g(\nabla_XY, \phi(Z)) - g(Y, \phi(\nabla_XZ)) = g(Y,(\nabla_X\phi)Z) = g(Y, \delta^2\alpha\{g(X, Z)\xi + \delta\eta(Z)X\})+ \beta\{\delta^2 g(\phi(X), Z)\xi + \delta\eta(Z)\phi(X)\}$
After simplification, we obtain (4.1).
(4.2) follows from (4.1) and the proof of (4.3) is obvious. This complete proof of Lemma 4.1
Putting $Z = \xi$ and $Y = X$ in (4.1) of Lemma 4.1, we get
$-(\nabla_X\Phi)(X, \xi) = \delta\alpha + \delta\beta\omega$, (4.4)
where $\omega = g(X, \Phi(X))$, X is orthogonal to ξ, and $g(X, X) = 1$. Further putting Y = X in (2.9) of Lemma 2.4, we get $\delta(\nabla_X\eta)(X) = \delta\alpha\omega + \delta\beta$(4.5)
Now eliminating α and β from (4.4) and (4.5), finally we have $\alpha = \frac{\delta\omega(\nabla_X\eta)(X)+(\nabla_X\Phi)(X,\xi)}{\delta(\omega^2-1)}$ (4.6)
$\beta = -\frac{\delta(\nabla_X\eta)(X)+\omega(\nabla_X\Phi)(X,\xi)}{\delta(\omega^2-1)}$ (4.7)
Provided $\omega^2 - 1 \neq 0$. Hence we state the following.

Theorem 4.2: In a δ-Lorentzian para trans-Sasakian manifold M the smooth functions α and β are given by (4.6) and (4.7) respectively provided $\omega^2 - 1 \neq 0$.

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