

R_i-Seperation Axioms On Infra Soft Topological Spaces

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Abstract

The aim of this study is to introduce and investigate two new classes of separation axioms called Infra soft R_0 and Infra soft R_1 space in the spaces of infra soft topologies by using the notions of infra soft open sets and infra soft closure operator. We discuss the basic properties and characterizations of them. We also study the relationships between these classes and some other supra soft separation axioms with many results and explanatory examples.

Keywords: Infra soft topological space, infra soft open set, Infra soft kernal, Infra soft R_0 space, Infra soft R_1 space.

1. INTRODUCTION

The classical mathematical approaches are often insufficient for modelling problems with uncertain data. There are several theories such as fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets and theory of rough sets which can be considered as tools for dealing with uncertainties. But these theories have their own difficulties due to the inadequacy of the parametrization tool of the theories as pointed out by Molodtsov. In 1999, Molodtsov [4] introduced soft set theory as a mathematical tool for solving complex problems dealing with uncertainties. Recently research works on soft set theory and its applications in various fields are advancing rapidly. In the year 2011, Shabir and Naz [6] introduced soft topological spaces which are defined over an initial universal set with a fixed set of parameters. Sabir Huzzain and Bashir Ahmad [9] discussed properties of soft interior, soft closure, soft exterior and soft boundary in 2011. Some topological properties are kept under conditions weaker than topology's conditions; also, some phenomena are described under structures relaxing a topology. The structures of supra soft topology [14] and infra soft topology [15] have been born with these goals. They have become two of the most interesting developments of soft topology in recent years. Supra topology is a class of subsets that extend the concept of topological space by dispensing with the postulate that the class is closed under finite intersections, whereas infra topology is a class of subsets that extend the concept of topological space by dispensing with the postulate that the class is closed under arbitrary unions. Generalized soft topology [16] and soft weak structure are other generalizations of soft topology. Recently, the concepts of compactness and connectedness have been studied through the frame of infra soft topology in [17], [18] respectively. In this work, we aim to introduce a new forms of separation properties called Infra soft R_0 and infra soft R_1 space by using infra soft open sets and infra soft closure operator. we tried to investigate some basic properties and some characterization of them. The relationship between these two new separation axioms and their relationship with some other properties are also studied. We begin with some basic definitions and results which are essential for our study.

2. Preliminaries

Definition 2.1.[6] Let τ be collection of soft sets (F,A) over X . Then τ is said to be a soft topology on X if

$$1 \quad \phi_A, X_A \in \tau$$

2 Soft union of any number of soft sets in τ belongs to τ

3 Soft intersection of two soft sets in τ belongs to τ

Then (X, τ, A) is called soft a topological space over X . The members of τ are called soft open sets.

Definition 2.2.[6] A soft set (F,A) over X is said to be a soft closed set if its relative complement $(F,A)' \in \tau$.

Definition 2.3.[19] The collection μ of soft sets over X under a fixed set of parameters E is said to be an infra soft topology on X if it is closed under finite soft intersection and the null soft set is a member of μ . The triple (X,μ,E) is called an infra soft topological space. Every member of μ is called an infra soft open set, and its relative complement is called an infra soft closed set

Definition 2.4. [19] We define the infra soft interior and infra soft closure of a soft subset (F, E) of (X, μ, E) which are, respectively, denoted by $\text{Int}_1(F, E)$ and $\text{Cl}_1(F, E)$ as follows:

- $\text{Int}_1(F,E)$ is the union of all infra soft open sets that are contained in (F, E) .
- $\text{Cl}_1(F, E)$ is the intersection of all infra soft closed sets containing (F, E) .

Proposition 2.1. [19] Let (F, E) be a soft subset of (X, μ, E) . Then, the following properties hold.

- If (F, E) is an infra soft open set, then $\text{Int}_1(F,E) = (F, E)$.
- If (F,E) is an infra soft closed set, then $\text{Cl}_1(F,E) = (F, E)$.

Proposition 2.2.[19] Let (F,E) be a soft subset of (X,μ,E) . Then, the following properties hold.

- A soft point $x_e \in \text{Int}_1(F,E)$ if and only if there exist an soft open soft set (G,E) such that $x_e \in (G,E) \subseteq (F,E)$.
- A soft point $x_e \in \text{Cl}_1(F,E)$ if and only if $(F,E) \cap (G,E) \neq \phi$ for every infra soft open soft set (G,E) containing x_e .

Definition 2.5.[19] Let (x,μ,E) be an infra soft topological space and let (F,E) be a soft set. Then,

- $\text{Int}(F,E)^C = \text{Cl}(F,E)^c$.
- $\text{Cl}(F,E)^C = \text{Int}(F,E)^c$.

Definition 2.6.[19] Let (X,μ,E) be an infra soft topological space and let (F,E) and (G,E) be soft subsets over X . Then, the following properties hold:

- $\text{Int}_1(X_E) = X_E$.
- $\text{Int}_1(F,E) \subseteq (F,E)$.
- $(F,E) \subseteq (G,E)$, then $\text{Int}_1(F,E) \subseteq \text{Int}_1(G,E)$.
- $\text{Int}_1 \text{Int}_1(F,E) = \text{Int}_1(F,E)$

Definition 2.7. [3] Let $S_E(X)$ and $S_E(Y)$ be families of soft sets over X and Y respectively. Let $u: X \rightarrow Y$ and $p: E \rightarrow K$ be mappings. We define a soft mapping $f_{pu}: S_E(X) \rightarrow S_K(Y)$ as follows:

- If $(F,E) \in S_E(X)$, then the image of (F,E) under f_{pu} , written as $f_{pu}(F,E)$ is a soft set in $S_K(Y)$ such that:

$$f_{pu}(F, E)(k) = \begin{cases} \bigcup \{u[F(e) : e \in p^{-1}(k)]\} & \text{if } p^{-1}(k) \neq \phi \\ \phi & \text{if } p^{-1}(k) = \phi \end{cases}$$

- If $(H,K) \in S_K(Y)$, then the inverse of (H,K) under f_{pu} , written as $f_{pu}^{-1}(H,K)$ is a soft set in $S_E(X)$ such that

$$f_{pu}^{-1}(H,K)(e) = u^{-1}[H(p(e))] \text{ for all } e \in E$$

Definition 2.8. [5] Let (X,τ_1,E) and (Y,τ_2,K) be soft topological spaces and let $f_{pu}: S_E(X) \rightarrow S_K(Y)$ be a soft map. Then f_{pu} is called continuous soft map if $f_{pu}^{-1}(G,K) \in \tau_1$, for all $(G,K) \in \tau_2$.

Definition 2.9. Let (X,μ,E) be an infra soft topological space and let x_e and y_r be soft points on X . Then

- (X,μ,E) is an infra soft T_0 space if there exist an infra soft open set (F,E) such that $x_e \in (F,E)$ and $y_r \notin (F,E)$ or $y_r \in (F,E)$ and $x_e \notin (F,E)$.
- (X,μ,E) is an infra soft T_1 space if there exist an infra soft open set (F,E) and (G,E) such that $x_e \in (F,E)$, $y_r \notin (F,E)$ and $y_r \in (G,E)$, $x_e \notin (G,E)$.
- (X,μ,E) is an infra soft T_2 space if there exist an infra soft open set (F, E) and (G, E) such that

$x_e \in (F, E)$ and $y_r \notin (G, E)$.

Theorem 2.10. A soft map $f_{pu}: (X, \mu_1, E) \rightarrow (Y, \mu_2, K)$ is infra soft continuous if and only if the inverse image of each infra soft open set is an infra soft open set.

3. R_i -Separation Axioms

We introduce new separation axioms called infra soft R_i separation axioms on infra soft topological space. Some characterization and properties are also given.

Definition 3.1. Let (X, μ, E) be an infra soft topological space, let (F, E) be soft set and x_e be soft point on X . Infra soft kernel of (F, E) is defined as $ISK(F, E) = \bigcap \{(G, E) \in \mu : (F, E) \subseteq (G, E)\}$ and $ISK(x_e) = \bigcap \{(G, E) \in \mu : x_e \in (G, E)\}$.

Proposition 3.1. Let (X, μ, E) be an Infra soft topological space and let (F, E) be a soft set over X . Then $ISK(F, E) = \bigcap \{x_e \in SP(X) : Cl_I(x_e) \cap (F, E) = \phi \tilde{E}\}$.

Proof. If $(F, E) = \phi$, then $ISK(F, E) = \phi \tilde{E} = \bigcap \{x_e \in SP(X) : Cl_I(x_e) \cap (F, E) = \phi \tilde{E}\}$.
 $\phi \tilde{E}$. so let, $(F, E) = \phi \tilde{E}$. Let $x_e \in ISK(F, E)$. Then $x_e \in (G, E), \forall (G, E) \in \mu$ with $(F, E) \subseteq (G, E)$.

Assume $Cl(x) \cap (F, E) = \phi \tilde{E}$. Then $(F, E) \subseteq Cl(x)^c = Int(x)^c$. So, there exist an Infra soft open set containing (F, E) , which is a contradiction. Hence $Cl_I(x_e) \cap (F, E) = \phi \tilde{E}$.

Conversely, assume $Cl(x_e) \cap (F, E) = \phi \tilde{E}$. If possible let $x_e \notin ISK(F, E)$. Then $x_e \in (G, E)$ for some $(G, E) \in \mu$ with $(F, E) \subseteq (G, E)$. Then $x_e \in (G, E)^c \Rightarrow Cl_I(x_e) \subseteq (G, E)^c$, a contradiction.

Hence, $Cl_I(x_e) \cap (F, E) = \phi \tilde{E}$. \square

Proposition 3.2. Let (X, μ, E) be an Infra soft topological space and let x_e be a soft point on X . Then, $x_e \in Cl_I(y_r)$ if and only if $y_r \in ISK(x_e)$.

Proof. It is clear from previous proposition. \square

Definition 3.2. Let (X, μ, E) be Infra soft topology on X and let x_e and y_r be soft points on X with $x_e \neq y_r$. Then,

- (X, μ, E) is Infra soft R_0 space (briefly ISR_0) if and only if $x_e \in Cl_I(y_r) \Rightarrow y_r \in Cl_I(x_e)$.
- (X, μ, E) is Infra soft R_1 space (briefly ISR_1) if and only if $x_e \neq y_r$ with $Cl_I(x_e) = Cl_I(y_r)$ then there exist soft open sets (F, E) and (G, E) such that $x_e \in (F, E)$ and $y_r \in (G, E)$ and $(F, E) \cap (G, E) \neq \phi \tilde{E}$.

Theorem 3.3. Every ISR_1 space is ISR_0 space.

Proof. Let $x_e \neq y_e$. Suppose $x_e \notin Cl_I(y_e)$. Then $Cl_I = Cl_I(y_e)$. Since (X, μ, E) is ISR_1 space, there exist infra soft open sets (F, E) and (G, E) such that $x_e \in (F, E), y_e \in (G, E)$ and $(F, E) \cap (G, E) = \phi \tilde{E}$. Since $x_e \notin (G, E), Cl_I(x_e) \subseteq (G, E)^c$. So that $y_e \notin Cl_I(x_e)$. Hence the theorem. \square

Remark 3.1. Converse of the above theorem need not be true.

Theorem 3.4. Let (X, μ, E) be an Infra soft topological space. Then (X, μ, E) is ISR_0 space if and only if $Cl_I(x_e) \subseteq (F, E)$, for any Infra soft open set (F, E) containing x_e .

Proof. Let (X, μ, E) be ISR_0 space. Assume $Cl_I(x_e) \not\subseteq (F, E)$ for some Infra soft open set containing x_e . Then there exist soft point y_r such that $y_r \in Cl_I(x_e)$, but $y_r \notin (F, E)$. Then $y_r \cap (F, E) = \phi \tilde{E}$. hence $x_e \notin Cl_I(y_r)$, which is a contradiction to (X, μ, E) is ISR_0 space.

Conversely, assume $Cl(x_e) \subseteq (F, E)$ for any $(F, E) \in \mu$ containing x_e . Let $x_e \notin Cl_I(y_r)$ where $x_e \neq y_r$. Then there exist infra soft open set (F, E) with $x_e \in (F, E)$ and $y_r \cap (F, E) = \phi \tilde{E}$. Then $y_r \notin (F, E)$. Hence $y_r \notin Cl_I(x_e)$, since $Cl_I(x_e) \subseteq (F, E)$ by our assumption.

Theorem 3.5. Let (X, μ, E) be ISR_0 space. Then $ISK(F, E) = (F, E)$ for any Infra soft closed set (F, E) . \square

Proof. Let $x_e \notin (F, E) \Rightarrow x_e \in (F, E)^c \Rightarrow Cl_I(x_e) \subseteq (F, E)$, by theorem 3.4. So that $Cl_I(x_e) \cap (F, E) = \phi \tilde{E}$. Hence $x_e \notin ISK(F, E)$ by proposition 3.1.

$x_e \in (F, E) \Rightarrow Cl_I(x_e) \subseteq (F, E)$, since (F, E) is infra soft closed soft set.

$$\Rightarrow Cl(x_e) \cap (F, E) = \phi \tilde{E}.$$

$$\Rightarrow x_e \in ISK(F, E).$$

Hence, $(F, E) = ISK(F, E)$. □

Remark 3.2. The converse above theorem need not be true.

Example 1. Let $X = \{a, b\}$ and $E = \{e_1, e_2\}$ and $\mu = \{\phi \tilde{E}, (F_1, E), (F_2, E)\}$, where

$$(F_1, E) = \{(e_1, \{a, b\}), (e_2, \{c\})\}.$$

$$(F_2, E) = \{(e_1, \{c\}), (e_2, \{b\})\}.$$

Then (x, μ, E) is an Infra soft topological space. We can see that $ISK(F, E) = (F, E)$ for any Infra soft closed soft set (F, E) . But (x, μ, E) is not an ISR_0 space because $a_{e_2} \in Cl(a_{e_1})$, but $a_{e_1} \notin Cl(a_{e_2})$.

Theorem 3.6. Let (x, μ, E) be an Infra soft topological space and let x_e be a soft point on X . Then (x, μ, E) is an ISR_0 space if and only if for any Infra soft closed soft set (F, E) not containing x_e , $Cl(x_e) \cap (F, E) = \phi \tilde{E}$.

Proof. Let (x, μ, E) be an ISR_0 space and let (F, E) be Infra soft closed soft set not containing x_e . Then $Cl(x_e) \subseteq (F, E)^c$, by theorem 3.4. Hence, $Cl(x_e) \cap (F, E) = \phi \tilde{E}$.

Conversely, assume for any Infra soft closed set (F, E) not containing x_e , $Cl(x_e) \cap (F, E) = \phi \tilde{E}$. Then it follows that for any Infra soft open set (F, E) containing x_e , $Cl(x_e) \subseteq (F, E)$. Hence (x, μ, E) is ISR_0 space by 3.4. □

Theorem 3.7. Let (x, μ, E) be an Infra soft topological space such that for any soft points x_e, y_r with $Cl(x_e) \neq Cl(y_r)$, $Cl(x_e) \cap Cl(y_r) = \phi \tilde{E}$. Then (x, μ, E) is a ISR_0 space.

Proof. Let $x_e \neq Cl(y_r)$. Then $Cl(x_e) \neq Cl(y_r)$. Hence, by our assumption

$$Cl(x_e) \cap Cl(y_r) = \text{space} \cdot \phi \tilde{E}. \text{ So that } y_r \notin Cl(x_e). \text{ Hence } (x, \mu, E) \text{ is an } ISR_0 \square$$

4. More properties and relation

In this section we tried to discuss relation between some other separation axioms and some more properties are also discussed.

Theorem 4.1. Every IST_i space is ISR_{i-1} space for $i=1, 2$.

Proof. Let (x, μ, E) be IST_1 space, let x_e be soft point and let (F, E) Infra soft open set containing x_e . Let $y_r \notin (F, E)$. Then $Cl(y_r) \subseteq (F, E)^c$, so that $x_e \notin Cl(y_r)$. So $x_e \neq y_r$. Since (x, μ, E) is IST_1 space, there exist Infra soft open set (G, E) such that $y_r \in (G, E)$ and $x_e \notin (G, E)$. Then $Cl(x_e) \subseteq (G, E)^c$.

Hence $y_r \notin Cl(x_e)$, so that $Cl(x_e) \subseteq (F, E)$. Therefore, (x, μ, E) is ISR_0 space by theorem 3.4.

The proof is trivial for $i= 2$. □

Remark 4.1. The converse of above theorem need not be true.

Example 2. Let $X = \{x, y\}$, $E = \{e_1, e_2\}$ and $\mu = \{\phi \tilde{E}, (F_1, E), (F_2, E)\}$, where $(F_1, E) = \{(e_1, X), (e_2, \phi)\}$

$$(F_2, E) = \{(e_1, \phi), (e_2, X)\}$$

Then (x, μ, E) is Infra soft topological space. We can easily verify that (x, μ, E) is both ISR_0 and ISR_1 space. But it is neither IST_1 nor IST_2 space.

Theorem 4.2. An Infra soft topological space is IST_1 if and only if it is both ISR_0 and IST_0 .

Proof. It is clear that if a Infra soft topological space is IST_1 then it is both ISR_0 and IST_0 , by theorem 4.1.

So let (x, μ, E) is both ISR_0 and IST_0 and let x_e, y_r be two soft points with $x_e \neq y_r$. Since (x, μ, E) is IST_0 , there exist an infra soft open set (F, E) containing only one of them. Suppose (F, E) contains x_e , but not y_r . Since (x, μ, E) is an ISR_0 space, $Cl(x_e) \subseteq (F, E)$.

$$y_r \in (F, E)^c \Rightarrow y_r \in (F, E)^c \subseteq Cl(x_e) \Rightarrow y_r \in Int(x_e) \Rightarrow y_r \in (G, E) \subseteq (x_e)^c \text{ for some } (G, E) \in \mu. \text{ Hence } (x, \mu, E) \text{ is an } IST_1 \text{ space.}$$

Theorem 4.3. A Infra soft topological space is IST_2 space if and only if it is both IST_0 and ISR_1 space.

Proof. It is clear by theorem 4.1 that if an Infra soft topological space is IST_2 space then it is both IST_0 and ISR_1 space. So let (x, μ, E) is both ISR_0 and IST_0 and let x_e, y_r be two soft points with $x_e \neq y_r$. Since (x, μ, E) is IST_0 space, there exist an Infra open set (F, E) containing only one of them. Suppose (F, E) contains x_e , but not y_r . Since (x, μ, E) is an ISR_1 space, $Cl_1(x_e) \subseteq (F, E)$ by theorems 3.4 and 3.3.

$y_e \in (F, E)^c \Rightarrow Cl_1(y_e) \subseteq (F, E)^c \Rightarrow Cl_1(x_e) = Cl_1(y_e)$. Hence, there exist two Infra soft open sets (F, E) and (G, E) such that $x_e \in (F, E), y_r \in (G, E)$ and $(F, E) \cap (G, E) = \phi \tilde{E}$ since (x, μ, E) is an ISR_1 space. Therefore, (x, μ, E) is an IST_2 space.

Proposition 4.1. Let $f_{pu}: (X, \mu_1, E) \rightarrow (Y, \mu_2, K)$ be bijective infra soft open map. If (X, μ_1, E) is ISR_i , then (Y, μ_2, K) is also ISR_i for $i = 0, 1$.

Proof. We will prove the case for $i=1$ and the proof is similar for $i=0$. Let

y_k, y_k' be two soft points on Y such that $y_k \neq y_k'$ and $Cl_1(y_k) = Cl_1(y_k')$. Since

f_{pu} is a bijective soft map, we have two distinct soft point x_e , and x_e' such that

$f_{pu}(x_e) = y_k, f_{pu}(x_e') = y_k'$ and $Cl_1(x_e) = Cl_1(x_e')$. Since (X, μ_1, E) is ISR_1 ,

there exist infra soft open sets $(f, E), (G, E)$ such that $x_e \in (f, E), x_e' \in (G, E)$

and $(f, E) \cap (G, E) = \phi \tilde{E}$. Since f_{pu} is infra soft open, $(H, K) = f_{pu}(f, E), (H, K) =$

$f_{pu}(G, E)$ are infra soft open and $y_k \in (H, K), y_k' \in (H, K), (H, K) \cap (H, K) =$

$\phi \tilde{K}$. Hence the proof. \square

Theorem 4.4. Let $f_{pu}: (X, \mu_1, E) \rightarrow (Y, \mu_2, K)$ be bijective infra soft continuous map. If (Y, μ_2, K) is ISR_i , then (X, μ_1, E) is also ISR_i for $i = 0, 1$.

Proof. We will prove the case for $i=1$ and the proof is similar for $i=0$. Let

x_e , and x_e' be two soft points on X such that $x_e \neq x_e'$ and $Cl_1(x_e) = Cl_1(x_e')$.

Since f_{pu} is a bijective soft map, we have two distinct soft points y_k, y_k' such that

$f_{pu}(x_e) = y_k, f_{pu}(x_e') = y_k'$ and $Cl_1(y_k) = Cl_1(y_k')$. Since (Y, μ_2, K) is ISR_1 ,

there exist infra soft open sets $(H, K), (H, K)$ such that $y_k \in (H, K), y_k' \in$

$(H, K), (H, K) \cap (H, K) = \phi \tilde{K}$. Since f_{pu} is infra soft continuous, $(f, E) =$

$f_{pu}(H, K), (G, E) = f_{pu}(H, K)$ are infra soft open and $x_e \in (f, E), x_e' \in (G, E)$

and $(f, E) \cap (G, E) = \phi \tilde{E}$. Hence the proof. \square

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