Structure of Prime Near Rings with Generalized Derivations satisfying some Identities

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ABSTRACT
The purpose of the present paper is to obtain the commutativity of a prime near Ωwith a generalized derivation τ associated with a nonzero derivation σ satisfying oneof the conditions: (i) $\tau([\lambda,\mu]) = \pm\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, (ii) $\tau(\lambda o\mu) = \pm\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$, (iii) $\tau([\lambda,\mu]) = \pm\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, (iv) $\tau(\lambda o\mu) = \pm\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$ and (v) $\tau([\lambda,\mu]) = \pm\lambda^{\alpha}(\lambda o\mu)\mu^{\beta}$ for all $\lambda,\mu\in\Omega$ and $\alpha, \beta \in \mathbb{N}$, the set of nonnegative integers.Moreover, we give an example which shows the necessity of primness hypothesis in the theorems.

Keywords: prime near ring, derivation, generalized derivation.

1. INTRODUCTION

A right near ring Ωis a triplet $(\Omega, +, \cdot)$where + and · are two binary operationssuch that (i)$(\Omega, +)$is a group (not necessarily a belian)(ii)$(\Omega,\cdot)$is a semigroup,and (iii)$(\lambda+\mu)\cdot\omega = \lambda\cdot\omega + \mu\cdot\omega$ for all $\lambda, \mu, \omega \in \Omega$.Analogously,ifinsteadof(iii),Ωsatisfiestheleftdistributive law then,Ωis said to be a left near ring. A near ring Ωis said to be zero-symmetric if $\lambda 0 = 0$ for all $\lambda \in \Omega$(right distributive yields that $0\lambda = 0$). Throughout thepaper, Ωrepresents a zero-symmetric right near ring with multiplicative center $Z(\Omega)$.For any pair of elements$\lambda, \mu \in \Omega$,The symbols $[\lambda, \mu]$and $(\lambda \circ \mu)$denote the Lie product$\lambda\mu - \mu\lambda$and the Jordan product $\lambda\mu + \mu\lambda$respectively. AnearringΩisknownasprimeif $\lambda\Omega\mu = 0$ forall$\lambda, \mu \in \Omega$impliesthat$\lambda = 0$or $\mu = 0$. A near ring Ωis known as 2-torsion free if (Ω, +) has no element of order 2. Throughout the paper ℕ represents the set of non-negative integers.

The notion of derivation in near rings was introduced by Bell and Mason [9].An additive mapping $\sigma: \Omega \to \Omega$is said to be a derivation on Ωif $\sigma(\lambda\mu) = \lambda\sigma(\mu) + \sigma(\lambda)\mu$ for all $\lambda, \mu \in \Omega$or equivalently in [28],$\sigma(\lambda\mu) = \sigma(\lambda)\mu + \lambda\sigma(\mu)$ forall$\lambda, \mu \in \Omega$.Motivated by the definition of derivation in near rings ,Gölbaşi [22]defined generalized derivation in near rings as follows: An additive mapping$\tau: \Omega \to \Omega$is said to be a right (resp. left) generalized derivation associated witha derivation σ on Ωif $\tau(\lambda\mu) = \tau(\lambda)\mu + \lambda\sigma(\mu)$ (resp. $\tau(\lambda\mu) = \sigma(\lambda)\mu + \lambda\tau(\mu)$) forall$\lambda, \mu\in\Omega$. Moreover,τ is said to be a generalized derivation with associated with a derivation σon Ωif it is both a right generalized derivation as well as a left generalized derivation on Ω. All derivations are generalized derivations. There has been a great deal of work by different authors with some suitable constraints on derivations and generalized derivations in prime and semi prime rings (see [1, 7, 8, 12, 13, 16, 17, 24]).A number of authors have been established some comparable results on nearrings,(c.f.[2,3,9, 11, 14, 19, 21, 27, 28, 29]).

Daif and Bell[18]proved that if R is a primering and I is a nonzero ideal of a R .If σ is a derivation on R such that $\sigma([\lambda,\mu]) = \pm[\lambda,\mu]$forall$\lambda,\mu\in I$, then Ris commutative.

Further, Dhara [7] proved that if R is a semiprime ring with a generalized derivation τ associated with a derivation σ satisfying $\tau([\lambda,\mu]) = \pm[\lambda,\mu]$ or $\tau(\lambda o\mu) = \pm(\lambda o\mu)$forall$\lambda,\mu\in I$, a nonzero ideal of R, then R must contain a nonzero central ideal, provided$\sigma(I) \neq \{0\}$. Moreover, he proved that in case R is a prime ring, R must be commutative, provided $\sigma \neq 0$. Motivated by above results, Boua and

Oukhtite[6] prove that a prime near ring Ω with a derivation $\sigma$is a commutative ring if one of the conditions holds: $\sigma([\lambda,\mu]) = \pm[\lambda,\mu]$ or $\sigma(\lambda o\mu) = \pm(\lambda o\mu)$forall$\lambda,\mu\in\Omega$.

Recently, Shang [29] considered the more general situations (i) $\tau([\lambda,\mu]) \pm \lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$; (ii) $\tau(\lambda o\mu) = \pm\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, for all$\lambda,\mu\in\Omega$; $\alpha \geq 0$, $\beta \geq 0$non negative integers and proved that the prime near ring Ω is a commutative ring if it satisfies one of the above conditions.

More recently, Miyan et. al [26] established the following results: A prime near ring Ωequipped with a generalized derivationτassociated with a nonzero derivation σ is a commutative ring if it satisfies any one of the following conditions:

(i) $[\tau(\lambda),\mu] = \pm\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, (ii)$[\lambda,\tau(\mu)] = \pm\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, (iii) $\tau(\lambda)\circ\mu = \pm\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$,(iv) $\lambda\circ\tau(\mu) = \pm\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$,(v) $\tau(\lambda)\circ\mu = \pm\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, (vi)$[\lambda,\tau(\mu)] = \pm\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$, (vii)$[\tau(\lambda),\mu] = \pm\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$and (viii) $\lambda\circ\tau(\mu) = \pm\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, for all $\lambda,\mu\in\Omega$ and for some nonnegative integers α and β.

In this line of investigation, we prove that, let Ω be a prime near ring and $\mathbb{N}$ be the set of nonnegative integers. If Ωadmits a generalized derivation τassociated with a nonzero derivation σ satisfying any one of the following conditions:

(i) $\tau([\lambda,\mu]) = \pm\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, (ii) $\tau(\lambda o\mu) = \pm\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$, (iii)$\tau([\lambda,\mu]) = \pm\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, (iv) $\tau(\lambda o\mu) = \pm\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$ and (v) $\tau([\lambda,\mu]) = \pm\lambda^{\alpha}(\lambda o\mu)\mu^{\beta}$

for all $\lambda,\mu\in\Omega$ and$\alpha,\beta\in\mathbb{N}$, the set of nonnegative integers, then Ω is a commutative ring.

2. Preliminaries

For developing the proof of our theorems, we shall need the following lemmas. These results appear in the case of left near rings and so it is easy to observe that they also hold for right near ring as well.

Lemma2.1.[10, Lemma 1.2]LetΩbeaprimenearring.

(i) If$\lambda\in Z(\Omega)/\{0\}$,then$\lambda$isnotazerodivisor.

(ii) If$Z(\Omega)/\{0\}$containsanelementλsuchthat$\lambda+\lambda\in Z(\Omega)$,then$(\Omega,+)$isabelian.

(iii) If $\lambda\in Z(\Omega)/\{0\}$, and μ is an element of Ω such that $\mu\lambda\in Z(\Omega)$, then $\mu\in Z(\Omega)$.

Lemma 2.2. [10, Theorem 2.1] If a prime near ring Ωadmits a nonzero derivation $\sigma$for which$\sigma(\Omega)\subseteq Z(\Omega)$, then $\Omega$is a commutative ring.

Lemma 2.3. [26, Lemma 2.2] LetΩ be a near ring admitting a generalized derivation τ associated with a derivation σ. Then

(i) $\lambda\big(\tau(\mu)\xi + \mu\sigma(\xi)\big) = \lambda\tau(\mu)\xi + \lambda\mu\sigma(\xi)$for all $\lambda,\mu,\xi\in\Omega$,

(ii) $\lambda(\mu\sigma(\xi) + \tau(\mu)\xi) = \lambda\mu\sigma(\xi) + \lambda\tau(\mu)\xi$ for all $\lambda,\mu,\xi\in\Omega$.

3. Main Results

Let$\mathbb{N}$ be the set of nonnegative integers throughout in this section.

Theorem 3.1. Let Ωbe a prime near ring. If there exist $\alpha,\beta\in\mathbb{N}$ such that Ω admits a right generalized derivationτassociated with a nonzero derivation σ satisfying either

(i) $\tau([\lambda,\mu]) = \lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, or

(ii) $\tau([\lambda,\mu]) = -\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$, for all $\lambda,\mu\in\Omega$, then $\Omega$is acommutativering.

Proof. (i) Suppose

$\tau([\lambda,\mu]) = \lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$ for all $\lambda,\mu\in\Omega$(3.1)

Replacing$\mu$by $\mu\lambda$in (3.1), we find that

$\tau([\lambda,\mu\lambda]) = \lambda^{\alpha}(\lambda o\mu\lambda)\lambda^{\beta}$

$\tau([\lambda,\mu]\lambda) = \lambda^{\alpha}(\lambda o\mu)\lambda^{\beta+1}$ which implies that

$\tau([\lambda,\mu])\lambda + [\lambda,\mu]\sigma(\lambda) = \lambda^{\alpha}(\lambda o\mu)\lambda^{(\beta+1)}$for all$\lambda,\mu\in\Omega$

Applying (3.1) in the above expression, we arrive at

$[\lambda,\mu]\sigma(\lambda) = 0$ for all $\lambda,\mu\in\Omega$.(3.2)

Now replacing $\mu$by$\mu\omega$, for any$\omega\in\Omega$in(3.2)and using(3.2) again, we obtain$[\lambda,\mu]\omega\,\sigma(\lambda) = \{0\}$for all $\lambda,\mu\in\Omega$. This meansthat$[\lambda,\mu]\Omega\sigma(\lambda) = \{0\}$for all$\lambda,\mu\in\Omega$. Since Ω is a prime nearring, then either $[\lambda,\mu] = 0$ or$\sigma(\lambda) = 0$ for all $\lambda,\mu\in\Omega$ .But σ is a nonzero derivation on Ω, then we get $[\lambda,\mu] = 0$forall$\lambda,\mu\in\Omega$. This imply that $\lambda\in Z(\Omega)$, for all$\lambda\in\Omega$. (3.3)

By a one-line calculation, we know that if $\lambda\in Z(\Omega)$, then $\sigma(\lambda)\in Z(\Omega)$. Hence (3.3) forces that for all $\lambda\in\Omega$, then $\sigma(\lambda)\in Z(\Omega)$, i.e., $\sigma(\Omega)\subseteq Z(\Omega)$. It then follows from

Lemma 2.2 that Ω is a commutative ring.
(ii) By hypothesis,
$\tau([\lambda,\mu]) = - \lambda^{\alpha}(\lambda o\mu)\lambda^{\beta}$ forall$\lambda,\mu \in \Omega$.(3.4)
Replacing$\mu$by $\mu\lambda$in (3.4), we find that
$\tau([\lambda,\mu\lambda]) = -\lambda^{\alpha}(\lambda o\mu\lambda)\lambda^{\beta}$
$\tau([\lambda,\mu]\lambda) = -\lambda^{\alpha}(\lambda o\mu)\lambda^{\beta+1}$ which implies that
$\tau([\lambda,\mu])\lambda + [\lambda,\mu]\sigma(\lambda) = -\lambda^{\alpha}(\lambda o\mu)\lambda^{(\beta+1)}$for all$\lambda,\mu \in \Omega$
Applying (3.4) in the above expression, we arrive at
$[\lambda,\mu]\sigma(\lambda) = 0$ for all $\lambda,\mu \in \Omega$. (3.5)
Equation (3.5) is the same as equation (3.2).Now arguing in the similar manner, as we have done in case of (i), we obtain the result.

Theorem 3.2. Let Ωbe a prime near ring. If there exist $\alpha, \beta \in \mathbb{N}$ such that Ω admits a right generalized derivation τ associated with an on zero derivation σ satisfying either
(i) $\tau(\lambda o\mu) = \lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$ or
(ii) $\tau(\lambda o\mu) = -\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$, for all $\lambda,\mu \in \Omega$, then Ω is a commutative ring.

Proof. (i) Suppose
$\tau(\lambda o\mu) = \lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$ for all $\lambda,\mu \in \Omega$(3.6)
Replacing $\mu$by $\mu\lambda$in (3.6), we find that
$\tau\big((\lambda o\mu)\lambda\big) = \lambda^{\alpha}[\lambda,\mu]\lambda^{\beta+1}$ which gives
$\tau(\lambda o\mu)\lambda + (\lambda o\mu)\sigma(\lambda) = \lambda^{\alpha}[\lambda,\mu]\lambda^{\beta+1}$for all $\lambda,\mu \in \Omega$ (3.7)
Using(3.6) in (3.7), weobtain
$(\lambda o\mu)\sigma(\lambda) = 0$forall$\lambda,\mu \in \Omega$. (3.8)
This implies that
$\lambda\mu\sigma(\lambda) = -\mu\lambda\sigma(\lambda)$forall$\lambda,\mu \in \Omega$
Substituting $\omega\mu$, where$\omega \in \Omega$in place ofμin the last expression and using it again, we get
$\lambda\omega\mu\sigma(\lambda) = (-\omega)(-\lambda)\mu\sigma(\lambda)$
which implies that
$(\lambda\,\omega - (-\omega(-\lambda)))\mu\sigma(\lambda) = 0$forall$\lambda,\mu,\omega \in \Omega$. (3.9)
Replcing$\lambda$by$-\lambda$ in (3.9), we get
$((-\lambda)\omega - (-\omega)(\lambda))\mu\sigma(-\lambda) = 0$
$((-\lambda)\omega + \omega\lambda))\mu\sigma(-\lambda) = 0$forall$\lambda,\mu,\omega \in \Omega$.
Which implies that $[\omega,\lambda]\mu\sigma(-\lambda) = 0$ forall$\lambda,\mu,\omega \in \Omega$,
i.e., $[\lambda,\omega]\,\Omega\,\sigma(\lambda) = \{0\}$, forall$\lambda,\omega \in \Omega$.
By virtue of the primeness of Ω, we have that for each $\lambda \in \Omega$,
$\sigma(\lambda) = 0$ or $\lambda \in Z(\Omega)$.
But σ is a nonzero derivation on Ω, then we get
$\lambda \in Z(\Omega)$, for all $\lambda \in \Omega$. (3.10)
Since equation (3.10) is the same as equation (3.3), arguing as in the proof of Theorem 3.1 (i) we obtain that $\Omega$is a commutative ring.
(ii) By hypothesis,
$\tau(\lambda o\mu) = -\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta}$, for all $\lambda,\mu \in \Omega$.(3.11)
Replacing $\mu$by $\mu\lambda$in (3.11), we find that
$\tau\big((\lambda o\mu)\mu\big) = -\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta+1}$ which gives
$\tau(\lambda o\mu)\lambda + (\lambda o\mu)\sigma(\lambda) = -\lambda^{\alpha}[\lambda,\mu]\lambda^{\beta+1}$forall$\lambda,\mu \in \Omega$.(3.12)
Using(3.11) in (3.12), we obtain
$(\lambda o\mu)\sigma(\lambda) = 0$forall$\lambda,\mu \in \Omega$. (3.13)
Equation (3.13) is the same as equation (3.8). Now arguing in the similar manner, as we have done in case of (i), we obtain the result.

Theorem 3.3. Let Ωbe a prime near ring. If there exist $\alpha, \beta \in \mathbb{N}$ such thatΩ admits a right generalized derivation τassociated with a nonzero derivation σ satisfying either
(i) $\tau([\lambda,\mu]) = \mu^{\alpha}(\lambda o\mu)\mu^{\beta}$ or
(ii) $\tau([\lambda,\mu]) = -\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, for all $\lambda,\mu \in \Omega$, then $\Omega$is a commutative ring.

Proof. (i) Suppose
$\tau([\lambda,\mu]) = \mu^{\alpha}(\lambda o\mu)\mu^{\beta}$for all $\lambda,\mu \in \Omega$.(3.14)

Substituting $\lambda\mu$ for λin (3.14), we get
$\tau([\lambda,\mu]\mu) = \mu^{\alpha}(\lambda o\mu)\mu^{\beta+1}$for all $\lambda,\mu \in \Omega$
Applying the definition of τ, we find that
$\tau[\lambda,\mu]\,\mu$+$[\lambda,\mu]\,\sigma(\mu)$=$\mu^{\alpha}(\lambda o\mu)\mu^{\beta+1}$for all $\lambda,\mu \in \Omega$. (3.15)
Using (3.14) in (3.15), we obtain
$[\lambda,\mu]\,\sigma(\mu)$=0 for all $\lambda,\mu \in \Omega$. (3.16)
This can be written as
$\lambda\mu\,\sigma(\mu) = \mu\,\lambda\,\sigma(\mu)$for all $\lambda,\mu \in \Omega$
Now replacing $\lambda$by$\omega\lambda$in the above expression and using it again, we arrive at
$[\mu,\omega]\,\Omega\,\sigma(\mu)$=0 for all $\mu,\omega \in \Omega$.
By virtue of the primeness of Ω, we have that for each$\mu \in \Omega$,
$\sigma(\mu) = 0$ or $\mu \in Z(\Omega)$.
But σ is a nonzero derivation on Ω, then we get
$\mu \in Z(\Omega)$, for all$\mu \in \Omega$. (3.17)
Since equation (3.17) is the same as equation (3.3), arguing as in the proof of Theorem 3.1 (i) we obtain thatΩis a commutative ring.
(ii) By hypothesis,
$\tau([\lambda,\mu]) = -\mu^{\alpha}(\lambda o\mu)\mu^{\beta}$, for all $\lambda,\mu \in \Omega$. (3.18)
Substituting$\lambda\mu$for λin (3.18), we get
$\tau([\lambda,\mu]\mu) = -\mu^{\alpha}(\lambda o\mu)\mu^{\beta+1}$for all $\lambda,\mu \in \Omega$
Applying the definition of τ, we find that
$\tau[\lambda,\mu]\,\mu$+$[\lambda,\mu]\,\sigma(\mu)$=$-\mu^{\alpha}(\lambda o\mu)\mu^{\beta+1}$for all $\lambda,\mu \in \Omega$. (3.19)
Using (3.19) in (3.18), we obtain
$[\lambda,\mu]\,\sigma(\mu)$=0 for all $\lambda,\mu \in \Omega$. (3.20)
Equation (3.20) is the same as equation (3.16). Now arguing in the similar manner, as we have done in case of (i), we obtain the result.

Theorem 3.4. Let Ωbe a prime near ring. If there exist α, $\beta \in \mathbb{N}$ such that Ω admits a right generalized derivationτ associated with a non zero derivation σ satisfying either
(i) $\tau(\lambda o\mu) = \mu^{\alpha}[\lambda,\mu]\mu^{\beta}$ or
(ii) $\tau(\lambda o\mu) = -\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$, for all $\lambda,\mu \in \Omega$, then Ω is a commutative ring.
Proof.(i) Suppose
$\tau(\lambda o\mu) = \mu^{\alpha}[\lambda,\mu]\mu^{\beta}$ for all $\lambda,\mu \in \Omega$ (3.21)
Replacing $\lambda$by $\lambda\,\mu$ in (3.21), we find that
$\tau((\lambda o\mu)\mu) = \mu^{\alpha}[\lambda,\mu]\mu^{\beta+1}$which gives
$\tau(\lambda o\mu)\mu + (\lambda o\mu)\sigma(\mu) = \mu^{\alpha}[\lambda,\mu]\mu^{\beta+1}$forall$\lambda,\mu \in \Omega$.(3.22)
Using(3.21) in (3.22), we obtain
$(\lambda o\mu)\sigma(\mu) = 0$for all$\lambda,\mu \in \Omega$. (3.23)
Which implies that
$\lambda\mu\sigma(\mu)$=$-\mu\lambda\,\sigma(\mu)$for all$\lambda,\mu \in \Omega$. (3.24)
Replacing$\lambda$by$\omega\lambda$in (3.24) and using (3.24), we arrive at
$(\omega(-\mu) + \mu\,\omega)\,\lambda\,\sigma(\mu)$=0, for all$\lambda,\mu,\omega \in \Omega$.
Substituting$-\mu$ for μ in the last expression, we obtain
$[\omega,\mu]\,\lambda\,\sigma(-\mu)$=0, forall$\lambda,\mu,\omega \in \Omega$,
i.e., $[\mu,\omega]\Omega\sigma(\mu)$={0}, for all$\mu,\omega \in \Omega$
By virtue of the primeness of Ω, we have that for each$\mu \in \Omega$,
$\sigma(\mu) = 0$ or $\mu \in Z(\Omega)$.
But σ is a nonzero derivation on Ω, then we get
$\mu \in Z(\Omega)$, for all$\mu \in \Omega$. (3.25)
Since equation (3.25) is the same as equation (3.3), arguing as in the proof of Theorem 3.1 (i) we obtain thatΩis a commutative ring.
(ii) By hypothesis,
$\tau(\lambda o\mu) = -\mu^{\alpha}[\lambda,\mu]\mu^{\beta}$, for all $\lambda,\mu \in \Omega$. (3.26)
Replacing $\lambda$by $\lambda\,\mu$ in (3.26), we find that
$\tau((\lambda o\mu)\mu) = -\mu^{\alpha}[\lambda,\mu]\mu^{\beta+1}$ which gives
$\tau(\lambda o\mu)\mu + (\lambda o\mu)\sigma(\mu) = -\mu^{\alpha}[\lambda,\mu]\mu^{\beta+1}$forall$\lambda,\mu \in \Omega$.(3.27)
Using(3.26) in (3.27), we obtain
$(\lambda o\mu)\sigma(\mu) = 0$for all$\lambda,\mu \in \Omega$.(3.28)

Equation (3.28) is the same as equation (3.23). Now arguing in the similar manner, as we have done in case of (i), we obtain the result.

Theorem 3.5. Let Ωbe a prime near ring. If there exist $\alpha, \beta \in \mathbb{N}$ such that Ω admits a right generalized derivation τ associated with a non zero derivation σ satisfying either
(i) $\tau([\lambda, \mu]) = \lambda^{\alpha}(\lambda o \mu)\mu^{\beta}$ or
(ii) $\tau([\lambda, \mu]) = -\lambda^{\alpha}(\lambda o \mu)\mu^{\beta}$, for all $\lambda, \mu \in \Omega$, then Ω is a commutative ring.
Proof.(i) Suppose
$\tau([\lambda, \mu]) = \lambda^{\alpha}(\lambda o \mu)\mu^{\beta}$ forall$\lambda, \mu \in \Omega$. (3.29)
Substituting$\lambda\mu$ for λin (3.29), then we find that
$\tau([\lambda, \mu]\mu)=\lambda^{\alpha}(\lambda o \mu)\mu^{\beta+1}$
which implies that
$\tau[\lambda, \mu]\mu+[\lambda, \mu]\sigma(\mu)= \lambda^{\alpha}(\lambda o \mu)\mu^{\beta+1}$for all$\lambda, \mu \in \Omega$. (3.30)
Using (3.29) in (3.30), we arrive at
$[\lambda, \mu]\sigma(\mu) = 0$for all$\lambda, \mu \in \Omega$. (3.31)
This can be written as
$\lambda\mu\, \sigma(\mu) = \mu\, \lambda\, \sigma(\mu)$for all $\lambda, \mu \in \Omega$.
Now replacing λ by $\omega\lambda$ in the above expression and using it again, we arrive at
$[\mu,\omega]\, \Omega\, \sigma(\mu)=0$ for all $\mu, \omega \in \Omega$.
By virtue of the primenessof Ω, we have that for each$\mu \in \Omega$,
$\sigma(\mu) = 0$ or $\mu \in Z(\Omega)$.
But σ is a nonzero derivation on Ω, then we get
$\mu \in Z(\Omega)$, for all$\mu \in \Omega$. (3.32)
Since equation (3.32) is the same as equation (3.3), arguing as in the proof of Theorem 3.1 (i), we obtain that$\Omega$is a commutative ring.
(ii) By hypothesis,
$\tau([\lambda, \mu]) = -\lambda^{\alpha}(\lambda o \mu)\mu^{\beta}$, for all $\lambda, \mu \in \Omega$. (3.33)
Substituting$\lambda\mu$ for λ in (3.32), then we find that
$\tau([\lambda, \mu]\mu)=-\lambda^{\alpha}(\lambda o \mu)\mu^{\beta+1}$forall$\lambda, \mu \in \Omega$.
Which implies that
$\tau[\lambda, \mu]\mu+[\lambda, \mu]\sigma(\mu)=-\lambda^{\alpha}(\lambda o \mu)\mu^{\beta+1}$forall$\lambda, \mu \in \Omega$. (3.34)
Using (3.33) in (3.34), we arrive at
$[\lambda, \mu]\sigma(\mu) = 0$forall$\lambda, \mu \in \Omega$. (3.35)
Equation (3.35) is the same as equation (3.31). Now arguing in the similar manner, as we have done in case of (i), we obtain the result.
The following example shows that the primness hypothesis in Theorems 3.1, 3.2, 3.3, 3.4 and3.5 cannot be omitted.

Example 3.1. Suppose that Υ is a zero-symmetric right near ring. Let us consider

$$\Omega=\left\{\begin{pmatrix} 0 & \lambda & \mu \\ 0 & 0 & 0 \\ 0 & \xi & 0 \end{pmatrix} \mid 0, \lambda, \mu, \xi \in \Upsilon\right\}.$$

It is easy to verify thatΩ is a non-prime zero-symmetric right near ring with respect to matrix addition and matrix multiplication.
Define mappings $\tau, \sigma: \Omega \rightarrow \Omega$ by

$$\tau\begin{pmatrix} 0 & \lambda & \mu \\ 0 & 0 & 0 \\ 0 & \xi & 0 \end{pmatrix}=\begin{pmatrix} 0 & \lambda & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{and} \sigma\begin{pmatrix} 0 & \lambda & \mu \\ 0 & 0 & 0 \\ 0 & \xi & 0 \end{pmatrix}=\begin{pmatrix} 0 & \lambda & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Then $\tau$is a nonzero generalized derivation associated with a nonzero derivation σ onΩ satisfying
(i) $\tau([\lambda, \mu]) = \pm\lambda^{\alpha}(\lambda o \mu)\lambda^{\beta}$, (ii) $\tau(\lambda o \mu) = \pm\lambda^{\alpha}[\lambda, \mu]\lambda^{\beta}$, (iii) $\tau([\lambda, \mu]) = \pm\mu^{\alpha}(\lambda o \mu)\mu^{\beta}$, (iv) $\tau(\lambda o \mu) = \pm\mu^{\alpha}[\lambda, \mu]\mu^{\beta}$ and (v) $\tau([\lambda, \mu]) = \pm\lambda^{\alpha}(\lambda o \mu)\mu^{\beta}$
for all $\lambda, \mu \in \Omega$ and$\alpha, \beta \in \mathbb{N}$, the set of nonnegative integers.
However, Ω is not commutative.

4.Concluding Remarks

In this paper, the structure of near rings involving generalized derivations satisfying some identities has been studied. We proved commutativity of prime near rings with identities on generalized derivations. This work can be further studied by considering generalized semiderivations and multiplicative generalized derivations on prime near rings and semiprime near rings along with examples that illustrates the necessity of the assumptions used which is left for future work.

CONFLICTS OF INTEREST
The authors declare that there is no conflict of interest.

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