Non Markovian retrial queue, balking, disaster under working breakdown and working vacation

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Abstract

Any arriving customer who arrives and finds that the server is free, enters the service station and the remaining customers connect into the orbit. When the normal busy server is running, the system may at any time become defective due to a disaster. All users are forced to quit the system due to a disaster, which also brings about the failure of the main server. When a primary server breaks, it is shipped out for repair, and the repair process starts instantly. The server stops running as soon as the orbit is empty at a typical service finish instant. During the working breakdown or working vacation, the replacement server offers arriving customers a lower level of service. The arriving customer receives service instantly if the server is idle. If not, he will choose whether to leave the system without service or returning to receive service. Using the supplementary variable technique, we calculate the steady state PGF for system and orbit sizes. We generate performance measures and particular cases. With the use of specific numerical examples, we analyse the model.
Keywords: Retrial queue, balking, disaster, working breakdown, working vacation.

Mathematics Subject Classification 2010: 60K25, 90B22

1 Introduction

Previously, various authors investigated queueing models with varying service rates. These models drive almost made the assistance rate subject to the framework's circumstance, like lines in irregular conditions, lines with breakdown, and working breakdown. Retrial lines with repeated tasks are distinguished in a retrial queueing system by the fact that an arriving customer sees the server busy upon arrival and is encouraged to leave the support area and join a retry line

1

known as orbit. After a specific measure of time has elapsed, the client in orbit might make another assistance demand. It makes no difference to the other customers in the orbit if any random customer in the orbit repeats the service request. Such queues assume a novel part in PC and broadcast communications frameworks. Rajadurai [8,9], estimated a Non-Markovian retrial queue including calamity and working breakdown. Kalidass and Ramanath (2012) pioneered "The concept of working breakdowns". If a regular busy server fails due to a disaster at any time, the system ought to be ready with a reinforcement (reserve) server in the event that the primary server falls flat. It makes no difference to the other customers in the orbit if any random customer in the orbit repeats the service request. The main server rejoins the system and becomes operational as soon as the repair is fulfilled. Furthermore, the operational breakdown service can reduce customer complaints as the principal server is being repaired, as well as the cost of customers who are waiting. As a result, a more sensible repair strategy for problematic queueing framework is the working breakdown service. Rajadurai et al [10], considered inconsistent queueing frameworks with different highlights, one of which is that when a server falls flat, it is sent for fix, during which time it stops offering support to essential clients until the assistance channel is fixed, and the client who was simply being served before the server disappointment trusts that the leftover help will finish.

2 Model Description

In this model the arrival follows Poisson process with rate λ and the service discipline is FIFO. Since there is no waiting area, this is assumed. When a customer arrives and determines that the server is busy, they are joined to the orbit. If an orbital customer is permitted access to the server. Laplace-Stieltjes Transforms (LST) represent inter retrial times as $\Upsilon^*(\theta)$ and have an arbitrary distribution function $\Upsilon(t)$. In normal service period (NS period), service time have general distribution function S(t), with LST as $S^*(\theta)$. We assume that the disaster occur only when the main service is in progress and disaster follows a negative exponential distribution with rate δ. When the disaster occurs all customers are clear out and the primary server is dispatched for maintenance. The repair time follows an exponential distribution with parameter η. The server gives a lower rate of service follows an arbitrary distribution function $S_w(t)$ to arriving customers during the working breakdown period, with LST as $S_w^*(\theta)$. The server resumes normal operation after the repair is finished. As soon as the service is finished and the orbit is empty, the server goes on vacation. The duration of the vacation period is determined by an exponential distribution with the parameter θ. If there are still users in the system at the time the vacation ends, the server will begin a new busy period. Otherwise, he awaits the arrival of a customer. The server gives a lesser rate of service follows an arbitrary distribution function $S_w(t)$ to arriving customers during the working vacation period, with LST as $S_w^*(\theta)$. A vacation interruption occurred if the server quits

his vacation to return to the normal busy period after discovering that there is a customer in the orbit. Working breakdown and working vacation are both regarded as low service in this situation (LS period). If the server is idle, the customer arrives and gets served instantly. If not, he will choose whether to leave the system with probability $(1-r)$ or joining the orbit with probability r.
Let $\Upsilon^0(t)$ denotes the elapsed retrial time, $S^0(t)$ denotes the elapsed service time in NS period , $S_w^0(t)$ denotes the elapsed service time in LS period .
Let $F(t)$ denotes the size of the orbit at time "t". and we use the subsequent random variable as follows.
Let's use the subsequent random variables.
$F(t)$- Size of the orbit at time "t".
At time "t" the four distinct states of the server are

$$\Theta(t) = \begin{cases} 0, & \text{if the server is idle in LS period} \\ 1, & \text{if the server is idle in NS period} \\ 2, & \text{if the server is busy in LS period} \\ 3, & \text{if the server is busy in NS period} \end{cases}$$

o generate bivariate Markov Process, $\{(F(t), \Theta(t)); t \geq 0\}$ further supplementary variables $\Upsilon^0(t)$, $S^0(t)$, and $S_w^0(t)$ are introduced. The sequence of periods at which a NS or LS periods completion occurs is $\{t_m, m = 1, 2, 3, ...\}$. The Markov chain that is formed by the random vector sequences $Z_m = \{F(t_m+), \Theta(t_m+)\}$ is incorporated into the retrial queueing system. The concerned embedded Markov chain is ergodic if and only if $\rho < \Upsilon^*(\lambda)\big[$ See Sennott et al.,[12]$\big]$ where $\rho = \dfrac{\lambda r}{\delta}(1 - S^*(\delta))$ pretaining to our model.
Following are the limiting probabilities

$$\begin{aligned} \Omega_{0,1} &= \lim_{t\to\infty} P\{\Theta(t) = 1,\ F(t)\ = 0\}, \\ \Omega_{0,2} &= \lim_{t\to\infty} P\{\Theta(t) = 0,\ F(t)\ = 0\}, \\ \Upsilon_m(x) &= \lim_{t\to\infty} P\{\Theta(t) = 1,\ F(t)\ = m,\ x \leq \Upsilon^0(t) < x + dx\}, \\ &\qquad x \geq 0,\ m \geq 1 \\ \Omega_{m,1}(x) &= \lim_{t\to\infty} P\{\Theta(t) = 3,\ F(t)\ = m,\ x \leq S^0(t) < x + dx\}; \\ &\qquad x \geq 0,\ m \geq 0, \end{aligned}$$

$$\begin{aligned} \Omega_{m,2}(x) &= \lim_{t\to\infty} P\{\Theta(t) = 2,\ F(t)\ = m,\ x \leq S_w^0(t) < x + dx\}; \\ &\qquad x \geq 0,\ m \geq 0. \end{aligned}$$

Following are the probability generating function

$$\Upsilon(z,x)=\sum_{m=1}^{\infty}\Upsilon_m(x)z^m; \qquad \Upsilon(z,0)=\sum_{m=1}^{\infty}\Upsilon_m(0)z^m;$$
$$\Upsilon^*(\theta)=\int_0^{\infty}e^{-\theta x}r(x)dx; \qquad \Omega_1(z,x)=\sum_{m=0}^{\infty}\Omega_{m,1}(x)z^m;$$
$$\Omega_1(z,0)=\sum_{m=0}^{\infty}\Omega_{m,1}(0)z^m; \qquad S^*(\theta)=\int_0^{\infty}e^{-\theta x}\mu(x)dx;$$
$$\Omega_2(z,x)=\sum_{m=0}^{\infty}\Omega_{m,2}(x)z^m; \qquad \Omega_2(z,0)=\sum_{m=0}^{\infty}\Omega_{m,2}(0)z^m;$$
$$S_w^*(\theta)=\int_0^{\infty}e^{-\theta x}\mu_w(x)dx;$$

We are using the following hazard rate functions. Let $r(x)$ denotes the conditional retrial completion rate of $\Upsilon(x)$

and $r(x)dx=\dfrac{d\Upsilon(x)}{1-\Upsilon(x)}$.
Let $\mu(x)$ denotes the conditional normal service completion rate of $S(x)$

and $\mu(x)dx=\dfrac{dS(x)}{1-S(x)}$.
Let $\mu_w(x)$ denotes the conditional lower service completion rate of $S_w(x)$

and $\mu_w(x)dx=\dfrac{dS_w(x)}{1-S_w(x)}$.
The system was demonstrated in steady state by the following differential difference equations:

$$\lambda\Omega_{0,1} = (\theta+\eta)\Omega_{0,2}, \tag{1}$$
$$(\lambda+\theta+\eta)\Omega_{0,2} = \int_0^{\infty}\Omega_{0,1}(x)\mu(x)dx+\int_0^{\infty}\Omega_{0,2}(x)\mu_w(x)dx +\delta\int_0^{\infty}\Omega_{m,1}(x)dx, m\geq 0, \tag{2}$$
$$\frac{d\Upsilon_m(x)}{dx} = -(\lambda+r(x))\Upsilon_m(x), m\geq 1, \tag{3}$$
$$\frac{d\Omega_{0,1}(x)}{dx} = -(\lambda+\delta+\mu(x))\Omega_{0,1}(x)+\lambda(1-r)\Omega_{0,1}(x), m=0 \tag{4}$$
$$\frac{d\Omega_{m,1}(x)}{dx} = -(\lambda+\delta+\mu(x))\Omega_{m,1}(x)+\lambda(1-r)\Omega_{m,1}(x) +\lambda r\Omega_{m-1,1}(x), m\geq 1, \tag{5}$$

4

$$\frac{d\Omega_{0,2}(x)}{dx} = -(\lambda+\eta+\theta+\mu_w(x))\Omega_{0,2}(x)+\lambda(1-r)\Omega_{0,2}(x), m=0, \quad (6)$$

$$\frac{d\Omega_{m,2}(x)}{dx} = -(\lambda+\eta+\theta+\mu_w(x))\Omega_{m,2}(x)+\lambda(1-r)\Omega_{m,2}(x) +\lambda r\Omega_{m-1,2}(x), m\geq 1. \quad (7)$$

At $x=0$,

$$\Upsilon_m(0) = \int_0^\infty \Omega_{m,1}(x)\mu(x)dx+\int_0^\infty \Omega_{m,2}(x)\mu_w(x)dx,\ m\geq 1, \quad (8)$$

$$\Omega_{0,1}(0) = \int_0^\infty \Upsilon_1(x)r(x)dx+(\theta+\eta)\int_0^\infty \Omega_{0,2}(x)dx+\lambda\Omega_{0,1},\ m=0, \quad (9)$$

$$\Omega_{m,1}(0) = \int_0^\infty \Upsilon_{m+1}(x)r(x)dx+(\theta+\eta)\int_0^\infty \Omega_{m,2}(x)dx +\lambda\int_0^\infty \Upsilon_m(x)dx,\ m\geq 1, \quad (10)$$

$$\Omega_{m,2}(0) = \begin{cases}\lambda\Omega_{0,2}, & m=0,\\ 0, & m\geq 1,\end{cases} \quad (11)$$

The normalizing condition is

$$1 = \Omega_{0,1}+\Omega_{0,2}+\sum_{m=0}^{\infty}\Big[\int_0^\infty \Omega_{m,1}(x)dx+\int_0^\infty \Omega_{m,2}(x)dx\Big] +\sum_{m=1}^{\infty}\int_0^\infty \Upsilon_m(x)dx$$

Multiply the equations (2) - (8) by the proper powers of z

$$\frac{d\Upsilon(z,x)}{dx}+(\lambda+r(x))\Upsilon(z,x) = 0 \quad (12)$$

$$\frac{d\Omega_1(z,x)}{dx}+(\lambda(1-rz)-\lambda(1-r)+\delta+\mu(x))\Omega_1(z,x) = 0 \quad (13)$$

$$\frac{d\Omega_2(z,x)}{dx}+(\lambda(1-rz)-\lambda(1-r)+\theta+\eta+\mu_w(x))\Omega_2(z,x) = 0 \quad (14)$$

$$\Upsilon(z,0) = \int_0^\infty \Omega_1(z,x)\mu(x)dx+\int_0^\infty \Omega_2(z,x)\mu_w(x)dx -\int_0^\infty \Omega_{0,1}(x)\mu(x)dx-\int_0^\infty \Omega_{0,2}(x)\mu_w(x)dx \quad (15)$$

Using the equation (2) in equation (15), we get

$$\Upsilon(z,0) = \int_0^\infty \Omega_1(z,x)\mu(x)dx+\int_0^\infty \Omega_2(z,x)\mu_w(x)dx+\delta\int_0^\infty \Omega_1(z,x)dx -(\lambda+\theta+\eta)\Omega_{0,2} \quad (16)$$

Multiply the equations $(10)-(11)$ by the proper powers of z

$$\begin{aligned}\Omega_1(z,0) &= \frac{1}{z}\int_0^\infty \Upsilon(z,x)r(x)dx + \lambda\int_0^\infty \Upsilon(z,x)dx + \lambda\Omega_{0,1} \\ &\quad +(\eta+\theta)\int_0^\infty \Omega_2(z,x)dx \quad (17)\\ \Omega_2(z,0) &= \lambda\Omega_{0,2} \quad (18)\end{aligned}$$

Solving the first order linear differential equations (13), (14), (15) which yields,

$$\begin{aligned}\Upsilon(z,x) &= \Upsilon(z,0)[1-\Upsilon(x)]e^{-\lambda x} \quad (19)\\ \Omega_1(z,x) &= \Omega_1(z,0)[1-S(x)]e^{-B(z)x} \quad (20)\\ \Omega_2(z,x) &= \Omega_2(z,0)[1-S_w(x)]e^{-B_w(z)x} \quad (21)\end{aligned}$$

where $B(z) = (\lambda r(1-z)+\delta)$, $B_w(z) = (\lambda r(1-z)+\theta+\eta)$.
Substituting the equations (19) and (21)in equation (17), we get

$$\Omega_1(z,0) = \frac{\Upsilon(z,0)}{z}\Big[\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda))\Big] + \lambda\Omega_{0,1} + \lambda\Omega_{0,2}U(z) \quad (22)$$

where, $U(z) = \dfrac{(\eta+\theta)(1-S_w^*(B_w(z))}{B_w(z)}$.
Substituting the equations (20) and (21) in equation (16), we get

$$\Upsilon(z,0) = \Omega_1(z,0)[S^*(B(z))+S(z)] + \Omega_2(z,0)S_w^*(B(z)) - (\lambda+\theta+\eta)\Omega_{0,2}. \quad (23)$$

where $S(z) = \dfrac{\delta(1-S^*(B(z)))}{\delta+\lambda r(1-z)}$
Using equations (18) and (22) in equation (23) and get

$$\begin{aligned}\Upsilon(z,0) &= \frac{z\Omega_{0,2}}{Dr_1(z)}\Big\{[\theta+\eta+\lambda U(z)][S^*(B(z))+S(z)] + \lambda(S_w^*(B_w(z))-1) \\ &\quad -(\theta+\eta)\Big\}, \quad (24)\end{aligned}$$

Substituting the equation (24) in equation (22), we get

$$\begin{aligned}\Omega_1(z,0) &= \frac{\Omega_{0,2}}{Dr_1(z)}\Big\{[\lambda(S_w^*(B_w(z))-1)-(\theta+\eta)][\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda))] \\ &\quad +z[\theta+\eta+\lambda U(z)])\Big\} \quad (25)\end{aligned}$$

where $Dr_1(z) = z - [S^*(B(z))+S(z)][\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda))]$. Using the equations (24), (25) and (18) in equations (19), (20) and (21), then the limiting PGF's are $\Upsilon(z,x)$, $\Omega_1(z,x)$, and $\Omega_2(z,x)$.

6

3 Steady state results

If $\rho < \Upsilon^*(\lambda)$, The PGF's are listed below.
(i) The amount of orbiting customers as the server is not being utilized

$$\begin{aligned} \Upsilon(z) &= \frac{(1-\Upsilon^*(\lambda))}{\lambda Dr_1(z)}\Big\{ z\Omega_{0,2}\big[(\lambda U(z)+\theta+\eta)[S^*(B(z))+S(z)] \\ &\quad +\lambda(S_w^*(B_w(z))-1)-(\theta+\eta)\big]\Big\} \end{aligned} \tag{26}$$

(ii) The amount of orbiting customers as the server is regularly busy

$$\begin{aligned} \Omega_1(z) &= \frac{(1-S^*(B(z)))}{B(z)Dr_1(z)}\Big\{\Omega_{0,2}\big[(\lambda U(z)+\theta+\eta)z+[\lambda(S_w^*(B_w(z))-1) \\ &\quad -(\theta+\eta)][\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda))]\big]\Big\} \end{aligned} \tag{27}$$

(iii) PGF is used to determine the total number of users in orbit ($C_s(z)$).

$$\begin{aligned} C_s(z) &= \Omega_{0,1}+\Omega_{0,2}+\Upsilon(z)+z(\Omega_1(z)+\Omega_2(z)), \\ C_s(z) &= \frac{\Omega_{0,2}}{Dr_1(z)}\Big\{B(z)\Big(z-(S^*(B(z))+S(z))(\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda)))\Big) \\ &\quad \times\Big(\frac{\eta+\theta}{\lambda}+1\Big)+\Big((U(z)+\frac{1}{\lambda}(\theta+\eta))(S^*(B(z))+S(z)) \\ &\quad +(S_w^*(B_w(z))-1)-\frac{1}{\lambda}(\theta+\eta)\Big)z(1-\Upsilon^*(\lambda))B(z)+(1-S^*(B(z)) \\ &\quad \times\Big((\lambda U(z)+\theta+\eta)z+(\lambda(S_w^*(B_w(z))-1)-(\theta+\eta))(\Upsilon^*(\lambda) \\ &\quad +z(1-\Upsilon^*(\lambda)))\Big)z+B(z)\Big(z-(S^*(B(z))+S(z))(\Upsilon^*(\lambda) \\ &\quad +z(1-\Upsilon^*(\lambda)))\Big)\frac{\lambda zU(z)}{(\theta+\eta)}\Big\}. \end{aligned}$$

(iv) PGF is used to determine the total number of users in orbit ($C_o(z)$).

$$\begin{aligned} C_o(z) &= \Omega_{0,1}+\Omega_{0,2}+\Upsilon(z)+\Omega_1(z)+\Omega_2(z), \\ C_o(z) &= \frac{\Omega_{0,2}}{Dr_1(z)}\Big\{B(z)\Big(z-(S^*(B(z))+S(z))(\Upsilon^*(\lambda)+z(1-\Upsilon^*(\lambda)))\Big) \\ &\quad \times\Big(\frac{\eta+\theta}{\lambda}+1\Big)+\Big((U(z)+\frac{1}{\lambda}(\theta+\eta))(S^*(B(z))+S(z)) \\ &\quad +(S_w^*(B_w(z))-1)-\frac{1}{\lambda}(\theta+\eta)\Big)z(1-\Upsilon^*(\lambda))B(z)+(1-S^*(B(z)) \\ &\quad \times\Big((\lambda U(z)+\theta+\eta)z+(\lambda(S_w^*(B_w(z))-1)-(\theta+\eta))(\Upsilon^*(\lambda) \\ &\quad +z(1-\Upsilon^*(\lambda)))\Big)+B(z)\Big(z-(S^*(B(z))+S(z))(\Upsilon^*(\lambda) \\ &\quad +z(1-\Upsilon^*(\lambda)))\Big)\frac{\lambda U(z)}{(\theta+\eta)}\Big\}. \end{aligned} \tag{28}$$

(v) The amount of orbiting customers as the server is lower speed service

$$\Omega_2(z) = \frac{\lambda\Omega_{0,2}U(z)}{\theta+\eta} \tag{29}$$

Using normalizing condition , we find $\Omega_{0,1}$, $\Omega_{0,2}$ by putting $z = 1$ and we apply L's hospital rule,
$\Omega_{0,1} + \Omega_{0,2} + \Upsilon(1) + \Omega_1(1) + \Omega_2(1) = 1$,

$$\Omega_{0,2} = \frac{\Upsilon^*(\lambda) - \frac{\lambda r}{\delta}(1 - S^*(\delta))}{\left[\begin{array}{r} \Upsilon^*(\lambda)(\frac{\eta+\theta}{\lambda}+1) + \frac{\lambda r}{\theta+\eta}(1 - S_w^*(\theta+\eta)) \\ +\frac{\lambda}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) + \frac{\eta+\theta}{\delta}\Upsilon^*(\lambda)(1-r) \\ \times(1-S^*(\delta)) - \frac{\lambda r}{\delta}S_w^*(\theta+\eta)(1-S^*(\delta)) - \frac{\lambda}{\delta}S_w^*(\theta+\eta) \\ \times\Upsilon^*(\lambda)(1-r) + \frac{\lambda}{\theta+\eta}\Upsilon^*(\lambda)(1-r)(1-S_w^*(\theta+\eta) \end{array}\right]} \tag{30}$$

$$\Omega_{0,1} = \frac{\Upsilon^*(\lambda) - \frac{\lambda r}{\delta}(1 - S^*(\delta))}{\frac{\lambda}{\eta+\theta}\left[\begin{array}{r} \Upsilon^*(\lambda)(\frac{\eta+\theta}{\lambda}+1) + \frac{\lambda r}{\theta+\eta}(1 - S_w^*(\theta+\eta)) \\ -\frac{\lambda r}{\delta}S_w^*(\theta+\eta)(1-S^*(\delta)) + \frac{\eta+\theta}{\delta}\Upsilon^*(\lambda)(1-r) \\ \times(1-S^*(\delta)) + \frac{\lambda}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) \\ +\frac{\lambda}{\theta+\eta}\Upsilon^*(\lambda)(1-r)(1-S_w^*(\theta+\eta) \\ -\frac{\lambda}{\delta}S_w^*(\theta+\eta)\Upsilon^*(\lambda)(1-r) \end{array}\right]} \tag{31}$$

4 System Performance Measures

When the server is not being utilized,the steady state probability is $\Upsilon(1)$

$$\Upsilon(1) = \frac{(1-\Upsilon^*(\lambda))\Omega_{0,2}\left[\begin{array}{r} (1 - S_w^*(\theta+\eta)[\frac{\lambda r}{\delta}(1-S^*(\delta)) + \frac{\lambda r}{\theta+\eta}] \\ +(\frac{(\eta+\theta)r}{\delta})(1-S^*(\delta) \end{array}\right]}{\Upsilon^*(\lambda) - \frac{\lambda r}{\delta}(1-S^*(\delta)} \tag{32}$$

When the server is busy, let $\Omega_1(1)$ be the steady state probability.

$$\Omega_1(1) = \frac{(1-S^*(\delta))\Omega_{0,2}\left[\begin{array}{r}(\eta+\theta)\Upsilon^*(\lambda) - \Upsilon^*(\lambda)(\lambda(S_w^*(\theta+\eta)-1)) \\ +\dfrac{\lambda^2 r}{\theta+\eta}(1-S_w^*(\theta+\eta))\end{array}\right]}{\delta[\Upsilon^*(\lambda) - \dfrac{\lambda r}{\delta}(1-S^*(\delta)]} \tag{33}$$

When the server is providing slower service, let $\Omega_2(1)$ be the steady state probability.

$$\Omega_2(1) = \frac{\lambda\Omega_{0,2}(1-S_w^*(\theta+\eta))}{\theta+\eta} \tag{34}$$

The busy cycle and busy period's expected durations are $E(T_b)$ and $E(T_c)$. Then

$$\begin{aligned} E(T_b) &= \frac{1}{\lambda}\left[\frac{1}{\Omega_{0,1}} - 1\right], \\ E(T_c) &= \frac{1}{\lambda\Omega_{0,1}}, \\ E(T_0) &= \frac{1}{\lambda}. \end{aligned}$$

where the duration of the system's empty state is indicated by the time T_0.

$$E(T_b) = \frac{\left[\begin{array}{r}\Upsilon^*(\lambda) + \dfrac{(\eta+\theta)r}{\delta}(1-S^*(\delta)) + \dfrac{\lambda r}{\theta+\eta}(1-S_w^*(\theta+\eta)) \\ +\dfrac{\eta+\theta}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) - \dfrac{\lambda r}{\delta}S_w^*(\theta+\eta) \\ \times(1-S^*(\delta)) + \dfrac{\lambda}{\theta+\eta}\Upsilon^*(\lambda)(1-r)(1-S_w^*(\theta+\eta) \\ +\dfrac{\lambda}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) - \dfrac{\lambda}{\delta}S_w^*(\theta+\eta)\Upsilon^*(\lambda)(1-r)\end{array}\right]}{(\theta+\eta)[\Upsilon^*(\lambda) - \dfrac{\lambda r}{\delta}(1-S^*(\delta))]} \tag{35}$$

$$E(T_c) = \frac{\left[\begin{array}{r}\Upsilon^*(\lambda)(\dfrac{\eta+\theta}{\lambda} + 1) + \dfrac{\lambda r}{\theta+\eta}(1-S_w^*(\theta+\eta)) \\ +\dfrac{\eta+\theta}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) - \dfrac{\lambda r}{\delta}S_w^*(\theta+\eta) \\ \times(1-S^*(\delta)) + \dfrac{\lambda}{\theta+\eta}\Upsilon^*(\lambda)(1-r)(1-S_w^*(\theta+\eta) \\ +\dfrac{\lambda}{\delta}\Upsilon^*(\lambda)(1-r)(1-S^*(\delta)) - \dfrac{\lambda}{\delta}S_w^*(\theta+\eta)\Upsilon^*(\lambda)(1-r)\end{array}\right]}{(\theta+\eta)[\Upsilon^*(\lambda) - \dfrac{\lambda r}{\delta}(1-S^*(\delta))]} \tag{36}$$

5 Particular Cases

Case(i) Assuming that $r = 1$ then our model reduces to a non Markovian retrial queue with single working vacation, vacation interruption, disaster and working breakdown.
Case(ii) Assuming that if there is no disaster, then our model reduces to a non Markovian retrial queue with Balking, single working vacation and vacation interruption.
Case(iii) Assuming that if there is no disaster, $r = 1$, and no vacation, then our model reduces to a non Markovian retrial queue.

6 Numerical results

In Figure 1 displays the appropriate line graphs and Table 1 contains the values of $E(T_b)$ by fixing the values of $\mu = 4$, $\mu_w = 2$, $\lambda = 3$, $\theta = 6$, and $r = 0.5$ subject to stability conditions and extending the value of η from 1 to 2 increased with 0.2 and θ from the graph suggests that $E(T_b)$ decreases as η increases as would be predicted.

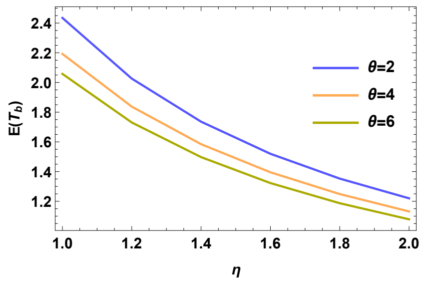

Figure 1: $E(T_b)$ with turn over of η

η	$\theta = 2$	$\theta = 4$	$\theta = 6$
1.0	8.0714	5.1388	4.1767
1.2	6.3738	3.9925	3.2156
1.4	5.3334	3.2963	2.6343
1.6	4.6285	2.8277	2.2444
1.8	4.1185	2.4905	1.9645
2.0	3.7321	2.2361	1.7537

Table 1: $E(T_b)$ with turn over of η

In Figure 2 displays the appropriate line graphs and Table 2 contains the values of $E(T_b)$ subject to stability conditions, by fixing the values of $\mu = 2$, $\mu_w = 1$, $\lambda = 4$, $\theta = 4$, and $r = 0.2$, and extending the values of δ from 1 to 2 increased-with 0.2 and η. The graph suggests that $E(T_b)$ decreases as expected when δ increases.

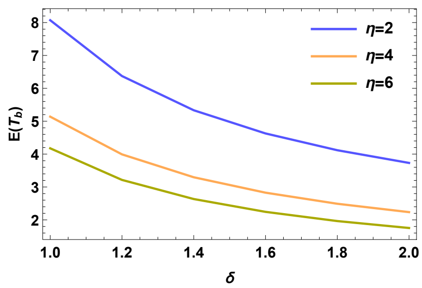

Figure 2: $E(T_b)$ with turn over of δ

δ	$\eta = 2$	$\eta = 4$	$\eta = 6$
1.0	8.0714	5.1388	4.1767
1.2	6.3738	3.9925	3.2156
1.4	5.3334	3.2963	2.6343
1.6	4.6285	2.8277	2.2444
1.8	4.1185	2.4905	1.9645
2.0	3.7321	2.2361	1.7537

Table 2: $E(T_b)$ with turn over of δ

In Figure 3 displays the appropriate line graphs and Table 3 contains the values of $E(T_b)$ by fixing the values of $\mu = 3$, $\mu_w = 2$, $\lambda = 3$, $\theta = 1$, and $r = 0.4$, subject to stability conditions, and extending the values of δ from 1 to 2 incremented with 0.2 and η. From the graph, it can be deduced that $E(T_c)$ decreases as expected when δ increases.

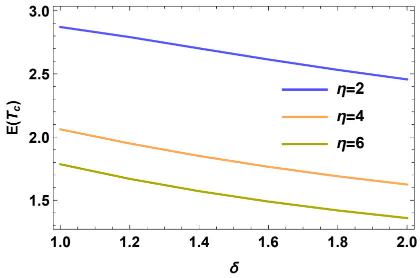

Figure 3: $E(T_c)$ with turn over of δ

δ	$\eta = 2$	$\eta = 4$	$\eta = 6$
1.0	2.8708	2.0610	1.7847
1.2	2.7911	1.9492	1.6691
1.4	2.7016	1.8505	1.5719
1.6	2.6137	1.7646	1.4896
1.8	2.5316	1.6898	1.4194
2.0	2.4564	1.6245	1.3589

Table 3: $E(T_c)$ with turn over of δ

In Figure 4 displays the appropriate line graphs and Table 4 contains the values of $E(T_b)$ subject to stability conditions, by fixing the values of $\mu = 3$, $\mu_w = 2$, $\lambda = 3$, $\theta = 1$, and $r = 0.4$, and extending the values of η from 1 to 2 increased with 0.2 and δ. The graph suggests that $E(T_c)$ decreases as η increases as would be predicted.

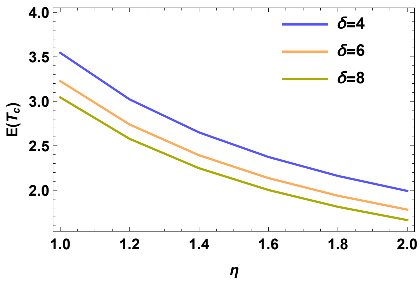

Figure 4: $E(T_c)$ with turn over of η

η	$\delta = 4$	$\delta = 6$	$\delta = 8$
1.0	3.5452	3.2272	3.0440
1.2	3.0212	2.7388	2.5773
1.4	2.6499	2.3931	2.2472
1.6	2.3738	2.1363	2.0022
1.8	2.1609	1.9385	1.8135
2.0	1.9920	1.7818	1.6641

Table 4: $E(T_c)$ with turn over of η

7 Conclusion

In this paper, non Markovian retrial queue, balking, disaster under working breakdown and working vacation is analysed. We discovered the PGF for the total and average number of people in invisible waiting area. We derived some performance measures and deduced some particular cases and illustrated some numerical results.

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