Goal programming approach to solve linear transportation problems with multiple objectives

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December 28, 2022

Abstract

In multi-objective transportation, due to the conflicting nature of objectives, no method is available to find the best compromise optimal solution. In this paper, we present a method to obtain a compromised solution for multi-objective transportation problems under a weighted environment. In which, a modified weighted model is presented that provides us with an efficient solution according to the priorities of the decision maker. To measure the efficiency of the method, a numerical example is included and the results are compared with previously reported work for the same numerical problems to illustrate the feasibility and the applicability of the proposed method.

Keywords- Multi-objective optimization; transportation problem; compromise solution; goal programming

1 Introduction

With the growing population of this competitive world the demand for the goods is growing day by day and to fulfil the demands, the businesses have to outperform themselves every time. Due to this, the management of the business faces a lot of challenges and single objective transportation is not enough to meet the needs of this competitive market. Just Minimizing the transportation cost cannot be the only objective, they must take other factors into consideration and solving such type of problem with multiple objectives which need to be fulfilled simultaneously gives birth to a new branch of transportation problem that we call multi-objective transportation problem (MOTP).

In a classic transportation problem, a product is to be transported from m sources to n destinations and there is a penalty p_{ij} associated with transporting a unit of product. This penalty may be cost or delivery time or safety of delivery, or something else depending upon the decision maker (DM). Over a period of time, many algorithms have been developed to obtain initial basic feasible solutions like the North-west corner rule, least cost method, and Vogel's approximation method. Veena Adlakha and Kowalski [2](1997) proposed a very effective algorithm (Absolute point method) that can be used to directly obtain optimal cost without using the MODI method. These methods are applicable when all the decision parameters are given in a precise way, but as already discussed in real life situations, not all transportation problems are single-objective.

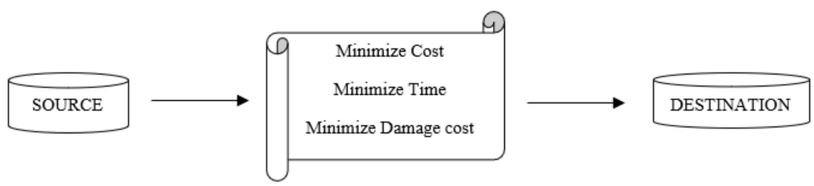
Past many years a lot of work has been done in developing an algorithm to solve multi-objective transportation problems. Every algorithm gives varying results and it is very difficult to say which is the best method to obtain a compromised solution (For multi-objective transportation, a compromised solution is a feasible solution that is favoured more by the DM over all other feasible solutions, taking into consideration all criteria contained in the multi-objective function).The quality of the solution totally depends on the DM.

Lee and Moore [7](1973) inspected the optimization of transportation problems with multiple objectives. Isermann and Diaz [6] (1979) formulated different algorithms for all the non-dominated solutions for linear multi-objective transportation problems. The fuzzy programming technique was applied by Bit, Biswal, and Alam [3] (1992) to solve the multi-objective transportation problem. For the first time in the early 1960s, Charnes and Cooper suggested the concept of goal programming (GP) and a very good literature review was given. It has been found extensive in various fields. Since 1960, numerous works have been done and a lot of applications have been proposed. A review of GP formulations and their applications was given by Lee and Olson [8](1999). Edward L. Hannan [4] (1981) illustrated GP with fuzzy goals having a linear membership function. Zangiabadi and Maleki [13](2013) presented the application of fuzzy goal programming to linear MOTP using a non-linear membership function. Despite its recognition and a huge variety of applications, there's no assurance that GP will offer Pareto an optimal solution.

In multi-objective problems, we can assign different weights to the objective according to the importance of the objective and obtain varying results for different weights assigned by the DM. Due to the overlapping existence of priorities, it is rare to find an optimal solution that optimizes all of them at the same. Here in this paper, we have discussed the weighted sum method and the algorithm proposed by Nomani [10](2016) and a comparison has been made with the proposed model with the help of numerical examples. The proposed model is a new weighted method that helps obtain compromised solutions according to the priorities given by the DM for different goals.

2 Multi-objective linear transportation problem (MOLTP)

In today's aggressive environment, a single-objective transportation assignment is insufficient to deal with all real-life decision-making issues. So, to address all real-life conditions on transportation problems, the DM regularly wishes to consider more than one non-commensurable or conflicting objective in transportation problems. The problem wherein more than one target is optimized concurrently is referred to as a multi-objective transportation problem (MOTP). The reason for defining the multi-objective transportation problem in the mathematical programming framework is to optimize numerous objectives concurrently subject to a set of constraints Other than transportation expense the objectives can include shipping time, degradation of goods, secure shipping of items, energy consumption, etc



The mathematical model of MOTP is written as follows:

$$Min\ F_k(x_ij) = \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^k x_{ij}, \qquad k = 1, 2, \ldots, K$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = s_i, \qquad i = 1, 2, 3, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = dj, \qquad j = 1, 2, 3, \ldots, n$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, \ldots m, and\ \ j = 1, 2, \ldots, n$$

Where m is no. of source, n is no. of destination, d_n is capacity of destination, s_m is capacity of sources, p_{ij}^k is penalty of k^{th} objective, F_k is k^{th} objective and x_{ij} is unknown qty to be shipped.

3 Methods for solving MOLTP

3.1 Weighted sum method

For solving a MOLTP the method of the weighted sum is highly used to obtain varying results for different weights. The basic idea of this method is to assign

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weight $n_k \geq 0$ to each objective function F_k and minimize the new objective function $\sum_{k=1}^{K} n_k F_k$ with respect to problem constraints. This method is very easy to use but the solution majorly depends on the weights given by the DM and it should be decided beforehand. Using the weighted sum method, the following normalized single-objective optimization problem is obtained:

$$Minimize \;\; F = n_1 F_1 + n_2 F_2 + \ldots + n_K F_K$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = s_i, \qquad i = 1, 2, 3, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = dj, \qquad j = 1, 2, 3, \ldots, n$$

$$x_{ij} \geq 0, \quad i = 1, 2, 3, \ldots m, and \;\; j = 1, 2, \ldots, n$$

Where the weights $n_k, k = 1, 2, \ldots K$, corresponding to the objective function satisfy the following conditions $n_1 + n_2 + \ldots + n_k = 1 \; k = 1, 2, \ldots K$
Using the above method, single solution points are obtained for different weights that reflect the preferences of the decision-maker. This method fails when DM have no idea about preference.

3.2 Method proposed by Nomani(2016)

In 2016, Mohammad Asim Nomani, Irfan and Ahmed proposed a model to obtain a compromised solution for a MOLTP. This model focuses on converting multiobjective optimization into a new single objective optimization where the objective is to $minimize \; \mu^{'} = \sum \mu(1 - n_k)$, where μ is the general deviation variable for all objectives and n_k is the weight assigned to the k^{th} objective.

Consider the following multi-objective optimization problem:

$$Minimize \; F(x) = [F_1(x), F_2(x), \ldots, F_K(x)]$$

$$Subject \; to \;\; x \in S$$

Where x is an n-dimensional decision maker variable and S is the set of feasible solutions. Each objective is transformed into constraints with an upper bound of $F_k^* + \mu(1 - n_k)$, where F_k^* is an ideal solution obtained when each objective $F_k, k = 1, 2, \ldots, K$ is solved independently of other objectives.
The problem reduces as:

$$Minimize \; \mu^{'} = \sum_{k=1}^{K} \mu(1 - n_k)$$

Subject to:

$$F_k \leq F_k^* + \mu(1 - n_k); \qquad x_{ij} \geq 0$$

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In this model, instead of using a deviation variable alone, a factor $(1 - n_k)$ has been introduced. This method is capable of providing a solution even if DM has no priority for objectives.

4 Proposed Method

In this section, we will discuss the proposed method and later we will see a comparison between the results obtained by these three methods. Let us consider a multi-objective optimization problem:

$$Minimize\ \ F(x) = [F_1(x), F_2(x), \ldots, F_K(x)]$$

Like Nomani's model, this model also focuses on converting the multi-objective problem into a single objective problem. The main idea is to minimize the deviation of each objective from its ideal solution. To do so a deviation variable μ was introduced.

The model is formulated as:

$$Minimize\ \mu^{'} = \sum_{k=1}^{K} \mu(1 - n_k)$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}^{k} x_{ij} \leq F_k^{*} + \frac{\mu(1 - n_k}{(F_u^k - F_l^k)} \quad k = 1, 2, \ldots, K$$

$$\sum_{j=1}^{n} x_{ij} = s_i, \qquad i = 1, 2, 3, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = dj, \qquad j = 1, 2, 3, \ldots, n$$

$$0 \leq n_k \leq 1, k = 1, 2, \ldots, K$$

$$x_{ij} \geq 0, i = 1, 2, \ldots, m \ \ and\ j = 1, 2, 3, \ldots, n$$

Here in this model a factor $\frac{1}{(F_u^k - F_l^k)}$ is introduced alongside with existing $\mu(1 - n_k)$. F_u^k and F_l^k represent upper and lower bounds in which the compromised solution will lie. The solution cannot exceed this range. For a k^{th} objective, this range can be obtained by using the ideal allocation. For upper bound max (Solution obtained by substituting others allocation in kth objective) and for lower bound the optimal solution of k^{th} objective is it's lower bound and this lower bound is the ideal solution F_k^{*}.

Step by Step method:

Step 1: Solve all the K objectives as a single objective problem without considering other objectives.
Step 2: Obtain the range for every objective as stated above.
Step 3: Now develop a model for the problem as described above and define weights for the objectives if DM has any.
Now simply evaluate and a compromised solution will be obtained.

5 Numerical illustration

The first example that we will be considering is used by many authors and they have obtained different solutions. Ringuest and Rinks [11](1987) used this problem to illustrate the MOLTP. In this paper, we have formulated the problem like a real-life problem to make a better understanding of the problem.

Example 1: Let us consider a problem in which Jethalal wants to transport TVs from its 3 Factories situated in Delhi, Mumbai and Bangalore, to the 4 warehouses at Bhopal, Dehradun, Kolkata and Chennai. The Factory capacity of Delhi is 8 thousand TVs, Mumbai is 19 thousand TVs and Bangalore is 17 thousand TVs. The warehouse requirement at Bhopal is 11 thousand TVs, Dehradun is 3 thousand TVs, Kolkata is 14 thousand TVs and Chennai is 16 thousand TVs. Jethalal wants to minimize the transportation cost as well as the safety cost for the TVs. The cost of transportation and safety per unit is given in the table below (in thousands)

Safety,Transportation	**Bhopal**	**Dehradun**	**Kolkata**	**Chennai**
Delhi	1,4	2,4	7,3	7,4
Mumbai	1,5	9,8	3,9	4,10
Banglore	8,6	9,2	4,5	6,1

Solution The first step is to obtain a solution for both the objectives separately ignoring the other objective. The solution obtained is as follows:
$X^1 = (x_{11} = 5, x_{12} = 3, x_{21} = 6, x_{24} = 13, x_{33} = 14, x_{34} = 3)$
$F_1(X^1) = 143(ideal solution), F_1(X^2) = 208,$
Upper and lower bounds of the objective function F_1 is $143 \leq F_1 \leq 208$
$X^2 = (x_{13} = 8, x_{21} = 11, x_{22} = 2, x_{23} = 6, x_{32} = 1, x_{34} = 16)$
$F_2(X^2) = 167(ideal solution), F_2(X^1) = 265,$
Upper and lower bounds of the objective function F_2 is $167 \leq F_2 \leq 265$
Now since we have the bounds, we can formulate the mathematical model of the problem using the proposed model.

$$Minimize\ \mu' = \mu(1 - n_1) + \mu(1 - n_2)$$

Subject to:

$$\sum_{i=1}^{3}\sum_{j=1}^{4} p_{ij}^1 x_{ij} \leq 143 + \frac{\mu(1 - n_1)}{(208 - 143)}$$

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$$\sum_{i=1}^{3}\sum_{j=1}^{4} p_{ij}^2 x_{ij} \leq 167 + \frac{\mu(1-n_2)}{(265-167)}$$

$$\sum_{j=1}^{4} x_{ij} = s_i, \quad i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = d_j, j = 1,2,3,4$$

$$n_1 + n_2 = 1$$

$$0 \leq n_k \leq 1$$

$$k = 1,2; \quad x_{ij} \geq 0$$

Now simply allot the weight to the objective function and solve the LPP. Make sure the weights are non-negative and their sum is exactly equal to 1. We have used Lingo 19.0 to solve the LLP.

	Weights (n_1, n_2)	**Proposed method**	**Nomani**	**Weighted sum**
1	$n_1 = 0.1, n_2 = 0.9$	197;169	186;171	208;167
2	$n_1 = 0.2, n_2 = 0.8$	186;171	176;175	186;171
3	$n_1 = 0.3, n_2 = 0.7$	176;175	172;180	176;175
4	$n_1 = 0.4, n_2 = 0.6$	172;180	168;185	176;175
5	$n_1 = 0.5, n_2 = 0.5$	168;185	164;190	176;175
6	$n_1 = 0.6, n_2 = 0.4$	148;180	160;195	156;200
7	$n_1 = 0.7, n_2 = 0.3$	160;195	156;200	156;200
8	$n_1 = 0.8, n_2 = 0.2$	156;200	154;210	156;200
9	$n_1 = 0.9, n_2 = 0.1$	152;220	150;230	143;265

Comparison of solution of Example 1 by the proposed method, Nomani method, weighted sum

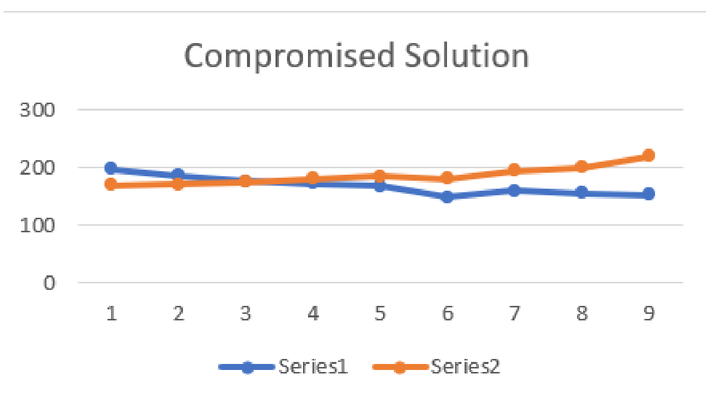

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Figure 1 Graphical representation of solution with different priorities

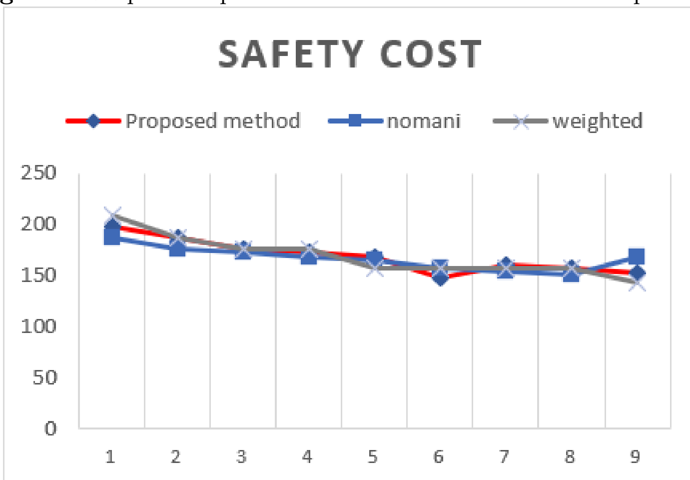

Figure 2 Comparison of safety costs obtained by different methods

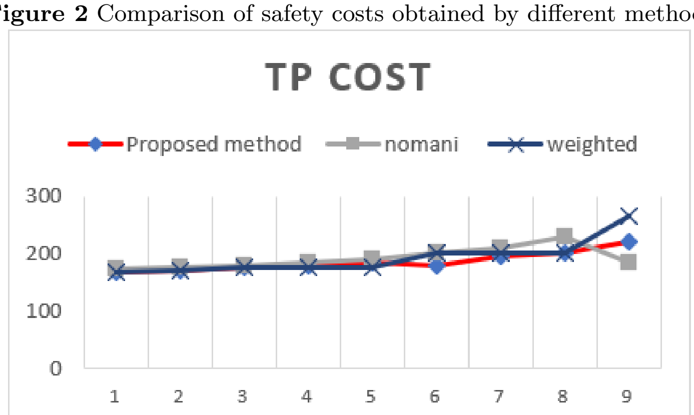

Figure 3 Comparison of TP cost obtained by different methods

Let us now consider a 3-objective problem. This example is already used by authors to compare varying results. Diaz [6](1979) used this example to illustrate the approach. Like the previous problem, we have formulated it like real life problem for better understanding.

Example 2: Madhavi Bhide have a business selling Pickel and she wants to deliver the pickle to various locations across India. She has manufacturing units in Mumbai, Ahmedabad, Chandigarh, and Mirzapur and needs to supply at Ratlam, Nagpur, Patna, Panji, and Kota. The supply capacity of Mumbai is 500 boxes, Ahmedabad is 400 boxes, Chandigarh is 200 boxes, and Mirzapur is 900 boxes. Demand at Ratlam is 400 boxes, Nagpur is 400 boxes, Patna is 600 boxes, Panji is 200 boxes, and Kota is 400 boxes. She wants to minimize the delivery time, transportation cost and packaging cost. The cost time and packing cost per unit are given below (in hundreds).

Cost,Time,Packing cost	Ratlam	Nagpur	Patna	Panji	Kota
Mumbai	9,2,2	12,9,4	9,8,6	6,1,3	9,4,6
Ahmedabad	7,1,4	3,9,8	7,9,4	7,5,9	5,2,2
Chandigarh	6,8,5	5,1,3	9,8,5	11,4,3	3,5,6
Mirzapur	6,2,6	8,8,9	11,6,6	2,9,3	2,8,1

Solution: The first step is to obtain a solution for all three objectives separately ignoring the other objectives. The solution obtained is as follows:
$X^1 = (x_{13} = 5, x_{22} = 3, x_{23} = 1, x_{31} = 1, x_{32} = 1, x_{41} = 3, x_{44} = 2, x_{45} = 4)$
$F_1(X^1) = 102, F_1(X^2) = 164, F_1(X^3) = 134,$
Upper bound = Max 164,134 = 164
Upper and lower bounds of the objective function F_1 is $102 \leq F_1 \leq 164$
$X^2 = (x_{11} = 3, x_{14} = 2, x_{21} = 1, x_{25} = 4, x_{32} = 2, x_{41} = 1, x_{42} = 2, x_{43} = 6)$
$F_2(X^2) = 72(ideal solution), F_2(X^1) = 141, F_2(X^3) = 122,$
Upper bound = Max 141,122 = 141
Upper and lower bounds of the objective function F_2 is $72 \leq F_2 \leq 141$
$X^3 = (x_{11} = 3, x_{12} = 2, x_{21} = 1, x_{23} = 3, x_{32} = 2, x_{43} = 3, x_{44} = 2, x_{45} = 4)$
$F_3(X^3) = 64(ideal solution), F_3(X^2) = 90, F_3(X^1) = 94,$
Upper bound = Max 90,94 = 94
Upper and lower bounds of the objective function F_3 is $64 \leq F_2 \leq 94$
Now since we have the bounds, we can formulate the mathematical model of the problem using the proposed model.

$$Minimize\ \mu' = \mu(1-n_1) + \mu(1-n_2) + \mu(1-n_3)$$

Subject to:

$$\sum_{i=1}^{4}\sum_{j=1}^{5} p_{ij}^1 x_{ij} \leq 102 + \frac{\mu(1-n_1)}{(164-102)}$$

$$\sum_{i=1}^{4}\sum_{j=1}^{5} p_{ij}^2 x_{ij} \leq 72 + \frac{\mu(1-n_2)}{(141-72)}$$

$$\sum_{i=1}^{4}\sum_{j=1}^{5} p_{ij}^3 x_{ij} \leq 64 + \frac{\mu(1-n_2)}{(94-64)}$$

$$\sum_{j=1}^{5} x_{ij} = s_i, \quad i = 1,2,3,4$$

$$\sum_{i=1}^{4} x_{ij} = d_j, \quad j = 1,2,3,4,5$$

$$n_1 + n_2 + n_3 = 1 \quad 0 \leq n_k \leq 1 \quad k = 1,2,3$$

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Now simply allot the weight to the objective function and solve the LPP. Make sure the weights are non-negative and their sum is exactly equal to 1. We have used Lingo 19.0 to solve the LLP.

	Weights (n_1, n_2, n_3)	**Proposed method**	**Nomani**	**Weighted sum**
1	$n_1 = 0.1, n_2 = 0.9, n_3 = 0.0$	147;76;94	147;76;94	157;72;86
2	$n_1 = 0.2, n_2 = 0.8, n_3 = 0.0$	142;78;98	142;78;98	157;72;86
3	$n_1 = 0.3, n_2 = 0.7, n_3 = 0.0$	134;85;96	134;85;96	142;78;98
4	$n_1 = 0.4, n_2 = 0.0, n_3 = 0.6$	114;99;89	124;109;78	129;126;64
5	$n_1 = 0.5, n_2 = 0.0, n_3 = 0.5$	119;101;91	118;110;83	105;128;84
6	$n_1 = 0.6, n_2 = 0.0, n_3 = 0.4$	117;106;88	117;108;84	105;128;84
7	$n_1 = 0.0, n_2 = 0.3, n_3 = 0.7$	134;93;83	139;99;74	153;89;75
8	$n_1 = 0.0, n_2 = 0.2, n_3 = 0.8$	135;97;78	141;102;72	134;122;64
9	$n_1 = 0.0, n_2 = 0.1, n_3 = 0.9$	141;102;72	140;110;68	134;122;64
10	$n_1 = 0.3, n_2 = 0.3, n_3 = 0.4$	126;92;94	124;99;87	127;104;76
11	$n_1 = 0.3, n_2 = 0.4, n_3 = 0.3$	126;92;94	129;95;87	141;86;82
12	$n_1 = 0.4, n_2 = 0.3, n_3 = 0.3$	124;97;91	124;99;87	112;110;88

Comparison of solution of Example 2 by the proposed method, Nomani method, weighted sum

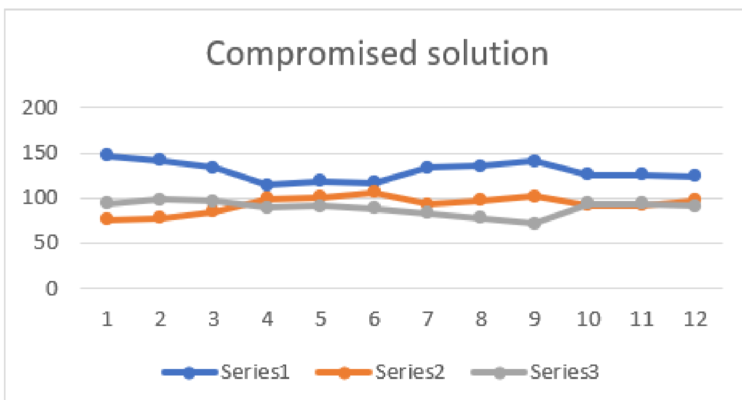

Figure 4 Graphical representation of solution with different priorities

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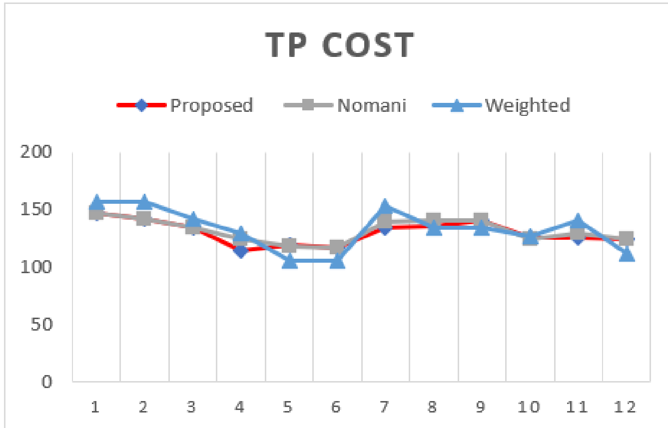

Figure 5 Comparison of transportation costs obtained by different methods

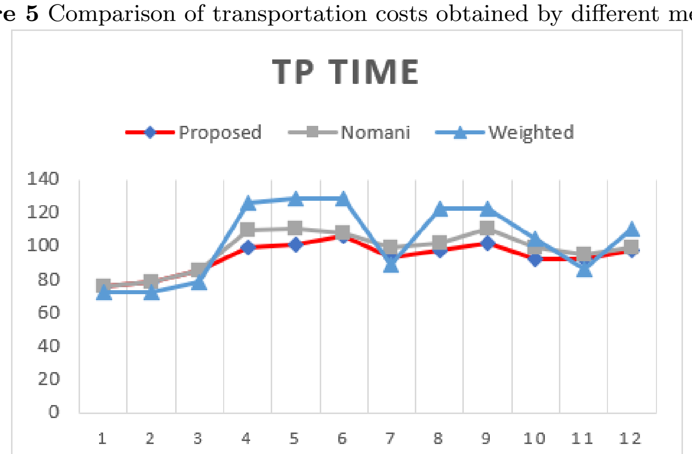

Figure 6 Comparison of transportation time obtained by different methods

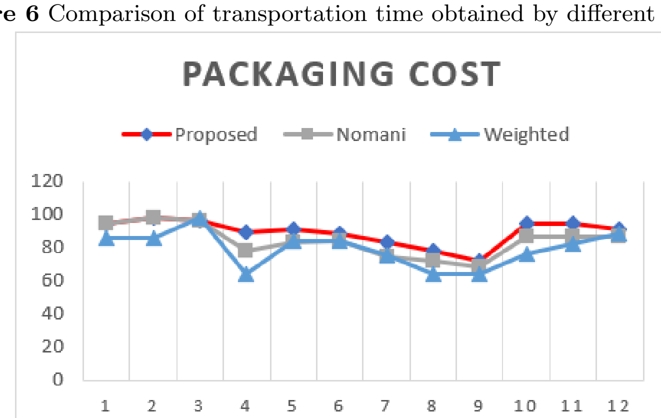

Figure 7 Comparison of packaging cost obtained by different methods

6 Conclusion

In this paper, we discussed a new modified goal programming model and a comparison was made with existing weighted models. The proposed model is capable of providing varying results for a MOTP. LINGO 19.0 was used to solve all the mathematical models. As further development, we plan to extend this method for Fractional MOTP, Rough MOTP and Fixed charge MOTP.

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