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SOME NEW RESULTS ON FRACTIONAL INTEGRALS INVOLVING SRIVASTAVA POLYNOMIALS, (p,q)-EXTENDED HYPERGEOMETRIC FUNCTION AND M-SERIES

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Abstract

Numerous prior publications on fractional calculus provide fascinating explanations of the theory and applications of fractional calculus operators throughout various mathematical analytic domains. In this paper, we introduce new fractional integral formulas using the Saigo-Maeda fractional integral operators and Appell's function F_3 along with the Srivastava polynomials, the (p,q)-extended Gauss hypergeometric function, and the M-Series. A few fascinating unusual cases of our main conclusions are also considered. This approach can be applied to explore a broad class of previously dispersed discoveries in the literature.

Key Words: (p, q)-Extended Gaussís hypergeometric function, Srivastava polynomials, (p, q)-Extended Beta function, S-Function, Generalized fractional integral operators.

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1 Introduction

Over the past three decades, the field of fractional calculus has dealt with derivatives and integrals of arbitrary orders, and it has been applied to nearly every branch of science and engineering. Recently, a large number of scholars have investigated higher transcendent hypergeometric type special functions [10, 11, 28] and associated extensions, generalizations, and unifications of Euler's Beta function (refer [1], [2], [3], [7], [8], [9], [12], [13], [16], [19], [21], [24]). In particular, Chaudhry et al. [2, p. 20, Equation (1.7)] represented the extension of the Beta function as

$$B(x,y;p) = \int_0^1 (\xi)^{x-1}(1-\xi)^{y-1}exp(\frac{-p}{\xi(1-\xi)})d\xi, \Re(p) > 0 \tag{1.1}$$

where for $p = 0$, $min(\Re(x), \Re(y)) > 0$

Chaudhry et al. [3] explored the relationships between the Beta function $B(\xi,\zeta;p)$, Error function, Whittaker function and Macdonald function (or modified Bessel function of the second kind). Moreover, the extended version of the Gaussian hypergeometric function was also derived by utilising (1.2) as -

$$F_p(a,b,c;\xi)=\sum_{n\geq 0}^{\infty}(a)_n\frac{B(b+n,c-b;p)}{B(b,c-b)}\frac{(\xi)^n}{n!} \tag{1.2}$$

$p>0$; for $p=0$, $|\xi|<1;\Re(c)>\Re(b)>0$

A recent expansion of $B(x,y;p)$ and $F_p(a,b,c;z)$ was presented by Choi et al. [4] in the following way

$$B(x,y;p,q)=\int_0^1(\xi)^{x-1}(1-\xi)^{y-1}exp(\frac{-p}{\xi}-\frac{q}{(1-\xi)})d\xi \tag{1.3}$$

provided $\min(\Re(p),\Re(q))\geq 0, \min(\Re(x),\Re(y))>0$
and

$$F_{p,q}(a,b;c;\xi)=\sum_{n\geq 0}^{\infty}(a)_n\frac{B(b+n,c-b;p,q)}{B(b,c-b)}\frac{\xi^n}{n!} \tag{1.4}$$

provided $p,q\geq 0$;for p=0, $|\xi|<1;\Re(c)>\Re(b)>0$
For further information on (1.3) and (1.4), see [23].

The goal of this inquiry is to comprehend the concept of the convolution of two analytical functions, generally known as the Hadamard product. A newly discovered function can be divided into two different functions. The Hadamard product series, in particular, defines a full function if a power series reflects an entire function. Let

$$f(z)=\sum_{n=0}^{\infty}a_n\xi^n,|\xi|<R_f \quad and \quad g(\xi)=\sum_{n=0}^{\infty}b_n\xi^n,|\xi|<R_g$$

Considering two given power series with, respectively, R_f and R_g as their radii of convergence. They produce a Hadamard product, which is a power series described by

$$(f*g)(\xi)=\sum_{n=0}^{\infty}a_nb_n\xi^n=(g*f)(\xi),|\xi|<R \tag{1.5}$$

where R is the radius of convergence

$$\frac{1}{R}=\lim_{n\to 0}sup(|a_nb_n|)^{\frac{1}{n}}\leq\lim_{n\to 0}sup(|a_n|)^{\frac{1}{n}}\lim_{n\to 0}sup(|b_n|)^{\frac{1}{n}}=\frac{1}{R_fR_g}$$

$$and\ so\ R>(R_fR_g)\ (see[15]).$$

Srivastava [22] defined polynomials of general class in the following way

$$S_w^u=\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}\,x^s,u\in\mathbb{N},s\in N_0 \tag{1.6}$$

Where $N_0 = \mathbb{N} \cup \{0\}$, and the coefficients $A_{w,s}$, $w, s \in N_0, w, s > 0$ are arbitrary constants either real or complex. The polynomial family $S_w^[x]$ exhibits several well-known polynomials in addition to its distinct cases when the coefficient $A_{w,s}$. is suitably specialised.

Parmar and Purohit [14] recently explored certain formulae for fractional integral connected to Saigo operators, Furthermore, Choi et al. [4] stated the extended form of hypergeometric functions $F_{p,q}(\xi)$ [26]. In this work, we explored some novel fractional integral formulas involving (p,q)–extended Beta function, (p,q)–extended Gauss's hypergeometric function, the general class of polynomial, and M-Series developed employing generalized fractional integral operators.

2 Fractional Integral Approach

Fractional integral operators involving several special functions [29,30] have been extensively researched in numerous mathematical tools (see, [6]). We explore here the Saigo and Maeda generalized fractional integral operators involving Appell function $F_3(.)$[18] in the kernel.

The extended fractional integrals incorporating Appell's function [22] are stated as, Assuming, $\mu, \mu', \nu, \nu', \tau \in \mathbb{C}$ and $x > 0$, then

$$(I_{0,+}^{\mu,\mu',\nu,\nu',\tau} f)(x) = \frac{x^{-\mu}}{\Gamma(\tau)} \int_0^x (x-\xi)^{\tau-1} \xi^{-\mu'} F_3(\mu,\mu',\nu,\nu';\tau;1-\frac{\xi}{x},1-\frac{x}{\xi}) f(\xi)dt; \Re(\tau) > 0 \tag{2.1}$$

and

$$(I_{0,-}^{\mu,\mu',\nu,\nu',\tau} f)(x) = \frac{x^{-\mu}}{\Gamma(\tau)} \int_x^\infty (\xi-x)^{\tau-1} \xi^{-\mu} F_3(\mu,\mu',\nu,\nu';\tau;1-\frac{x}{\xi},1-\frac{\xi}{x}) f(\xi)dt; \Re(\tau) > 0 \tag{2.2}$$

We will first express a few image formulas related to (2.1) and (2.2), which are given in the lemma that follows.

Lemma 1

Let $\mu, \mu', \nu, \nu', \tau \in \mathbb{C}$ and $x > 0$, Then
(a) If $\Re(\varepsilon) > \max(0, \Re(\mu+\mu'+\nu'-\tau), \Re(\mu'-\nu'))$ and $\Re(\tau) > 0$

$$(I_{0,+}^{\mu,\mu',\nu,\nu',\tau} x^{\varepsilon-1})(x) = x^{\varepsilon-\mu-\mu'+\tau-1} \Gamma \left[\begin{array}{c} \varepsilon, \varepsilon+\tau-\mu-\mu'-\nu, \varepsilon+\nu'-\mu' \\ \varepsilon+\nu', \varepsilon+\tau-\mu-\mu', \varepsilon+\tau-\mu'-\nu \end{array} \right] \tag{2.3}$$

(b) If $\Re(\varepsilon) < 1 + \min(\Re(-\nu), \Re(\mu+\mu'+\nu'-\tau), \Re(\mu'-\nu'))$ and $\Re(\tau) > 0$

$$(I_{0,-}^{\mu,\mu',\nu,\nu',\tau} x^{\varepsilon-1})(x) = x^{\varepsilon-\mu-\mu'+\tau-1} \Gamma \left[\begin{array}{c} 1-\varepsilon-\nu, 1-\varepsilon+\mu+\mu', 1-\varepsilon-\tau+\mu+\nu' \\ 1-\varepsilon, 1-\varepsilon+\mu+\mu'+\nu'-\tau, 1-\varepsilon+\mu-\nu \end{array} \right] \tag{2.4}$$

The symbols occurring in (2.3) and (2.4) are presented as

$$\Gamma\left[\begin{array}{c} a_1,a_2,a_3 \\ a_4,a_5,a_6 \end{array}\right]=\frac{\Gamma(a_1)\Gamma(a_2)\Gamma(a_3)}{\Gamma(a_4)\Gamma(a_5)\Gamma(a_6)}$$

The composition formulas for generalized fractional integrals (2.3) and (2.4) now involve generalized Gauss hypergeometric type functions $F_{p,q}(a,b;c;\xi)$, and the general class of polynomials [25], M-Series[20] are given in Theorem 3.1 and 3.2.
The M-Series is defined as

$$M^{\Im,\varepsilon}_{p',q'}=\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)} \tag{2.5}$$

Here α_j,β_j and $(\alpha_j)_l,(\beta_j)_l$ are pochhammer symbols. the series is defined by (2.5), where neither a negative integer nor zero can be found in any of the denominator parameters β_j, $j=1,2,\dots q'$. If any parameter $\alpha_j<0, j=1,2,...,p'$, then the series converts to a polynomial in x. The series is convergent for all x when $q'\geq p'$, and $p'=q'+1, \quad \mathrm{mod}\, x<1$. The series is divergent when $p'\geq q'+1$.

3 Main Results

In this paper, two new results are developed via generalized fractional operators, which involve Srivastava Polynomials [25], (p,q) - extended Gaussís Hypergeometric function [17, 22] and M-Series [20]. Further, their consequences are also mentioned in the form of simpler functions as special cases of these results

Theorem 3.1

Let $\mu,\mu',\nu,\nu',\tau,\varepsilon\in\mathbb{C}$ be such that $min(\Re(p,q))>0,\Re(\tau)>0$ and $\Re(\varepsilon+s)>max[0,\Re(\mu+\mu'+\nu-\tau),\Re(\mu'-\nu')]$ then for $x>0$

$$\left(I^{\mu,\mu',\nu,\nu',\tau}_{0,+}\left[\xi^{\varepsilon-1}S^u_W(\sigma\xi)F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};e\xi\right]M^{\Im,\varepsilon}_{p',q'}(\xi^{\lambda}\right]\right)(x)$$

$$=x^{\varepsilon+\lambda l-\mu-\mu'+\tau-1}\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma x)^s\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times\frac{\Gamma(\varepsilon+\lambda l+s)\Gamma(\varepsilon+\lambda l+\tau-\mu-\mu'-\nu+s)\Gamma(\varepsilon+\lambda l+\nu'-\mu'+s)}{\Gamma(\varepsilon+\lambda l+\nu'+s)\Gamma(\varepsilon+\lambda l+\tau-\mu-\mu'+s)\Gamma(\varepsilon+\lambda l+\tau-\mu'-\nu+s)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right]*_4F_3\left[\begin{array}{c} 1,\theta_1,\theta_2,\theta_3 \\ \theta_4,\theta_5,\theta_6 \end{array};ex\right] \tag{3.1}$$

where

$$\begin{aligned}\theta_1 &= \varepsilon+\lambda l+s, \theta_2=\varepsilon+\lambda l+\tau-\mu-\mu'-\nu+s,\\ \theta_3 &= \varepsilon+\lambda l+\nu'-\mu'+s, \theta_4=\varepsilon+\lambda l+\tau-\mu-\mu'+s,\\ \theta_5 &= \varepsilon+\lambda l+\tau-\mu'-\nu+s, \theta_6=\varepsilon+\lambda l+\nu'+s\end{aligned}$$

and $'*'$ signifies the Hadamard product

Proof.
Applying (1.4) and (1.6) to (2.1) and changing the order of integration and summation, which is valid under the given conditions here, and using (2.3), we find the LHS of (3.1) (say L) as

$$L=\sum_{s=0}^{[\frac{w}{u}]}\sum_{n=0}^{\infty}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma)^s(a)_n\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha'_p}{(\beta_1)_l...(\beta'_q)_l}\frac{1}{\Gamma(\Im l+\varepsilon)}\frac{B_{p,q}(b+n,c-b)}{B(b,c-b)}\frac{e^n}{n!}$$

$$\times\left(I_{0,+}^{\mu,\mu',\nu,\nu',\tau}(t)^{\varepsilon+n+\lambda l+s-1}\right)(x)$$

$$= \quad (x)^{\varepsilon+\lambda l-\mu-\mu'+\tau-1}\sum_{s=0}^{[\frac{w}{u}]}\sum_{n=0}^{\infty}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma)^s(a)_n\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}\frac{B_{p,q}(b+n,c-b)}{B(b,c-b)}$$

$$\times\frac{\Gamma(\varepsilon+\lambda l+n+s)\Gamma(\varepsilon+\lambda l+n+\tau-\mu-\mu'-\nu+s)\Gamma(\varepsilon+\lambda l+n+\nu'-\mu'+s)}{\Gamma(\varepsilon+\lambda l+n+\nu'+s)\Gamma(\varepsilon+\lambda l+n+\tau-\mu-\mu'+s)\Gamma(\varepsilon+\lambda l+n+\tau-\mu'-\nu+s)}\frac{(ex)^n}{n!} \tag{3.2}$$

Expressing the last summation in (3.2) in terms of the Hadamard product with the functions $F_{p,q}(.)$ mentioned in (1.4) and generalized hypergeometric function [22, 24], we obtain the RHS of (3.1).

Theorem 3.2

Let $\mu,\mu',\nu,\nu',\delta,\rho\in\mathbb{C}$ be such that $min(\Re(p),\Re(q))>0,\Re(\delta)>0$ and $\Re(\rho)<1+min[\Re(-\nu),\Re(\mu+\mu'-\delta),\Re(-\mu-\nu-\delta)]$ then for $x>0$

$$\left(I_{0,-}^{\mu,\mu',\nu,\nu',\tau}\left[\xi^{\varepsilon-1}S_W^u(\sigma\xi)F_{p,q}\left[\begin{matrix}a,b\\c\end{matrix};\frac{e}{\xi}\right]M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda}\right]\right)(x)$$

$$=x^{\varepsilon+\lambda l-\mu-\mu'+\tau-1}\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma,s)^s\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times\frac{\Gamma(1-\nu-\varepsilon-\lambda l-s)\Gamma(1+\mu+\nu'-\varepsilon-\lambda l-s)\Gamma(1+\mu+\nu'-\varepsilon-\lambda l-s)}{\Gamma(1-\varepsilon-\lambda l-s,)\Gamma(1+\mu+\mu'+\nu'-\tau-\varepsilon-\lambda l-s)\Gamma(1+\mu-\nu-\varepsilon-\lambda l-s)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] *_4 F_3\left[\begin{array}{c} 1,\delta_1,\delta_2,\delta_3 \\ \delta_4,\delta_5,\delta_6 \end{array};\frac{e}{x}\right] \tag{3.3}$$

where

$$\begin{aligned}
\delta_1 &= 1-\nu-\varepsilon-\lambda l-s, \delta_2 = 1+\mu+\nu'-\varepsilon-\lambda l-s,\\
\delta_3 &= 1+\mu+\nu'-\varepsilon-\lambda l-s, \delta_4 = 1-\varepsilon-\lambda l-s\\
\delta_5 &= 1+\mu+\mu'+\nu'-\tau-\varepsilon-\lambda l-s, \delta_6 = 1+\mu-\nu-\varepsilon-\lambda l-s
\end{aligned}$$

and $'*'$ signifies the Hadamard product

Proof. Applying a similar argument as in the proof of Theorem 3.1 by using (1.4) and (1.6) to (2.2), and using (2.4), we obtain the RHS of (3.3).

4 Special Cases

(i) If we set $\mu' = \nu' = 0, \nu = -\eta, \mu = \mu+\nu, \delta = \mu, \Re(\mu) > 0$ in the operators (2.1) and (2.2), then we arrive at Saigo hypergeometric fractional integral operators [6]

$$\left(I_{0,+}^{\mu,\nu,\eta} f(\xi) \right)(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)} \int_0^x (x-\xi)^{\mu-1} {}_2F_1\left(\mu+\nu, -\eta; \mu; (1-\tfrac{\xi}{x}) \right) f(\xi) d\xi \tag{4.1}$$

and

$$\left(I_{0,-}^{\mu,\nu,\eta} f(\xi) \right)(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)} \int_x^\infty (\xi-x)^{\mu-1} {}_2F_1\left(\mu+\nu, -\eta; \mu; (1-\tfrac{x}{\xi}) \right) f(\xi) d\xi \tag{4.2}$$

Corollary 1. Let $\mu, \mu', \eta, \rho \in \mathbb{C}$ be such that $\min(\Re(p), \Re(q)) > 0, \Re(\mu) > 0$ and $\Re(\rho) > \max[0, \Re(\nu-\eta)], x > 0$

then the result (3.1) reduced as

$$\left(I_{0,+}^{\mu,\nu,\eta} \left[t^{\varepsilon-1} S_w^u(\sigma\xi) F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array}; e\xi\right] M_{p',q'}^{\Im,\varepsilon}(\xi^\lambda \right] \right)(x)$$

$$= x^{\varepsilon+\lambda l-\nu-1} \sum_{s=0}^{[\frac{w}{u}]} \frac{(-w)_{us}}{s!} A_{w,s}(\sigma x)^s \sum_{l=0}^\infty \frac{(\alpha_1)_l \ldots (\alpha_{p'})_l}{(\beta_1)_l \ldots (\beta_{q'})_l} \frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times \frac{\Gamma(\varepsilon+\lambda l+s)\Gamma(\varepsilon+\lambda l-\nu+\eta+s)}{\Gamma(\varepsilon+\lambda l+\mu+\eta+s)\Gamma(\varepsilon+\lambda l-\nu+s)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array}; ex\right] *_3 F_2\left[\begin{array}{c} 1,\varepsilon+\lambda l+s,\varepsilon+\lambda l-\nu+\eta+s \\ \varepsilon+\lambda l-\nu+s,\varepsilon+\lambda l+\mu+\eta+s \end{array}; ex\right] \tag{4.3}$$

Corollary 2 Let $\mu,\nu,\eta,\rho\in\mathbb{C}$ be such that $\min(\Re(p),\Re(q))>0,\Re(\mu)>0$ and $\Re(\rho)<1+\min[\Re(\eta),\Re(\nu)]$ as

$$\left(I_{0,-}^{\mu,\nu,\eta}\left[\xi^{\varepsilon-1}S_{W}^{u}(\sigma\xi)F_{p,q}\left[\begin{matrix} a,b \\ c\end{matrix};\frac{e}{\xi}\right]M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda} \right]\right)(x)$$

$$=x^{\varepsilon+\lambda l-\nu-1}\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma x)^{s}\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times\frac{\Gamma(1-\varepsilon-\lambda l+n+\nu-s)\Gamma(1-\varepsilon-\lambda l+\eta-s)}{\Gamma(1-\varepsilon-\lambda l-s)\Gamma(1-\varepsilon-\lambda l+\eta+\mu+\nu-s)}$$

$$\times F_{p,q}\left[\begin{matrix} a,b \\ c\end{matrix};\frac{e}{x}\right]*_3F_2\left[\begin{matrix} 1,1-\varepsilon-\lambda l+\nu-s,1-\varepsilon-\lambda l+\eta-s \\ 1-\varepsilon-\lambda l-s,1-\varepsilon-\lambda l+\eta+\mu+\nu-s\end{matrix};\frac{e}{x}\right] \tag{4.4}$$

(ii) The operator $I_{0,+}^{\mu,\nu,\eta}(.)$ contains both the Riemann-Liouville $I_{0-}^{\mu}(.)$ and the Erdélyi-Kober [5] $I_{\eta,\mu}^{+}(.)$ fractional integral operators which can be defined as

$$\left(I_{0,+}^{\mu}f(\xi)\right)(x)=\left(I_{0,+}^{\mu,-\mu,\eta}f(\xi)\right)(x)=\frac{1}{\Gamma(\mu)}\int_0^x(x-\xi)^{\mu-1}f(\xi)d\xi \tag{4.5}$$

and

$$\left(I_{\eta,\mu}^{+}f(\xi)\right)(x)=\left(I_{0,+}^{\mu,0,\eta}f(\xi)\right)(x)=\frac{(x)^{-\mu-\eta}}{\Gamma(\mu)}\int_0^x(x-\xi)^{\mu-1}(\xi)^{n}f(\xi)d\xi \tag{4.6}$$

Corollary 3. Let $\mu,\eta,\rho\in\mathbb{C}$ be such that $min(\Re(p),\Re(q))>0,\Re(\mu)>0$ and $\Re(\rho+s)>\Re(-\eta),x>0$

By using (4.6), the result (3.1) reduced as

$$\left(I_{\eta,\mu}^{+}\left[\xi^{\varepsilon-1}S_{w}^{u}(\sigma\xi)F_{p,q}\left[\begin{matrix} a,b \\ c\end{matrix};e\xi\right]M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda} \right]\right)(x)$$

$$=x^{\varepsilon+\lambda l-1}\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma x)^{s}\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times\frac{\Gamma(\varepsilon+\lambda l+\eta+s)}{\Gamma(\varepsilon+\lambda l+\mu+\eta+s)}\times F_{p,q}\left[\begin{matrix} a,b \\ c\end{matrix};ex\right]*_2F_1\left[\begin{matrix} 1,\varepsilon+\lambda l+\eta+s \\ \varepsilon+\lambda l+\mu+\eta+s,\end{matrix};ex\right] \tag{4.7}$$

(iii) It can be observed that the operator (4.2) unifies the Erdélyi-Kober fractional operators with the Weyl type as follows:

$$\left(I_{-}^{\mu} f(\xi) \right)(x) = \left(I_{-}^{\mu,-\mu,\eta} f(\xi) \right)(x) = \frac{1}{\Gamma(\mu)} \int_{x}^{\infty} (\xi - x)^{\mu-1} f(\xi) d\xi \tag{4.8}$$

and

$$\left(K_{\eta,\mu}^{-} f(\xi) \right)(x) = \left(I_{-}^{\mu,0,\eta} f(\xi) \right)(x) = \frac{(x)^{-\mu-\eta}}{\Gamma(\mu)} \int_{x}^{\infty} (\xi - x)^{\mu-1} (\xi)^{-\mu-\eta} f(\xi) d\xi \tag{4.9}$$

Corollary 4. Let $\mu, \eta, \rho \in \mathbb{C}$ be such that $min(\Re(p), \Re(q)) > 0, \Re(\mu) > 0$ and $\Re(\rho + s) < 1 + \Re(\eta)$,$x > 0$ By using (4.9), the result (3.3) reduced as

$$\left(K_{\eta,\mu}^{-} \left[\xi^{\varepsilon-1} S_{w}^{u}(\sigma\xi) F_{p,q} \left[\begin{array}{c} a, b \\ c \end{array} ; e\xi \right] M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda} \right] \right)(x)$$

$$= x^{\varepsilon+\lambda l-1} \sum_{s=0}^{[\frac{w}{u}]} \frac{(-w)_{us}}{s!} A_{w,s} (\sigma x)^{s} \sum_{l=0}^{\infty} \frac{(\alpha_1)_l \ldots (\alpha_{p'})_l}{(\beta_1)_l \ldots (\beta_{q'})_l} \frac{1}{\Gamma(\Im l + \varepsilon)}$$

$$\times \frac{\Gamma(\varepsilon + \lambda l + \eta + s)}{\Gamma(\varepsilon + \lambda l + \mu + \eta + s)} \times F_{p,q} \left[\begin{array}{c} a, b \\ c \end{array} ; ex \right] *_2 F_1 \left[\begin{array}{c} 1, \varepsilon + \lambda l + \eta + s \\ \varepsilon + \lambda l + \mu + \eta + s, \end{array} ; ex \right] \tag{4.10}$$

(iv) Additionally, on replacing ν by $-\mu$ in Corollary 1 and 2 and making use of the relations (4.5) and (4.8) give the other Riemann-Liouville and Weyl fractional integrals of the extended hypergeometric function in (1.4) are provided by the following corollaries

Corollary 5. Let $\mu, \rho \in \mathbb{C}$ be such that $min(\Re(p), \Re(q)) > 0, \Re(\mu) > 0$, $x > 0$ By using (4.5), the result (3.1) reduced as

$$\left(I_{0,+}^{\mu} \left[\xi^{\varepsilon-1} S_{w}^{u}(\sigma\xi) F_{p,q} \left[\begin{array}{c} a, b \\ c \end{array} ; e\xi \right] M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda} \right] \right)(x)$$

$$= x^{\varepsilon+\lambda l+\mu-1} \sum_{s=0}^{[\frac{w}{u}]} \frac{(-w)_{us}}{s!} A_{w,s} (\sigma x)^{s} \sum_{l=0}^{\infty} \frac{(\alpha_1)_l \ldots (\alpha_{p'})_l}{(\beta_1)_l \ldots (\beta_{q'})_l} \frac{1}{\Gamma(\Im l + \varepsilon)}$$

$$\times \frac{\Gamma(\varepsilon + \lambda l + \mu + \eta + s)}{\Gamma(\varepsilon + \lambda l + \mu + s)} \times F_{p,q} \left[\begin{array}{c} a, b \\ c \end{array} ; ex \right] *_2 F_1 \left[\begin{array}{c} 1, \varepsilon + \lambda l + s \\ \varepsilon + \lambda l + \mu + s, \end{array} ; ex \right] \tag{4.11}$$

Corollary 6. Let $\mu, \rho \in \mathbb{C}$ be such that $min(\Re(p), \Re(q)) > 0, \Re(\mu) > 0$, $x > 0$ By using (4.8), the result (3.3) reduced as

$$\left(I_{-}^{\mu} \left[\xi^{\varepsilon-1} S_{w}^{u}(\sigma\xi) F_{p,q} \left[\begin{array}{c} a, b \\ c \end{array} ; \frac{e}{\xi} \right] M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda} \right] \right)(x)$$

$$= x^{\varepsilon+\lambda l+\mu-1}\sum_{s=0}^{[\frac{w}{u}]}\frac{(-w)_{us}}{s!}A_{w,s}(\sigma x)^{s}\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{1}{\Gamma(\Im l+\varepsilon)}$$

$$\times\frac{\Gamma(1-\varepsilon-\lambda l-\mu-s)\Gamma(1-\varepsilon-\lambda l+\eta-s)}{\Gamma(1-\varepsilon-\lambda l-s)\Gamma(1-\varepsilon-\lambda l-\eta-s)}$$

$$\times F_{p,q}\left[\begin{matrix} a,b \\ c\end{matrix};\frac{e}{\xi}\right]*_3F_2\left[\begin{matrix}1,1-\varepsilon-\lambda l-\mu-s,1-\varepsilon-\lambda l+\eta-s \\ 1-\varepsilon-\lambda l-s,1-\varepsilon-\lambda l-\eta-s\end{matrix};\frac{e}{\xi}\right] \tag{4.12}$$

(v) If we emphasise in our conclusion that the general class of polynomials yields numerous well-known classical orthogonal polynomials as its particular cases when appropriate unique values are provided for the coefficient $A_{w,s}$.More specifically, if we set $w = 0$, $A_{0,0} = 1$ then $S_w^u = 1$ in (3.1) and (3.3), we gain the fresh findings claimed in corollaries 7 and 8 as

Corollary 7. Let $\mu,\mu',\nu,\nu',\delta,\rho\in\mathbb{C}$ be such that $min(\Re(p),\Re(q))>0,\Re(\delta)>0$ and $\Re(\rho+s)>max[0,\Re(\mu+\mu'+\nu-\delta),\Re(\mu'-\nu')],x>0$
then the result (3.1) reduced as

$$\left(I_{0,+}^{\mu,\mu',\nu,\nu',\tau}\left[\xi^{\varepsilon-1}F_{p,q}\left[\begin{matrix}a,b \\ c\end{matrix};e\xi\right]M_{p',q'}^{\Im,\varepsilon}(\xi^{\lambda})\right]\right)(x)$$

$$= x^{\varepsilon+\lambda l-\mu-\mu'+\tau-1}\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{(\varepsilon')_{l\tau,k}}{\Gamma_k(\sigma l+\eta)}\frac{1}{l!}$$

$$\times\frac{\Gamma(\varepsilon+\lambda l)\Gamma(\varepsilon+\lambda l+\tau-\mu-\mu'-\nu)\Gamma(\varepsilon+\lambda l+\nu'-\mu')}{\Gamma(\varepsilon+\lambda l+\tau-\mu-\mu')\Gamma(\varepsilon+\lambda l+\tau-\mu'-\nu)\Gamma(\varepsilon+\lambda l+\nu')}$$

$$\times F_{p,q}\left[\begin{matrix}a,b \\ c\end{matrix};ex\right]*_4F_3\left[\begin{matrix}1,\theta_7,\theta_8,\theta_9 \\ \theta_{10},\theta_{11},\theta_{12}\end{matrix};ex\right] \tag{4.13}$$

where
$\theta_7=\varepsilon+\lambda l,\theta_8=\varepsilon+\lambda l+\tau-\mu-\mu'-\nu,$
$\theta_9=\varepsilon+\lambda l+\nu'-\mu',\theta_{10}=\varepsilon+\lambda l+\tau-\mu-\mu',$
$\theta_{11}=\varepsilon+\lambda l+\tau-\mu'-\nu,\theta_{12}=\varepsilon+\lambda l+\nu'$

Corollary 8. Let $\mu,\mu',\nu,\nu',\delta,\rho\in\mathbb{C}$ be such that $min(\Re(p),\Re(q))>0,\Re(\delta)>0$ and $\Re(\rho)<1+min[\Re(-\nu),\Re(\mu+\mu'-\delta),\Re(\mu-\nu'-\delta)],x>0$ then the result (3.3) reduced as

$$\left(I_{-}^{\mu,\mu',\nu,\nu',\tau}\left[\xi^{\varepsilon-1}F_{p,q}\left[\begin{matrix}a,b \\ c\end{matrix};\frac{e}{\xi}\right]M_{p',q'}^{\gamma,\zeta}(\xi^{\lambda})\right]\right)(x)$$

$$= x^{\varepsilon+\lambda l-\mu-\mu'+\tau-1}\sum_{l=0}^{\infty}\frac{(\alpha_1)_l...(\alpha_{p'})_l}{(\beta_1)_l...(\beta_{q'})_l}\frac{(\varepsilon')_{l\tau,k}}{\Gamma_k(\sigma l+\eta)}\frac{1}{l!}$$

$$\times\frac{\Gamma(1+\mu+\mu'-\tau-\varepsilon-\lambda l)\Gamma(1+\mu+\nu'-\tau-\varepsilon-\lambda l)\Gamma(1-\nu-\varepsilon-\lambda l)}{\Gamma(1-\varepsilon-\lambda l)\Gamma(1+\mu+\mu'+\nu'-\tau-\varepsilon-\lambda l)\Gamma(1+\mu-\nu-\varepsilon-\lambda l)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] *_4F_3\left[\begin{array}{c} 1,\theta_{13},\theta_{14},\theta_{15} \\ \theta_{16},\theta_{17},\theta_{18} \end{array};\frac{e}{x}\right] \tag{4.14}$$

where
$\theta_{13}=1+\mu+\mu'-\tau-\varepsilon-\lambda l, \theta_{14}=1+\mu+\nu'-\tau-\varepsilon-\lambda l,$
$\theta_{15}=1-\nu-\varepsilon-\lambda l, \theta_{16}=1-\varepsilon-\lambda l,$
$\theta_{17}=1+\mu+\mu'+\nu'-\tau-\varepsilon-\lambda l, \theta_{18}=1+\mu-\nu-\varepsilon-\lambda l$

(vi) Also, it is interesting to note that if we set $w=0$, $A_{0,0}=1$ and $S_W^u=1$, the results obtained in Corollaries 1 to 6, yield corresponding results given due to Parmar and Purohit [14]. If we set $u=2$ and $A_w,s=(-1)^s$, then the general class of polynomials become

$$S_w^u[x] \rightarrow x^{u/2}\, H_w(\frac{1}{2\sqrt{x}})$$

$H_w(x)$ denotes the well known Hermite polynomials and are defined by

$$H_w(x)=\sum_{s=0}^{[u/2]}(-1)^s\frac{w}{(w-2s)!s!}(2x)^{w-2s}$$

Conclusion

Our findings are significant because of their broad applicability. Given the universality of Srivastava's polynomial and hypergeometric function, we can obtain multiple results comprising a fairly large variety of useful functions and their various special instances by specialising the various parameters. As a result, the main result described in this article would yield a very large number of results containing a wide range of simpler special functions happening in scientific and technological disciplines all at once. This study involves the establishment of two formulas involving specific special functions through the use of generalised fractional integral operators. Several novel and well-known results are produced as a result of their further ramifications, and it is projected that these results will have an impact on numerous applied science fields.

Conflict of Interests

None

References

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