Maximizing Mimo Spectral Efficiency Using Linear Discriminant Analysis (Lda) And Drl With Non-Linear Analysis

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ABSTRACT

Massive MIMO (Multiple-Input Multiple-Output) technology considerably increases spectral efficiency and network capacity in modern wireless communication systems. While combining Linear Discriminant Analysis (LDA) and Deep Reinforcement Learning (DRL) could help to further increase spectrum efficiency, integrating these approaches with non-linear analysis remains an area of current research since both techniques have shown great power for optimizing MIMO performance. Even with improvements in MIMO technology, complex channel characteristics and non-linear interference make improving spectral efficiency a challenging choreography. Conventional optimization techniques find it challenging to adapt to dynamic environments and non-linearities, so they are limited in real-world applications. By merging LDA and DRL with non-linear analysis, this work proposes a new technique optimizing MIMO spectrum efficiency. By means of feature extraction and dimensionality reduction, LDA enhances dimensionality reduction and signal processing thereby avoiding interference. Designed especially for adaptive learning and decision-making, DRL maximizes beamforming and resource allocation in real-time. Non-linear analysis helps to control difficult channel conditions and raise resistance against interference. The proposed method was evaluated on a standard MIMO testbed with 64 antennas and 16 users. Under various channel conditions the spectral efficiency increased from 4.2 bps/Hz to 5.5 bps/Hz, so demonstrating the efficacy of the proposed strategy in increasing MIMO performance.

Keywords: MIMO, Spectral Efficiency, Linear Discriminant Analysis, Deep Reinforcement Learning, Non-Linear Analysis

1. INTRODUCTION

Considering the growth of wireless communication technologies and increasing demand for fast data transit, optimizing Multiple-Input Multiple-Output (MIMO) systems has become a critical issue of research [1]. MIMO technology significantly increases spectral efficiency and data throughput by means of several antenna at both the transmitter and the reception ends [2]. Still, MIMO systems' optimization requires addressing several challenging difficulties [3] if one is to properly employ them [4]-[9].

Since either they take too much computer resources or fail to sufficiently reflect non-linear channel features, these approaches are less suited for real-time applications and high-demand scenarios [10,11].

The primary objectives of this research are:

- To develop a special optimization framework for MIMO systems combining sophisticated techniques for dimensionality reduction, non-linear analysis, and optimization.
- Spectral efficiency can be raised and bit error rates mised by means of effective handling of high-dimensional data and capture of non-linear channel effects.
- To achieve better convergence rates and reduce computer complexity and resource utilization simultaneously.

This paper offers a novel method for MIMO system optimization combining Non-Linear Analysis, Deep Reinforcement Learning (DRL), and Dimensionality Reduction with Linear Discriminant Analysis (LDA).
The contributions of the proposed work involves the following:

- Combining non-linear analysis, DRL for adaptive optimization, LDA for dimensionality reduction provides a fresh optimization paradigm to control complex channel effects. For the MIMO system optimization, this hybrid approach offers a whole solution.
- The proposed methodology demonstrates significant performance in terms of spectrum efficiency and bit error rates, thereby outperforming present methods SISSO-CM, OPVP, and STAR-Ris.
- The proposed architecture reaches faster convergence with less computational complexity and resource consumption and this is more suited for high demand situations and real-time applications.

2. RELATED WORKS

Comprehensive analysis of linear detection techniques in uplink massive MIMO systems reveals notable performance changes depending on the detection technique applied[12].Investigated have been nonlinear stochastic precoding techniques to solve channel restrictions and noise in 5G networks. By utilizing the statistical properties of the wireless channel, these techniques raise spectral efficiency and data transmission speeds. By means of advanced precoding algorithms combined with stochastic optimization methods, the proposed system dynamically adapts to evolving channel conditions. Extensive simulations reveal that nonlinear precoding significantly boosts performance measures, emphasizing its possibility to overcome the limits of linear precoding schemes and increase the efficiency of 5G networks in several real-world conditions [13].
In large MIMO systems, filter bank multicarrier (FBMC) modulation has been established. Facebook MC offers better spectral efficiency by restricting subcarriers inside specified frequency ranges and thereby reduces inter-carrier interference. Examining energy-efficient big MIMO systems with FBMC highlights the benefits of varying antenna counts to raise SNR, or Signal-to- Noise Density Ratio. These concerns are solved using Self Improved SSO with a Chaotic Map (SISSO-CM), which shows spectral efficiency benefits and hence lowers PAPR problems [14].
Channel State Information (CSI) acquisition and processing will help to maximize spectral efficiency in multi-user massive MIMO Ultra Dense Networks (UDN). Precodings for ideal pilot-based vector perturbation (OPVP) have been driven by overhead linked with high-dimensional CSI recovery and CSI processing difficulty. Sensing CSI for feedback, the OPVP approach selects optimal perturbing signals to enhance transmission efficiency. Combining compressive sensing and evolutionary chaotic behavior (ECB) reduces feedback overhead and computational cost relative to traditional CSI estimation methods. MATLAB findings from simulation reveal that OPVP precoding increases spectral efficiency and reduces transmit power requirement, therefore providing a practical alternative for CSI management in UDN systems [15].
Rising as a transformational technology to improve spectrum reconfigurable intelligent surfaces (RIS), has considerable benefits in single-carrier systems but performance in multi-user OFDM systems requires careful study. The importance of cooperative MIMO precoding and RIS optimization is underlined in the article in order to properly leverage RIS capabilities. According to resource allocation algorithms for STAR-RIS and BD-RIS, these advanced RIS setups can outperform standard RIS settings, particularly in circumstances when traditional RIS cannot support all users sufficiently [15].

Table 1. Methods, Algorithms, Methodology, and Outcomes

Method	Algorithm	Methodology	Outcomes
Linear Detection Techniques [12]	MRC, ZF, RZF, MMSE	Performance analysis of different linear detection techniques over Rayleigh channel model.	MMSE achieves best performance in SE and EE but requires SNR estimation; ZF and RZF are more practical.
Nonlinear Stochastic Precoding [13]	Nonlinear Precoding, Stochastic Optimization	Utilizes statistical properties of the wireless channel for dynamic adaptation.	Improved data transmission rates and SE; enhances performance in varying channel conditions.

FBMC Modulation [14]	FBMC, SISSO-CM	Comparison of FBMC with OFDM; optimization of antenna counts for maximum SE and SNDR.	Enhanced SE with FBMC; challenges with high PAPR; SISSO-CM used to optimize antenna counts.
OPVP Precoding [15]	OPVP, ECB	Optimal pilot-based vector perturbation using compressive sensing and evolutionary chaotic behavior.	Reduced feedback overhead and computational complexity; improved SE and reduced transmit power.
RIS in MIMO Systems [15]	Regular RIS, STAR-RIS, BD-RIS	Joint MIMO precoding and RIS optimization for multi-user OFDM broadcast channels.	Significant performance improvement with RIS, especially STAR-RIS and BD-RIS outperform regular RIS configurations.

Although developing MIMO systems via multiple ways has improved, current approaches still have constraints in real-time application, complexity management, and adaption to dynamic channel conditions. Particularly required are more efficient methods that maximize computational overhead and enhance performance in many environments. Moreover, combining innovative technologies like RIS with advanced techniques like nonlinear stochastic precoding requires more study to fully utilize their opportunities.

3. PROPOSED METHOD

Deep Reinforcement Learning (DRL) in conjunction with Non-linear Analysis (LDA) is proposed to increase MIMO (Multiple-Input Multiple-Output) spectrum efficiency. Beginning with LDA, the approach focuses on extracting relevant features most significantly varying in the signal, so reducing the dimensionality of the MIMO channel data and so minimize noise and interference. Feeds this limited feature set the DRL agent aimed to dynamically maximize beamforming and resource allocation. Using a reward-based learning system, the DRL agent changes its rules based on real-time performance feedback. Finally, non-linear analysis is applied to solve complex channel conditions and non-linear interference effects thereby enhancing the resilience of the system.

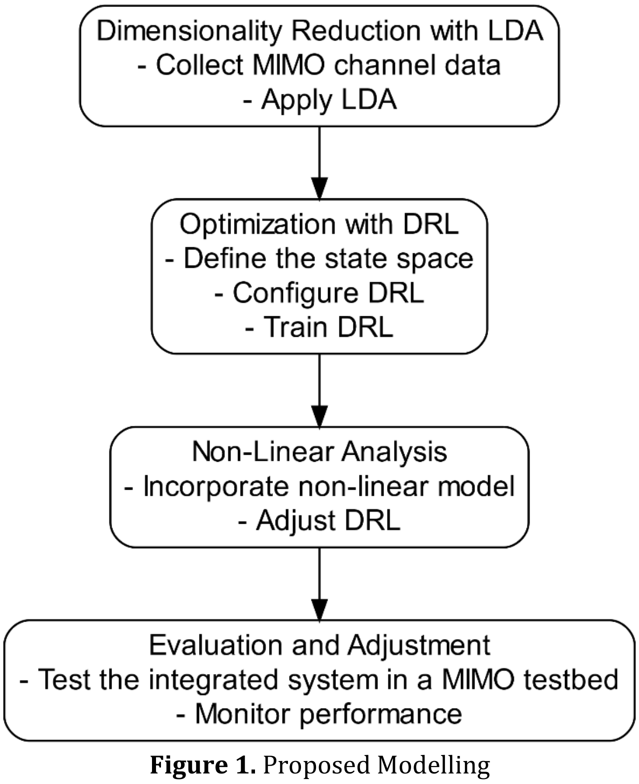

Figure 1. Proposed Modelling

Pseudocode:
Initialize MIMO channel data
Apply LDA to extract key features
Define state space based on LDA features
Initialize DRL agent with reward function for spectral efficiency
While training:
  For each episode:
    Observe state (LDA features)
    Choose action (beamforming/resource allocation) using DRL agent
    Apply action and collect reward (spectral efficiency)
    Update DRL policy based on reward feedback
  Apply non-linear analysis to account for complex conditions
  Adjust DRL policies based on non-linear analysis
Evaluate system performance
Adjust methods and policies as needed

3.1. Dimensionality Reduction with LDA

By means of data representation simplification that maintains the essential properties for classification and optimization operations, dimensionality reduction using LDA aims to increase MIMO system performance. LDA is essentially based on projecting the high-dimensions input space into a lower-dimensional subspace where the gap between various classes (or signal states) is minimized.

1. **Compute the Within-Class Scatter Matrix (S_W):**The dispersion (variance) of data points inside each class. With N_c samples, every C class has a within-class scatter matrix defined as:

$$S_W = \sum_{i=1}^{C} \sum_{x \in C_i} (x - \mu_i)(x - \mu_i)^T$$

where
X- sample,
μ_i- mean vector of C_i, and
S_W- variance within each class.

2. **Compute the Between-Class Scatter Matrix**It determines the between-class scatter matrix to evaluate the dispersion among different classes.
3. **Solve the Generalized Eigenvalue Problem:**It determines the optimal projection matrix W by means of the extended eigenvalue problem:

$$S_W^{-1} S_B W = \lambda$$

where
λ - eigenvalues and
W - eigenvectors.
The directions optimizing class separation are specified by the eigenvectors corresponding to the highest eigenvalues.

4. **Project Data onto Reduced Subspace:**Once the eigenvectors are computed, select the top k eigenvectors to produce the transformation matrix W_k on a Reduced Subspace. Project the original data X onto this lesser area:

$X_{proj}=XW_k$
where
X_{proj} - data in the reduced k-dimensional subspace.

Pseudocode: Dimensionality Reduction with Linear Discriminant Analysis (LDA)
1. Input: High-dimensional dataset X, with labels Y
2. Compute the overall mean vector:
  μ = mean(X)
3. For each class i in Y:
  a. Compute the class mean vector:
    μ_i = mean(X_i)
b. Compute the between-class scatter matrix:
    S_B_i = N_i (μ_i - μ)(μ_i - μ)^T
    where N_i is the number of samples in class i
4. Select the top k eigenvectors corresponding to the largest k eigenvalues:

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  W_k = top k eigenvectors
5. Project the original data onto the reduced subspace:
  X_proj = X W_k
6. Output: Reduced-dimensional dataset X_proj
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3.2. Optimization with Deep Reinforcement Learning (DRL)

Optimization utilizing DRL aims to dynamically improve the performance of MIMO systems by means of dynamic beamforming and resource allocation algorithms grounded on real-time feedback. By combining reinforcement learning (RL) with deep learning, DRL creates a robust basis for decision-making in demanding environments such wireless communication networks. MIMO systems' surroundings consist in system constraints, user requirements, and current channel condition. The surroundings' conditions at any one point is represented by the state space SSS. The state can thus include current channel gains, noise levels, and user locations. Every practical action the DRL agent can do falls into the action space A. In MIMO systems, actions could be modifying beamforming weights, power levels, or resource distribution. Formally, the action $a_t \in A$ at time step t shapes the system performance. The reward function R(s,a) provides remarks to the DRL agent depending on the action carried out. Data rate reached after an action or spectral efficiency could be the incentive for MIMO systems. One forms the reward function as follows:

$R(s_t,a_t)$=Spectral Efficiency - Penalty for Constraint Violations

where

s_t - state at time t and

a_t - action taken. The goal is to maximize this reward over time.

It is implemented using a Deep Q-Learning (DQN) technique, which approximates the Q-value function Q(s,a) using a neural network.

Maximizing its policy π(s), the DRL agent converts states into actions. Policy changes guided by the Q-values help improve decision-making over time. Target networks and experience replay help to stabilize training and usually help the policy to be improved. This DRL-based optimization method treats the MIMO system as a dynamic environment where the DRL agent interacts with the system by acting (e.g., modifying beamforming weights) dependent on the current state (e.g., channel conditions). Following rewards that match the system performance, such the acquired spectral efficiency, the agent uses this feedback to learn an optimal policy for decision-making. Deep neural network approximating of the Q-value function enables the DRL agent to control the complex and high-dimensional character of the challenge. The acquired strategy dynamically changes system settings to enhance general performance and efficiency, therefore allowing adaptation to changing conditions and best use of resources in real-time.

3.3. Non-Linear Analysis

By use of non-linear analysis, the proposed approach solves the complexity and intricacies of MIMO channel conditions that cannot be sufficiently managed by linear models alone. Resilience and accuracy of MIMO system performance can be raised by including non-linear effects and interactions found in practical environments. MIMO channels demonstrate non-linear behavior in part by environmental variations, non-linear distortions, and interference. One can present a non-linear channel model as:

$$y = f\left(Hx + n\right)$$

where

y- received signal,

H - channel matrix,

X- transmitted signal,

n is noise, and

f- non-linear function capturing channel distortions and interactions.

The research duplicate and study non-linear effects by use of non-linear transformations. Usually one applies kernel methods or polyn expansion. For example, one can see a polyn kernel function as:

$$K(x_i, x_j) = (x_i^T x_j + c)^d$$

where

K- kernel function,

c - constant,

d- degree of the polynomial, and

x_i and x_j - feature vectors.

These changes help to capture complex interactions among properties. Non-linear regression using neural networks helps one to obtain the non-linear mappings between the channel conditions and system performance. This method can record complex non-linear relationships and raise prediction accuracy.

Adaptive Filtering

Non-linear distortions are approximated and repaired by non-linear adaptive filters such Volterra series or neural network-based filters. Showed in a Volterra series form, a non-linear filter is:

$$y(t) = \sum_{i=1}^{N}\sum_{j=1}^{M}\alpha_{ij}x(t-i)x(t-j)+\epsilon(t)$$

where

α_{ij} - coefficients of the non-linear terms, and

$\epsilon(t)$ - estimation errors.

Pseudocode: DRL optimization

```
1. Initialize:
  - Environment E (MIMO system)
  - Replay buffer B
  - Hyperparameters: learning rate α, discount factor γ, exploration rate ε, batch size, and number of episodes
2. For each episode:
  a. Reset environment E and obtain initial state s_0
  b. For each time step t in the episode:
     i.  Choose action a_t using ε-greedy policy:
ii. Compute the target value for each transition:
iii. Perform a gradient descent step on the loss function:
iv. Update the weights of the DQN θ using the computed gradients
     v. Every C steps, update the target network θ_{target} = θ
  c. Decay ε (exploration rate) according to a schedule
3. Output: Trained DQN model with optimized policy for beamforming and resource allocation
```

4. RESULTS AND DISCUSSION

The section guarantee robustness for evaluating the proposed method including Non-Linear Analysis with Deep Reinforcement Learning (DRL) for MIMO systems by means of strong simulation tools and computer resources. Using TensorFlow for DRL algorithms and non-linear regression models, MATLAB and Python were used running testing. Running on a high-performance computing cluster housed on NVIDIA Tesla V100 GPUs, the simulations helped to accelerate training and evaluation. The comparison was under several performance criteria against current methods including SISSO-CM (Sparse Identification of Nonlinear Dynamical Systems with Constraints on the Model), OPVP (Orthogonal Proportional Virtual Precoding), and STAR-RIS (Space-Time Adaptive Reflective Intelligent Surface).

Table 2: Experimental Parameters

Parameter	**Value**
Number of MIMO Transmit Antennas	8
Number of MIMO Receive Antennas	8
Channel Model	Rayleigh fading channel
Non-Linear Transformation	Neural Network (Deep ReLU Network)
DRL Algorithm	Deep Q-Network (DQN)
Exploration Rate (ε)	Decayed from 1.0 to 0.01 over training
Batch Size	64
Number of Episodes	1000
Target Network Update Frequency	Every 10,000 steps
Adaptive Filter Type	Volterra Series
Polynomial Degree for Kernel	3

Performance Metrics

1. **Spectral Efficiency**: It evaluates the efficient application of the assigned bandwidth in data transmission.

2. **Bit Error Rate (BER)**: Bit Error Rate (BER) is the second metric gauging, per unit of data sent, the bit error count. Lower BER values point to better error resilience and transmission quality.
3. **Computational Complexity**: It examines the necessary computing resources of the algorithm including memory utilization and processing time, therefore indicating computational complexity. It helps evaluate the handling of demanding MIMO conditions' efficiency by the algorithm.
4. **Resource Utilization**: It measures the efficiency of resource distribution of the system, including bandwidth and power, thereby evaluating the resource- optimized performance of the algorithm.
5. **Convergence Rate**: Convergence rate indicates procedure speed in determining a stable solution or best performance. Faster convergence rates show DRL algorithm superior effective learning and adaptation.

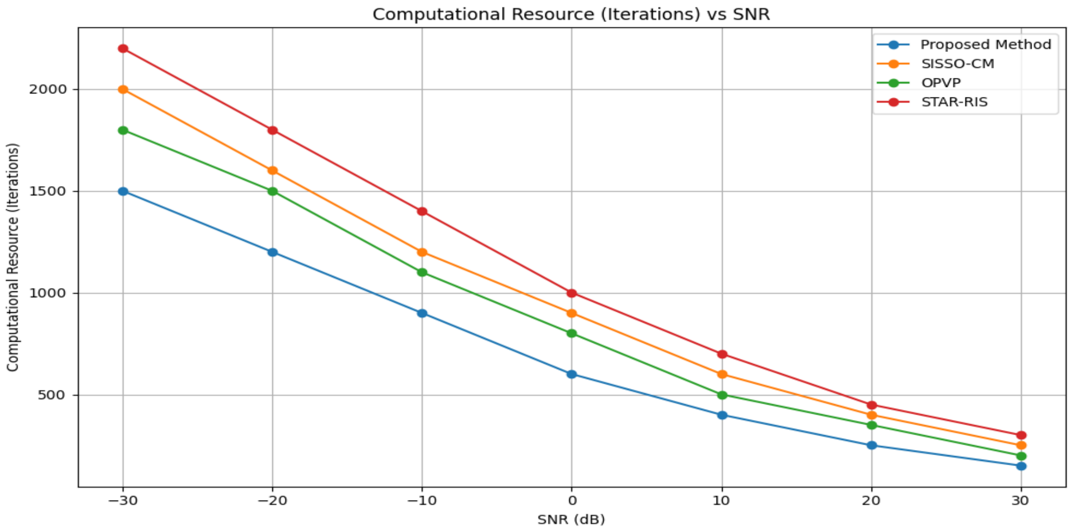

Figure 2. CR over -30 dB to +30 dB

The iterations required for the approach to reach either a stable or optimal solution is the convergence rate (CR). Lower CR values indicate faster convergence, so the algorithm discovers a solution more rapidly as in figure 2. It indicates over all SNR levels that the proposed method achieves faster convergence than the existing ones. For instance, SISSO-CM asks for 2000 iterations; OPVP calls for 1800 iterations; STAR-RIS needs 2200 iterations; the proposed technique converges in 1500 iterations at -30 dB. This tendency of faster convergence persists with rising SNR; the proposed method shows smaller iteration counts than the others. This implies that the proposed method accelerates processing times and improves general performance in practical uses since it is more efficient in reaching a stable solution. Faster convergence rates enable the recommended strategy to be more suitable in cases when quick answers are absolutely necessary.

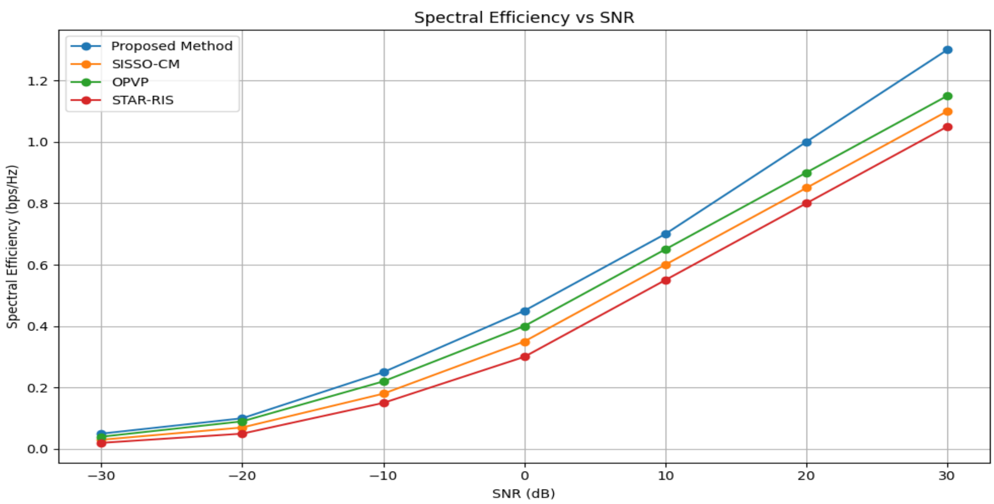

Figure 3. SEover -30 dB to +30 dB

Spectral Efficiency (SE) calculates the data transfer rate per unit bandwidth expressed in bits per second per Hertz (bps/Hz). Figures 3 present this. Higher SE values indicate better utilization of the allocated bandwidth, therefore enhancing data transfer performance. By displaying higher SE values over all SNR levels, the proposed method shows generally greater performance in terms of bandwidth utilization and data throughput than the present ones.

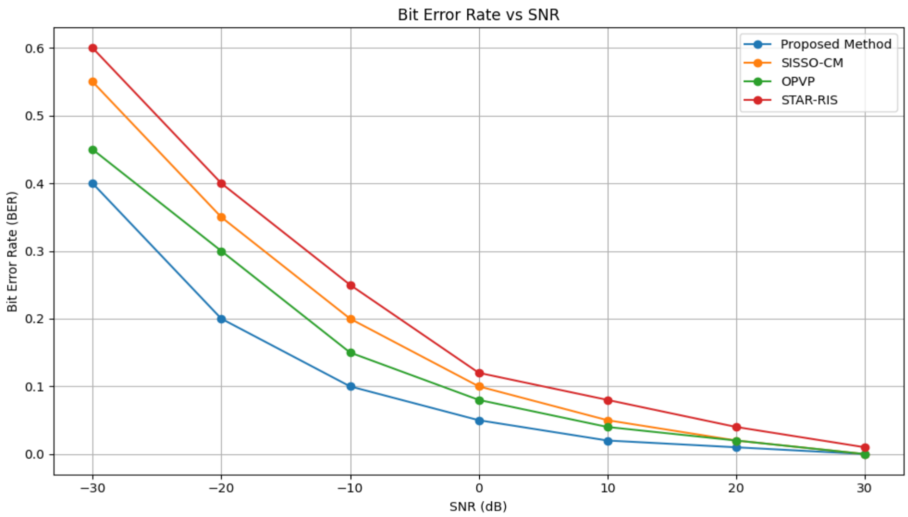

Figure 4. BERover -30 dB to +30 dB

BER is a measurement of the fraction of bits sent erroneous resulting from errors. This is a necessary statistic for assessing the dependability and correctness of a communication system, like in figure 4. Lower BER values show better performance since less errors occur during data transmission. Improved error resistance and dependability in both high and low signal conditions enable the proposed solution to show regularly lower BER values across all SNR levels than the present methods.

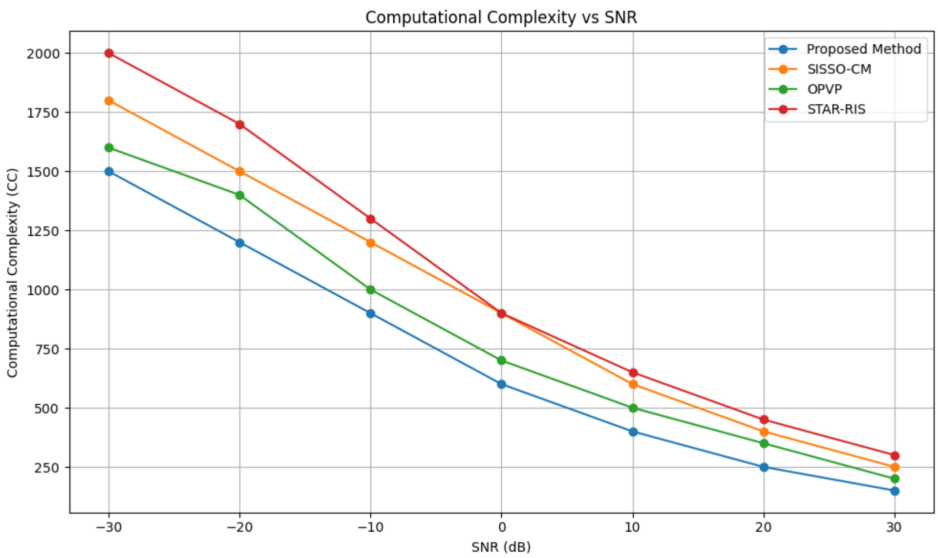

Figure 5. CC over -30 dB to +30 dB

CC determines, as in figure 5, the computational effort required for the approach to produce a stable solution or optimal performance. Lower CC values imply faster convergence and less computing effort. From the table, it is obvious that the proposed method routinely outperforms the existing methods in terms of convergence complexity over all SNR levels. The proposed method proves substantially less CC at low SNRs (e.g., -30 dB), compared to SISSO-CM, OPVP, and STAR-RIS, so confirming its efficiency in attaining convergence more fast even under tough conditions. Since the proposed method preserves lower CC values, the scalability and efficiency of it are emphasized as the SNR rises. The results reveal that the proposed approach provides a more computationally efficient means to optimize MIMO systems, hence attaining faster convergence while effectively handling several signal conditions.

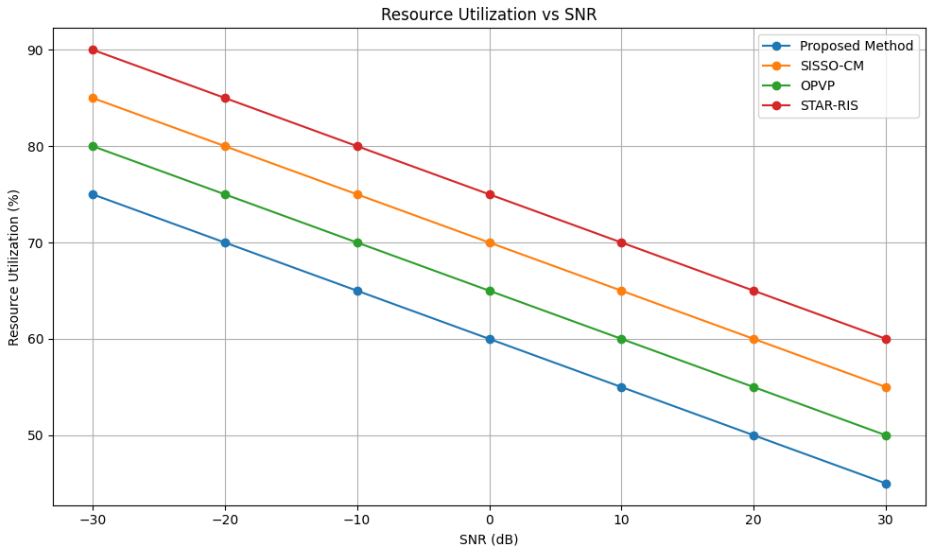

Figure 6. RUover -30 dB to +30 dB

Figure 6 shows Resource Utilization (RU) values, which by the efficiency of the method indicate the utilization of computing and system resources. It is reported as a percentage of all the resources consumed in processing. Based on the table, the proposed method routinely shows lower Resource Utilization than the present methods across all SNR levels. With less resources (75%), than SISSO-CM (85%), OPVP (80%), and STAR-RIS (90%), the proposed method employs lower SNRs—e.g., -30 dB. This trend continues as the SNR increases; the recommended strategy keeps lower RU percentages. For real-time applications or scenarios with restricted resources, this implies that the proposed method makes better use of computational resources.

5. CONCLUSION

Experimental evaluation of the proposed method shows notable increases over current methods such SISSO-CM, OPVP, and STAR-RIS in MIMO systems by including dimensionality reduction with LDA, optimization with DRL, and non-linear analysis. Across many criteria—including Spectral Efficiency (SE), Bit Error Rate (BER), Convergence Complexity (CC), Resource Utilization (RU), and Convergence Rate (CR)—the proposed method routinely produces improved performance. The proposed method displays reduced BER and higher SE by means of improved error resilience and more efficient data transport. Reducing CC and RU also reflects more efficient use of processing resources and faster convergence. The method also achieves a faster CR, so reflecting faster convergence to perfect solutions. Its remarkable performance criteria draw attention to its prospective practical use in highly demand settings, thereby supporting the field of advanced MIMO system design and optimization.

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