Takabayashi String Cosmological Model Coupled With Perfect Fluid Distribution In F(R,T) Gravity

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ABSTRACT
In this paper, we have investigated the Bianchi type VI_0 space-time in presence of perfect fluid distribution contained with one dimentional cosmic string within the context of *f(R,T)* gravity. The exact solutions to the non linear differential field equations have been obtained by considering the expansion scalar Θ is proportional to the shear scalar σ, power law form of an average scale factor and the takabayashi string's equation of state.The constructed model's kinematical and physical properties have been examined and graphically presented. Remarkably, the resulting model is similar to the latest observational data.

Keywords: Bianchi Type VI_0 Space-time, Perfect Fluid, Cloud String, $f(R,T)$ Gravity.

1. INTRODUCTION

The universe is well believed to be undergoing a late-time cosmic acceleration, as shown by many high redshift supernovae observation [1-3] . This accelerated expansion is supposed to be caused by the fluid known as dark energy. Recent astronomical observations indicate that 70% of the universe consists of dark energy with negative pressure. Many theoretical and experimental developments have been made to investigate the mysterious facets of the cosmos. In recent years, a number of prominent researchers have dedicated their efforts to conceptually understanding these unknown characteristics of the universe by building several cosmological models based on gravity theories. Amongst these modified gravity theories like $f(R), f(T), f(R,T), f(G), f(Q)$ and many more are providing satisfactory solutions to cosmological problems. Recently, the $f(R,T)$ gravity was developed by Harko et al.[4] as a generalization of $f(R)$ gravity. Together with a trace of the energy-momentum tensor *T*, the theory includes an arbitrary function of the Ricci scalar *R*. They have acquired both the test particle's equation of motion and the gravitational field equation in metric formalism. The $f(R,T)$ gravity models could justify the late time cosmic accelerated enlargement of the universe. Thakre et al. [5] studied behaviour of quark and strange quark matter for higher dimensional bianchi type-I universe in $f(R,T)$ gravity. Analysis of marder's space-time tsallis holographic dark energy cosmological model in $f(R,T)$ theory of gravity investigated by Ugale et al. [6] . Pawar et al. [7] have studied Kaluza–Klein string cosmological model in $f(R,T)$ theory of gravity.String fluid cosmological models are now generating a lot of interest since they are thought to play an essential role in the universe's early evolution and late-time accelerated expansion. The strings are only one-dimensional hypothetical topological defects that arise during the phase transition from a temperature in the early stages of evolution of the universe. Anisotropy in space-time is caused by strings, even if they are now undetectable. Many authors have recently conducted extensive research on string cosmological models of the cosmos because of the crucial role that strings play in explaining the evolution of the early stages of our universe. The universe is homogenous and anisotropic, and the isotropy process of these models can be explored throughout time, according to current observational data that supports Bianchi type cosmological models. Also, from a theoretical perspective, anisotropic universes are more general than isotropic model universes. Considering Bianchi type-II, -VI_0, -VIII and –IX space-time, The letelier string cosmological model in different context to obtain the exact solutions of the model studied by krori et al. [8]. Magnetized Bianchi type III string cosmological model for anti-stiff fluid in general relativity investigated by Chhajed et al. [9]. Magnetized string cosmology for perfect fluid distribution using Bianchi type III space-time explain by Bali & Pareek [10]. Katore et al. [11] inspected massive string-magnetized perfect fluid universe in the bimetric theory of gravitations. The accelerating, expanding, and anisotropic cosmic model was developed utilising perfect fluid coupled string cosmology and the Bianchi-type I metric by Chirde et al. [12]. Sahoo and Mishra [13] studied cosmic acceleration may result not only due to geometric contribution to the matter but also depends on matter contents of the universe in $f(R,T)$ gravity. The Bianchi-type I metric was used by Gaikwad et al. [14] to analyse the massive string magnetised barotropic perfect fluid cosmological model. Rani et al. [15] studied accelerating Bianchi type III perfect fluid string cosmological model within the context of $f(R,T)$ gravity. Pawar et al. [16] looked at the Perfect fluid and heat flow in $f(R,T)$ theory. Bali et al. [17] have investigated massive string magnetized barotropic perfect fluid cosmological model in general relativity using the Bianchi type I metric. Capozziello et al.[18] investigated (n + 1)-dimensional string-dilaton cosmology with an effective dilaton potential in the presence of perfect-fluid matter. Pawar and Dabre [19] studied bulk viscous string cosmological model with constant deceleration parameter in teleparallel gravity, By adopting a hybrid expansion law, Ram & Chandel [20] investigated the dynamics of a magnetised string universe. The string of cloud in presence of perfect fluid and decaying vacuum energy density Λ have been analyzed by Pradhan et al. [21]. Hatkar and Dudhe [22] studied bulk viscus dark fluid in research paper dark energy scenario in Metric $f(R)$ formalism. Mete & Dudhe [23] investigated Bianchi type I cosmological model with perfect fluid in modified $f(T)$ gravity. Again, in the context of perfect fluids and/or string fluids, the significant work conducted by several distinguished researchers [24-28]

In the present work , we are interested in studying Bianchi type VI_0 space-time to construct the takabayashi string cosmological model for perfect fluid distribution within the context of $f(R,T)$ gravity. The paper is organized as follows: Sec. 2, the basic concepts of $f(R,T)$ gravity are introduced. In Sec. 3, we have obtained the corresponding field equations by considering Bianchi type-VI_0 space-time. In Sec. 4, we discover the non-linear field equations together with solutions and calculated the different physical and kinematical quantities to study the cosmological implications and presented them graphically. Lastly, in Sec. 5, we have concluded the investigations.

2. BRIEF REVIEW OF $f(R,T)$ GRAVITY

The $f(R,T)$ gravity theory is the modification or generalization of Einstein's General Theory of Relativity, which has been proposed by Harko et al. [4]. The action for the modified $f(R,T)$ gravity is

$$S=\frac{1}{16\pi}\int\sqrt{-g}\,f(R,T)\,\mathrm{d}^4x+\int\sqrt{-g}\,L_m\,d^4x, \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, the trace T of the stress-energy tensor of the matter T_{ij} and L_m is the matter Lagrangian density. The stress-energy tensor T_{ij} for matter is defined as

$$T_{ij}=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{ij}}, \tag{2}$$

and its trace given by $T_{ij}=g^{ij}T_{ij}$.

By assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} rather than its derivative, so equation (2) leads to

$$T_{ij}=g_{ij}L_m-2\frac{\partial(L_m)}{\partial g^{ij}}. \tag{3}$$

By varying the action S in equation (1) with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ gravity theory are obtained as

$$f_R(R,T)\mathrm{R}_{ij}-\frac{1}{2}f(R,T)\mathrm{g}_{ij}+f_R(R,T)\left(\mathrm{g}_{ij}\Box-\nabla_i\nabla_j\right)=8\pi\mathrm{T}_{ij}-f_T(R,T)\mathrm{T}_{ij}-f_T(R,T)\Theta_{ij}, \tag{4}$$

with $\Theta_{ij}=g^{lm}\left(\frac{\delta T_{lm}}{\delta g^{ij}}\right)$, which follows from the relation $\delta\left(\frac{g^{lm}T_{lm}}{\delta g^{ij}}\right)=T_{ij}+\Theta_{ij}$, and $\Box=\nabla^i\nabla_i$,

$f_R(R,T)=\frac{\partial f(R,T)}{\partial R}$, $f_T(R,T)=\frac{\partial f(R,T)}{\partial T}$, and ∇_i are the covariant derivatives. The contraction of equation (4) yields

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)=\left(8\pi-f_T(R,T)\right)T-f_T(R,T)\Theta, \tag{5}$$

with $\Theta=g^{ij}\Theta_{ij}$. Eliminating $\Box f_R(R,T)$ from equations (4) and (5) we get,

$$f_R(R,T)\left(R_{ij}-\frac{1}{3}R\,g_{ij}\right)+\frac{1}{6}f(R,T)g_{ij}=F_1+F_2, \tag{6}$$

where $F_1=\left(8\pi-f_T(R,T)\right)\left(T_{ij}-\frac{1}{3}T\,g_{ij}\right)$, $F_2=-f_T(R,T)\Theta_{ij}+\frac{1}{3}f_T(R,T)\Theta g_{ij}+\nabla_i\nabla_j f_R(R,T)$.

From equation (2) we have

$$\frac{\delta T_{ij}}{\delta g^{lm}}=\left(\frac{\delta g_{ij}}{\delta g^{lm}}+\frac{1}{2}g_{ij}g_{lm}\right)L_m-2\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}-\frac{1}{2}g_{ij}T_{lm}. \tag{7}$$

Using the relation $\frac{\delta_{ij}}{\delta g^{lm}}=-g_{i\gamma}g_{j\sigma}\delta_{lm}^{\gamma\sigma}$ with $\delta_{lm}^{\gamma\sigma}=\frac{\delta T^{\gamma\sigma}}{\delta g^{lm}}$, Θ_{ij} is obtained as

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}. \tag{8}$$

We now consider the matter as perfect fluid and thus the stress-energy tensor of the matter Lagrangian is given by

$$T_{ij}=(\rho+p)u_i u_j-pg_{ij}, \tag{9}$$

where ρ and p are the respective energy density and pressure, $u^i=(1,0,0,0)$ are the co-moving coordinates for four velocities satisfying the conditions $u^iu_i=1$, and $u^i\nabla_j u_i=0$. Using (8) we have obtained the expression for the variation of stress energy of perfect fluid as

$$\Theta_{ij}=-2T_{ij}-pg_{ij}. \tag{10}$$

In this paper, we consider the cosmological consequences of the model proposed by Harko *et al.*, as

$$f(R,T)=R+2f(T), \tag{11}$$

where $f(T)$ is an arbitrary function of the trace of the stress-energy tensor of matter.

Combining equations (10) and (11), the field equation (4) of $f(R,T)$ gravity leads to

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}+2f'(T)T_{ij}+\left[2pf'(T)+f(T)\right]g_{ij}. \tag{12}$$

where the overhead prime denotes the differentiation with respect to the argument.

3. METRIC AND FIELD EQUATIONS

We consider Bianchi type-VI$_0$ metric as

$$ds^2=dt^2-A^2dx^2-B^2e^{2x}dy^2-C^2e^{-2x}dz^2, \tag{13}$$

where the scale factors *A, B,* and *C* are functions of cosmic time *t* only.

The energy-momentum tensor for string of clouds with perfect fluid distribution is given as

$$T_\mu^\nu=(\rho+p)u_\mu u^\nu-pg_\mu^\nu-\lambda x_\mu x^\nu, \tag{14}$$

in which u^μ denotes a four-velocity vector and x^μ denotes a unit space-like vector of the cloud string satisfying the conditions, $u^\mu u_\mu=1=-x^\mu x_\mu$ and $u^\mu x_\nu=0$, for $\mu\neq\nu$ and ρ is the proper energy density of the particle, *p* is the isotropic pressure, λ is the strings tension density.

In a co-moving coordinate system, we have

$$u^\mu=(0,0,0,1),\quad x^\mu=\left(A^{-1},0,0,0\right), \tag{15}$$

If the configuration of particle density is indicated by ρ_p, then we assume

$$\rho=\rho_p+\lambda. \tag{16}$$

The energy condition leads to $\rho\geq 0$ and $\rho_p\geq 0$, leaving the sign of λ unrestricted.

We consider the function $f(T)$ given by Harko et al. [4] as

$$f(T)=\mu T, \tag{17}$$

where μ is a constant. In the co-moving coordinate system, we obtained the field equations for Bianchi type-VI$_0$ space-time in the framework of $f(R,T)$ gravity as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}+\frac{1}{A^2}=8\pi\left(p-\lambda\right)+\mu\left(3p-3\lambda-\rho\right), \tag{18}$$

$$\frac{\dot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=8\pi p+\mu\left(3p-\lambda-\rho\right), \tag{19}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=8\pi p+\mu\left(3p-\lambda-\rho\right), \tag{20}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=-8\pi\rho+\mu\left(p-\lambda-3\rho\right), \tag{21}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{22}$$

where the overhead dot (.) denotes the derivative with respect to cosmic time t. Here we have five highly non-linear differential field equations with six unknowns, namely; $A, B, C, p, \lambda, \rho$.

We find some kinematical space-time quantities, as follows:

The average scale factor a and the spatial volume V are respectively defined as

$$a=\sqrt[3]{ABC}, \quad V=a^3. \tag{23}$$

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter H given by

$$H=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{1}{3}\left(H_1+H_2+H_3\right), \tag{24}$$

in which $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, and $H_3=\frac{\dot{C}}{C}$ denotes the directional Hubble parameters.

Using equations (23) and (24), we have obtained the expansion scalar Θ, mean anisotropy parameter Δ, shear scalar σ, and deceleration parameter q respectively as

$$\Theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=3H, \tag{25}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{26}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-\Theta^2\right), \tag{27}$$

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{28}$$

4. SOLUTION OF FIELD EQUATIONS

From equation (22), we get

$$B=\kappa C \tag{29}$$

where κ is an integrating constant but without loss of generality we consider $\kappa=1$.

Now using equation (29) and subtracting equation (18) from equation (19), we obtain

$$\left(8\pi+2\mu\right)\lambda=\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}-\frac{2}{A^2}. \tag{30}$$

Using equation (29) and subtracting equation (21) from equation (20), we obtain

$$\left(8\pi+2\mu\right)\left(p+\rho\right)=\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}. \tag{31}$$

For the deterministic solutions, we consider the shear scalar σ is proportional to the expansion scalar Θ which lead to the following analytic relation

$$A=B^n, \tag{32}$$

where n is a constant.

We consider the power law form of an average scale factor as

$$a=a_0t^m, \tag{33}$$

where a_0 and m are an arbitrary constants.
From equations (23), (29) and (32) we get the value of metric potential functions as

$$A=\left(a_0t^m\right)^{\frac{3n}{n+2}}, \quad B=C=\left(a_0t^m\right)^{\frac{3}{n+2}}. \tag{34}$$

Using equation (34) in equation (13), we get

$$ds^2=dt^2-\left(a_0t^m\right)^{\frac{6n}{n+2}}dx^2-\left(a_0t^m\right)^{\frac{6}{n+2}}\left(e^{2x}dy^2+e^{-2x}dz^2\right), \tag{35}$$

The spatial volume V, the mean Hubble's parameter H, the expansion scalar Θ, the mean anisotropy parameter Δ, the shear scalar σ, and the deceleration parameter q are obtained as

$$V=a_0^3t^{3m}, \tag{36}$$

$$H=\frac{m}{t}\quad, \tag{37}$$

$$\Theta=\frac{3m}{t}, \tag{38}$$

$$\Delta=\frac{2(n-1)^2}{(n+2)^2}, \tag{39}$$

$$\sigma^2=\frac{3m^2(n-1)^2}{t^2(n+2)^2}, \tag{40}$$

$$q=-1+\frac{1}{m}. \tag{41}$$

The Ratio

$$\frac{\sigma^2}{\Theta^2}=\frac{1}{3}\frac{(n-1)^2}{(n+2)^2}. \tag{42}$$

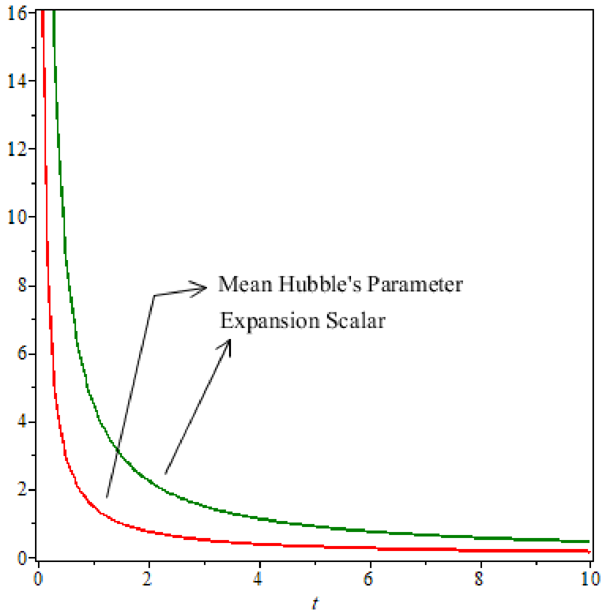

Figure 1. The variation of $H\,\&\,\Theta$ versus t for m = 1.5

From equation (36), it is observed that the volume is an power function of cosmic time that is ultimately zero in the begining but as time increases the volume increases continously and diverges at infinite time indicating the expansion of universe. The parameters discussed in equations (37) & (38) i.e. Hubble's parameter and expansion scalar whose graphical behaviour has been dipicted in Figure 1 has the infinitely large value in the begining but as time increases both the parameters drastically declines and dissapearse at infinite time. It indicates that the rate of expansion of the universe is high in the begining but with time it slows down. We are well known with the fact that the deceleration parameter symbolizes the inflation for $q<0$, deflation for $q>0$ and constant rate expansion for $q=0$. The equation (41) demonstrates the value of the deceleration parameter whose graphical behaviour with m has been discussed in Figure 2, in which when $0\le m<1$ we have observed the deflation phase, for $m=1$ constant rate evolution of universe while for the rest values of m we observed the inflationary cosmic accelerating phase. Furthermore, the mean anisotropy parameter and the ratio in equation (42) shows that the discussed model doesn't approach isotropy except for $n=1$.

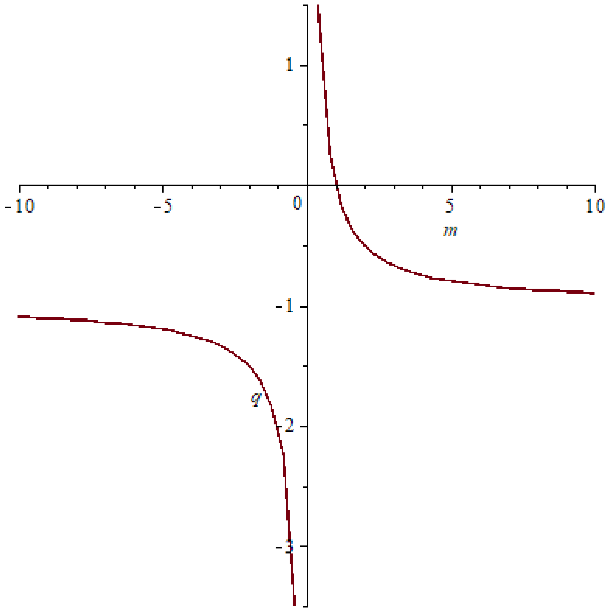

Figure 2. The variation of q versus m

From equation (30), we have obtained the tension density as

$$\lambda=\frac{3m(3m-1)(n-1)-2t^2(n+2)\left(a_0t^m\right)^{-\frac{6n}{n+2}}}{(8\pi+2\mu)(n+2)t^2}. \tag{43}$$

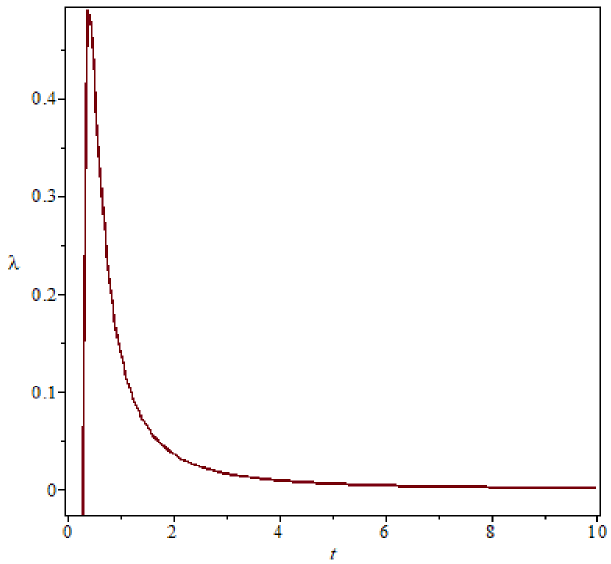

Figure 3. The variation of λ versus t for $m=1.5$, $\mu=0.7$, $n=2$, $a_0=2.2$

A positive value of string tension density shows the existence of the universe's string phase, whereas a negative value of λ suggests the disappearance of the universe's string phase, meaning that the universe is dominated by the cosmological constant [29]. Figure 3 presents the graphical representation of tension density versus time. From figure it is been observed that the tension density initially is a super-increasing function from negative to positive until 0.6, after which it immediately drops and gradually diminishes and vanishes. It demonstrates that the cosmos is first dominated by the cosmological constant for a short period of time before entering the string phase.

Now we consider the Takabayashi equation of state for the string cloud model as

$$\rho=(1+\omega)\lambda. \tag{44}$$

where ω is constant such that $\omega>0$.

Then the energy density is obtained as

$$\rho=\frac{(1+\omega)\left[3m(3m-1)(n-1)-2t^2(n+2)\left(a_0t^m\right)^{-\frac{6n}{n+2}}\right]}{(8\pi+2\mu)(n+2)t^2}. \tag{45}$$

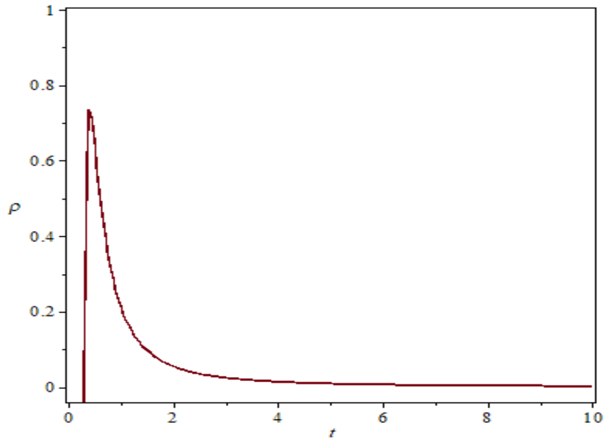

Figure 4. The variation of ρ Vs. t for $m=1.5$, $\mu=0.7$, $n=2$, $\omega=0.5$, $a_0=2.2$

The graphical representation of energy density versus time has been demonstrated in Figure 4, in which it has been observed that the energy density rises immediately from negative to positive in the initial phase of evolution, but thereafter declines immediately, diminishes and vanishes.

From equation (31) using equation (45) the pressure is obtained as

$$p=\frac{2t^2(n+2)^2(1+\omega)\left(a_0t^m\right)^{-\frac{6n}{n+2}}-3m\left[(n+2)(n-1)(3m-1)\omega+6m(n-1)+2(n+2)\right]}{(8\pi+2\mu)(n+2)^2t^2}. \tag{46}$$

Figure 5 shows the graphical representation of pressure versus time. It can be seen that pressure was positive for a brief period of time at first, but that it quickly decreased from positive to negative until it reached a certain extent, after which it rise in negative and disappeared.

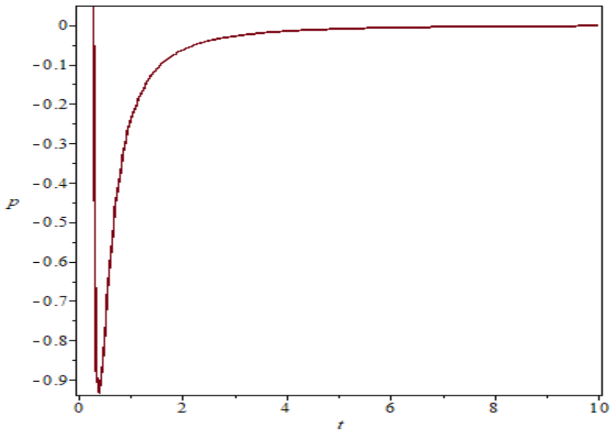

Figure 5. The variation of p versus t for $m=1.5$, $\mu=0.7$, $n=2$, $\omega=0.5$, $a_0=2.2$

5. CONCLUSIONS

Along the paper, the Bianchi type-VI$_0$ space-time in presence of string of clouds coupled with perfect fluid distributionhave been studied the context of $f(R,T)$ gravity. In order to derive exact solutions for the highly non-linear differential field equations, we have taken into consideration the fact that the shear scalar σ is proportional to the expansion scalar Θ, the power law form of an average scale factor and the Takabayashi equation of state for the string cloud model.

The constructed model is free from an initial singularity and anisotropic except for $n=1$. Furthermore, it is observed that the model is in decelerating phase for $0 \le m < 1$, for $m=1$ it is in constant rate evolution and having inflationary cosmic accelerating phase for the rest values of m.

From the observations of tension density, the deriverd model is first dominated by the cosmological constant for a short period of time but then enters in string phase. Additionally it has been observed that the energy density rises immediately from negative to positive in the initial phase of evolution, but thereafter declines immediately, diminishes, and vanishes. However the pressure was positive for a brief period of time at first, but that it quickly decreased from positive to negative until it reached a certain extent, after which it rise in negative and disappeared.

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