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Reachable sets for semilinear integrodifferential control systems

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Abstract

In this paper, we consider a control system for semilinear integrodifferential equations in Hilbert spaces with Lipschitz continuous nonlinear term. Our method is to find the equivalence of approximate controllability for the given semilinear system and the linear system excluded the nonlinear term, which is based on results on regularity for the mild solution. Finally, we give a simple example to which our main result can be applied.

Keywords: approximate controllability, semilinear control system, lipschtiz continuity, approximate controllability, reachable set

AMS Classification Primary 35B37; Secondary 93C20

1 Introduction

Let H and V be real Hilbert spaces such that V is a dense subspace in H. In this paper, we are concerned with the control results for the following retarded semilinear control system in Hilbert space H:

$$\begin{cases} x'(t) = Ax(t) + g(t, x(t), \int_0^t k(t, s, x(s))ds)) + Bu(t), \quad t > 0, \\ x(0) = x_0, \end{cases} \tag{1.1}$$

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where $t > 0$, B is a bounded linear controller, and $u(t)$ is an appropriate control functions. Let A be the operator associated with a bounded sesquilinear form defined in $V \times V$ satisfying Gårding inequality. Then it is well known that $S(t)$ generated by A is an analytic semigroup in both H and V^*, where V^* is the dual space of V, and so the system (1.1) may be considered as an system in both H and V^*. g is is a nonlinear mapping as detailed in Section 2.

Whether the reachable set associated with control space in dense subset of H. This is called an approximate controllability problem. As for linear evolution systems in general Banach, there are many papers and monographs, see [1, 2], Triggiani [3], Curtain and Zwart [4] and references and therein.

The controllability for nonlinear control systems has been studied by many authors, for example, control of nonlinear infinite dimensional systems in [5], controllability for parabolic equations with uniformly bounded nonlinear terms in [6], local controllability of neutral functional differential systems in [7].

Recently, the approximate controllability for semilinear control systems can be founded in [8, 9, 10], their results give sufficient condition on strict assumptions on the control action operator B. Similar considerations of semilinear systems have been dealt with in many references [11, 12, 13, 14].

We investigate the equivalence of approximate controllability for (1.1) such that excluded the nonlinear term and the controller. The solution mapping from the initial space to the solution space is Lipschitz continuous in $[0, T]$. We no longer require the strict range condition on B, and the uniform boundedness in [6] but instead we need the regularity and a variation of solutions of the given equations. For the basis of our study we construct the fundamental solution and establish variations of constant formula of solutions for the linear systems, see [15, 16].

Based on L^2-regularity properties of semilinear integrodifferential equations in Hilbert space and the regularity of solutions discussed in Section 2. We will obtain the relations between the reachable set of the semilinear system and that of its corresponding linear system in Section 3. Finally, a simple example to which our main result can be applied is given.

2 Regularity for retarded semilinear equations

If H is identified with its dual space we may write $V \subset H \subset V^*$ densely and the corresponding injections are continuous. The norms on V, H and V^* will be denoted by $||\cdot||$, $|\cdot|$ and $||\cdot||_*$, respectively. The duality pairing between the element v_1 of V^* and the element v_2 of V is denoted by (v_1, v_2), which is the ordinary inner product in H if $v_1, v_2 \in H$.

For $l \in V^*$ we denote (l, v) by the value $l(v)$ of l at $v \in V$. The norm of l as element of V^* is given by

$$||l||_* = \sup_{v \in V} \frac{|(l, v)|}{||v||}.$$

Therefore, we assume that V has a stronger topology than H and, for brevity, we may regard that

$$||u||_* \leq |u| \leq ||u||, \quad \forall u \in V. \tag{2.1}$$

Let $a(\cdot, \cdot)$ be a bounded sesquilinear form defined in $V \times V$ and satisfying Gårding's inequality

$$\text{Re } a(u, u) \geq \omega_1 ||u||^2 - \omega_2 |u|^2, \tag{2.2}$$

where $\omega_1 > 0$ and ω_2 is a real number. Let A be the operator associated with this sesquilinear form:

$$(Au, v) = -a(u, v), \quad u, \ v \in V. \tag{2.3}$$

Then A is a bounded linear operator from V to V^* by the Lax-Milgram Theorem. The realization of A in H which is the restriction of A to

$$D(A) = \{u \in V : Au \in H\}$$

is also denoted by A. It is well known that A generates an analytic semigroup in both of H and V^*(see [17]).

From the following inequalities

$$\omega_1 ||u||^2 \leq \text{Re}\, a(u, u) + \omega_2 |u|^2 \leq |Au|\,|u| + \omega_2 |u|^2 \leq \max\{1, \omega_2\} ||u||_{D(A)} |u|,$$

where

$$||u||_{D(A)} = (|Au|^2 + |u|^2)^{1/2}$$

is the graph norm of $D(A)$, it follows that there exists a constant $C > 0$ such that

$$||u|| \leq C ||u||_{D(A)}^{1/2} |u|^{1/2}. \tag{2.4}$$

Thus we have the following sequence

$$D(A) \subset V \subset H \subset V^* \subset D(A)^*, \tag{2.5}$$

where each space is dense in the next one, which is continuous injection.

Lemma 2.1. *With the notations (2.1), (2.4), and (2.5), we have*

$$(V, V^*)_{1/2,2} = H,$$
$$(D(A), H)_{1/2,2} = V,$$

where $(V, V^*)_{1/2,2}$ denotes the real interpolation space between V and V^*(Section 1.3.3 of [18]).

Assumption (K). Let $k : \mathbb{R}^+ \times [-h, 0] \times V \to H$ be a nonlinear mapping satisfying the following:

(K1) For any $x \in V$ the mapping $k(\cdot, \cdot, x)$ is measurable;

(K2) There exist positive constants K_0, K_1 such that

$$|k(t, s, x) - k(t, s, y)| \le K_1 ||x - y||,$$
$$|k(t, s, 0)| \le K_0$$

for all $(t, s) \in \mathbb{R}^+ \times [-h, 0]$ and $x, y \in V$.

Assumption (G). Let $g : \mathbb{R}^+ \times V \times H \to H$ be a nonlinear mapping satisfying the following:

(G1) For any $x \in V$, $y \in H$ the mapping $g(\cdot, x, y)$ is measurable;

(G2) There exist positive constants L_0, L_1, L_2 such that

$$|g(t, x, y) - g(t, \hat{x}, \hat{y})| \le L_1 ||x - \hat{x}|| + L_2 |y - \hat{y}|,$$
$$|g(t, 0, 0)| \le L_0$$

for all $t \in \mathbb{R}^+$, $x, \hat{x} \in V$, and $y, \hat{y} \in H$.

For $x \in L^2(-h, T; V)$, $T > 0$ we set

$$G(t, x) = g(t, x(t), \int_0^t k(t, s, x(s))ds).$$

The above operator g is the semilinear case of the nonlinear part of quasilinear equations considered by Yong and Pan [19]. The mild solution of (1.1) is represented by

$$x(t) = S(t)x_0 + \int_0^t \big\{G(s, x(s)0 + Bu(s)\big\}ds, \quad t \ge 0.$$

Lemma 2.2. *Let $x \in L^2(0, T; V)$, $T > 0$. Then $G(\cdot, x) \in L^2(0, T; H)$ and*

$$||G(\cdot, x)||_{L^2(0,T;H)} \le (L_0 + K_0 L_2)\sqrt{T} + (L_1 + L_2 K_1 T)||x||_{L^2(0,T;V)}.$$

Moreover if $x_1,\ x_2 \in L^2(0, T; V)$, then

$$||G(\cdot, x_1) - G(\cdot, x_2)||_{L^2(0,T;H)} \le (L_1 + L_2 K_1 T)||x_1 - x_2||_{L^2(0,T;V)}. \tag{2.6}$$

Proof. Hence, from (K2), (G2) and the above inequality it is easily seen that

$$\begin{aligned}
||G(\cdot,x)||_{L^2(0,T;H)} &\le ||G(\cdot,0)|| + ||G(\cdot,x) - G(\cdot,0)|| \\
&\le L_0\sqrt{T} + L_1||x||_{L^2(0,T;V)} + L_2||\int_0^{\cdot} k(\cdot,s,x(s))ds||_{L^2(0,T;H)} \\
&\le L_0\sqrt{T} + L_1||x||_{L^2(0,T;V)} + L_2K_1T||x||_{L^2(0,T;V)} + K_0L_2\sqrt{T} \\
&\le (L_0 + K_0L_2)\sqrt{T} + (L_1 + L_2K_1T)||x||_{L^2(0,T;V)}
\end{aligned}$$

Similarly, we can prove (2.6). □

In view of Lemma 2.2, we can apply the regularity results of Theorem 3.1 of [10] to (1.1), and so we obtain the following results.

Proposition 2.1. *1) Let $x_0 \in H$ and $k \in L^2(0,T;V^*)$, $T > 0$. Then there exists a unique solution x of (2.7) belonging to*

$$L^2(0,T;V) \cap W^{1,2}(0,T;V^*) \subset C([0,T];H)$$

and satisfying

$$||x||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)} \le C_1(|x_0| + ||k||_{L^2(0,T;V^*)}), \tag{2.7}$$

where C_1 is a constant depending on T.

2) If $x_0 \in H$ and $k \in L^2(0,T;V^*)$, then the mapping

$$H \times L^2(0,T;V^*) \ni (x_0,k) \mapsto x \in L^2(0,T;V) \cap W^{1,2}(0,T;V^*)$$

is Lipschitz continuous.

Here, we note that by using interpolation theory, we have that for $z \in L^2(0,T;V) \cap W^{1,2}(0,T;V^*)$, there exists a constant $C_2 > 0$ such that

$$||z||_{C([0,T];H)} \le C_2||z||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)}. \tag{2.8}$$

3 Approximately reachable sets

Let U be a Banach space and the controller operator B is bounded linear operator from another Banach space U to X.

Let $S(t)$ be an analytic semigroup generation by A. Then we may assume that there exists a positive constant C_0 such that

$$||S(t)|| \leq C_0, \quad ||AS(t)|| \leq C_0/t(t > 0). \tag{3.1}$$

The solution $x(t) = x(t; x_0, G, u)$ of initial value problem (1,1) is the following form:

$$x(t; x_0, G, u) = S(t)x_0 + \int_0^t S(t-s)\{G(t, x(s)) + Bu(s)\}ds, \quad t > 0,$$

For $T > 0$, $x_0 \in H$ and $u \in L^2(0, T; U)$ we define reachable sets as follows.

$$\begin{aligned} L_T(x_0) &= \{x(T; x_0, 0, u) : u \in L^2(0, T; U)\}, \\ R_T(x_0) &= \{x(T; x_0, G, u) : u \in L^2(0, T; U)\}, \\ L(x_0) &= \bigcup_{T>0} L_T(x_0), \quad R(x_0) = \bigcup_{T>0} R_T(x_0). \end{aligned}$$

Definition 3.1. *(1) System (1.1) is said to be H-approximately controllable for initial value x_0 (resp. in time T) if $\overline{R(x_0)} = H$ (resp. $\overline{R_T(x_0)} = H$).*
(2) The linear system corresponding (1.1) is said to be H-approximately controllable for initial value x_0 (resp. in time T) if $\overline{L(x_0)} = H$ (resp. $\overline{L_T(x_0)} = H$).

Remark 3.1. *Since A generate an analytic semigroup, the following (1)-(4) are equivalent for the linear system (see [2, Theorem 3.10]).*

(1) $\overline{L(x_0)} = H \quad \forall x_0 \in H.$

(2) $\overline{L(0)} = H.$

(3) $\overline{L_T(x_0)} = H \quad \forall x_0 \in H.$

(4) $\overline{L_T(0)} = H.$

Theorem 3.1. *For any $T > 0$ we have*

$$\overline{R_T(0)} \subset \overline{L_T(0)}.$$

Proof. Let $z_0 \notin \overline{L_T(0)}$. Since $\overline{L_T(0)}$ is a balanced closed convex subspace, we have $\alpha z_0 \notin \overline{L_T(0)}$ for every $\alpha \in \mathbb{R}$, and

$$\inf\{||z_0 - z|| : z \in \overline{L_T(0)}\} = d.$$

By the formula (2.7) we have

$$||x(\cdot; 0, G, u)||_{L^2(0,T;V)} \leq C_1||B||||u||_{L^2(0,T;U)}, \tag{3.2}$$

where C_1 is the constant in Proposition 2.1. For every $u \in L^2(0,T;U)$, we choose a constant $\alpha > 0$ such that

$$C_0\{(L_0 + K_0L_2)\sqrt{T} + (L_1 + L_2K_1T)C_1||B||||u||_{L^2(0,T;U)}\} < \alpha d. \tag{3.3}$$

Hence form (3.2), (3.3) and by using Hölder inequality, it follows that

$$\begin{aligned}
&|x(T;0,G,u) - \alpha z_0| \\
&\geq |\int_0^T S(T-s)Bu(s)ds - \alpha z_0| - |\int_0^T S(T-s)G(s,x(s))ds| \\
&\geq \alpha d - C_0\{(L_0 + K_0L_2)\sqrt{T} + (L_1 + L_2K_1T)||x||_{L^2(0,T;V)}\} \\
&\geq \alpha d - C_0\{(L_0 + K_0L_2)\sqrt{T} + (L_1 + L_2K_1T)C_1||B||||u||_{L^2(0,T;U)}\} > 0.
\end{aligned}$$

Thus, we have $\alpha z_0 \notin \overline{R_T(0)}$. □

Lemma 3.1. *Suppose that $k \in L^2(0,T;H)$ and $x(t) = \int_0^t S(t-s)k(s)ds$ for $0 \leq t \leq T$. Then there exists a constant C_3 such that*

$$||x||_{L^2(0,T;D(A))} \leq C_1||k||_{L^2(0,T;H)}, \tag{3.4}$$

$$||x||_{L^2(0,T;H)} \leq C_3T||k||_{L^2(0,T;H)}, \tag{3.5}$$

and

$$||x||_{L^2(0,T;V)} \leq C_3\sqrt{T}||k||_{L^2(0,T;H)}. \tag{3.6}$$

Proof. The assertion (3.4) is immediately obtained by (2.7). Since

$$\begin{aligned}
||x||^2_{L^2(0,T;H)} &= \int_0^T |\int_0^t S(t-s)k(s)ds|^2dt \leq C_0\int_0^T(\int_0^t |k(s)|ds)^2dt \\
&\leq C_0\int_0^T t\int_0^t |k(s)|^2dsdt \leq C_0\tfrac{T^2}{2}\int_0^T |k(s)|^2ds
\end{aligned}$$

it follows that

$$||x||_{L^2(0,T;H)} \leq T\sqrt{C_0/2}||k||_{L^2(0,T;H)}.$$

From (2.4), (3.4), and (3.5) it holds that

$$||x||_{L^2(0,T;V)} \leq C\sqrt{C_1T}(M/2)^{1/4}||k||_{L^2(0,T;H)}.$$

So, if we take a constant $C_3 > 0$ such that

$$C_3 = \max\{\sqrt{C_0/2}, C\sqrt{C_1}(C_0/2)^{1/4}\},$$

the proof is complete. □

Theorem 3.2. *Under Assumptions (K) and (G), for any $x_0 \in H$ we have*

$$\overline{L_T(x_0)} \subset \overline{R_T(x_0)}.$$

.

Proof. Let $u \in L^2(0,T;U)$ be arbitrary fixed. Then by (2.7) we have

$$||x_u||_{L^2(0,T;V)} \leq C_1(|x_0| + ||B||||u||_{L^2(0,T;U)}),$$

where x_u is the solution of (1.1) corresponding to the control u. For any $\epsilon > 0$, we can choose a constant $\delta > 0$ satisfying

$$\begin{aligned} \min\{\sqrt{\delta}, \delta\} < \min\big[&\big\{7C_3(L_1 + L_2K_1T))\big\}^{-1}, \\ &\epsilon\big\{C_3(L_0 + K_0L_2\sqrt{T})\big\}^{-1}, \\ &\epsilon\big\{C_3(L_1 + L_2K_1T)(C_1C_2||x_u||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)} + \epsilon)\big\}^{-1}, \\ &\epsilon\big\{C_3(C_0||x_u||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)} + \epsilon)(L_1 + L_2K_1T)\big\}^{-1}, \\ &\epsilon\big\{(C_3^2(L_0 + K_0L_2)\sqrt{T} + \epsilon)(L_1 + L_2K_1T)\big\}^{-1}\big]/6. \end{aligned} \tag{3.7}$$

Set

$$\begin{aligned} x_1 := x(T-\delta; x_0, G, u) = S(T-\delta)x_0 + \\ + \int_0^{T-\delta} S(T-\delta-s)G(s, x_u(s))ds + \int_0^{T-\delta} S(T-\delta-s)Bu(s)ds, \end{aligned}$$

where $x_u(t) = x(t; x_0, G, u)$ for $0 < t \leq T$. Consider the following problem:

$$\begin{cases} y'(t) = Ay(t) + Bu(t), \quad \delta < t \leq T, \\ y(T-\delta) = x_1, \quad y(s) = 0 \quad -h \leq s \leq 0. \end{cases} \tag{3.8}$$

The solution of (3.8) with respect to the control $w \in L^2(T-\delta, T; U)$ is denoted by

$$\begin{aligned} y_w(T) &= S(\delta)x_1 + \int_{T-\delta}^{T} S(T-s)Bw(s)ds \\ &= S(T)x_0 + S(\delta)\int_0^{T-\delta} S(T-\delta-s)G(s, x_u(s))ds \\ &\quad + S(\delta)\int_0^{T-\delta} S(T-\delta-s)Bu(s)ds + \int_{T-\delta}^{T} S(T-s)Bw(s)ds. \end{aligned} \tag{3.9}$$

Then since $z \in \overline{L_T(x_0)}$, and $\overline{L_T(x_0)} = \overline{L(0)}$ is independent of the time T and initial data x_0(see Remark 2.1), there exists $w_1 \in L^2(T-\delta, T; U)$ such that

$$\sup_{T-\delta \le t \le T} |y_{w_1}(t) - z| < \frac{\epsilon}{6}, \tag{3.10}$$

and hence, by (3.9),

$$|\int_{T-\delta}^{t} S(T-s)Bw_1(s)ds| \le C_0 ||x_u||_{L^2(0,T-\delta;V)} + \frac{\epsilon}{6}, \quad t-\delta \le t \le T. \tag{3.11}$$

Now, we set

$$v(s) = \begin{cases} u & \text{if } 0 \le s \le T-\delta, \\ w_1(s) & \text{if } T-\delta < s < T. \end{cases}$$

Then $v \in L^2(0, T; U)$. Observing that

$$x_v(t; G, v) = S(t)x_0 + \int_0^t S(t-\tau)\{G(\tau, x_v(\tau)) + Bv(\tau)\}d\tau,$$

from (3.9) and (3.10) we obtain that

$$\begin{aligned} &|x(T; x_0, G, v) - z| \le |y_{w_1}(T) - z| + |x(T; x_0, G, v) - y_{w_1}(T)| \\ &\le |y_{w_1}(T) - z| \\ &\quad + \Big| \int_0^T S(T-s)G(s, x_v(s))ds - S(\delta)\int_0^{T-\delta} S(T-\delta-s)G(s, x_u(s))ds \Big| \\ &\quad + \Big| \int_0^T S(T-s)Bv(s)ds - S(\delta)\int_0^{T-\delta} S(T-\delta-s)Bu(s)ds \\ &\quad - \int_{T-\delta}^T S(T-s)Bw_1(s)ds \Big| \\ &\le \frac{\epsilon}{6} + \Big| \int_{T-\delta}^T S(T-s)G(s, x_{w_1}(s))ds \Big| \\ &\le \frac{\epsilon}{6} + II. \end{aligned} \tag{3.12}$$

Here, we remind that the x_{w_1} is represented by

$$\begin{aligned} x_{w_1}(t) =& S(t)x(T-\delta; x_0, G, u) \\ &+ \int_{T-\delta}^t S(T-s)G(s, x_{w_1}(s))ds + \int_{T-\delta}^t S(T-s)Bw_1(s))ds \end{aligned}$$

for $T-\delta < t \le T$. Here, by (2.7) we have

$$\begin{aligned} ||S(\cdot)x(T-\delta; x_0, G, u)||_{L^2(0,T;V)} &\le C_1|x(T-\delta; x_0, G, u)| \\ &\le C_1C_2||x_u||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)}. \end{aligned} \tag{3.13}$$

Put

$$p(t) = \int_{T-\delta}^{t} S(t-s)G(s, x_{w_1}(s))ds, \quad T-\delta < t \le T,$$

and

$$q(t) := \int_{t-\delta}^{T} S(t-s)Bw_1(s)ds \quad T-\delta < t \le T.$$

Then with aid of (3.6) of Lemma 3.1 and Lemma 2.2, we have

$$\begin{aligned} &||p||_{L^2(T-\delta,T;V)} \le C_3\sqrt{\delta}||G(\cdot, x_{w_1})||_{L^2(T-\delta,T;V)} \\ &\le C_3\sqrt{\delta}\{(L_0 + K_0L_2)\sqrt{T} + (L_1 + L_2K_1T)||x_{w_1}||_{L^2(T-\delta,T;V)}\}, \end{aligned} \tag{3.14}$$

and by (3.11),

$$||q||_{L^2(T-\delta,T;V)} \le \sqrt{\delta}(C_0||x_u||_{L^2(0,T-\delta;V)} + \frac{\epsilon}{6}). \tag{3.15}$$

Since $C_3\sqrt{\delta}(L_1 + L_2K_1T)) < 1$ by virtue of (3.7), by (3.13)-(3.15), we get

$$\begin{aligned} ||x_{w_1}||_{L^2(T-\delta,T;V)} \le &\{C_1C_2||x_u||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)} \\ &+ \sqrt{\delta}(C_0||x_u||_{L^2(0,T-\delta;V)} + \frac{\epsilon}{6}) \\ &+ C_3\sqrt{\delta T}(L_0 + K_0L_2)\}\{1 - C_3\sqrt{\delta}(L_1 + L_2K_1T))\}^{-1}. \end{aligned} \tag{3.16}$$

Hence, with aid of (3.6), (3.7), (3.16), and by using the Hölder inequality, we have

$$\begin{aligned} II =& |\int_{T-\delta}^{T} S(T-s)G(s, x_{w_1}(s))ds| \\ &\le C_3\sqrt{\delta T}\{(L_0 + K_0L_2) + (L_1 + L_2K_1T)||x_{w_1}||_{L^2(T-\delta,T;V)}\} \\ &\le C_3\sqrt{\delta T}(L_0 + K_0L_2) + C_3\sqrt{\delta}(L_1 + L_2K_1T)\big\{C_1C_2||x_u||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)} \\ &+ \sqrt{\delta}(C_0||x_u||_{L^2(0,T-\delta;V)} + \frac{\epsilon}{6}) \\ &+ C_3\sqrt{\delta T}(L_0 + K_0L_2)\big\}\big\{1 - C_3\sqrt{\delta}(L_1 + L_2K_1T))\big\}^{-1} < \frac{5\epsilon}{6}. \end{aligned} \tag{3.17}$$

Therefore, by (3.12) and (3.17), we have

$$||x(T; x_0, G, v) - z||_H < \epsilon,$$

that is, $z \in \overline{R_T(x_0)}$ and the proof is complete. □

Remark 3.2. *Noting that $H([0,T];U)$ is dense in $L^2(0,T;U)$, we can obtain the same results of Theorem 3.2 corresponding to (1.1) with control space*

$$H([0,T];U) = \{w : [0,T] \to U : |w(t) - w(s)| \le H_0|t-s|^\theta,\ 0 < \theta < 1,\ H_0 > 0\}$$

instead of $L^2(0,T;U)$

From Theorems 3.1-2, we obtain the following control results of (1.1).

Corollary 3.1. *Under Assumptions (K) and (G), for $T > 0$ we have*

$$\overline{L_T(x_0)} = H \iff \overline{R_T(x_0)} = H.$$

Therefore, the approximate controllability of linear system (1.1) with $g = 0$ is equivalent to the condition for the approximate controllability of the nonlinear system (1.1).

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