Certain Integrals Involving the Incomplete Fox-Wright Functions

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Abstract

Hundreds of special functions have been employed in applied mathematics and computing sciences for many centuries due to their outstanding features and wide range of applications. When considering the relevance of these consequences in the evaluation of generalized integrals, applied physics, and many engineering areas, the illustration of image formulas involving one or more variable special functions is significant under various definite integrals. In this paper, it is devoted to study the various integral identities involving incomplete Fox-Wright functions and Srivastava's polynomials. It is shown that the integrals of the Fox-Wright functions are also the Fox-Wright functions but of greater order. Due to the fact that our results are unified, a substantial number of new results can be constructed as special instances from our leading results. The results obtained in this work are general in nature and very useful in science, engineering and finance.

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1 Introduction and Preliminaries

Several integral formulas have been established that include variety of special functions and play a major role in certain physical problems [21]. Such functions, in addition, are frequently connected to a variety of issues in various branches of mathematics. Therefore, a notable researcher has produced several integral formulas in turning a particular category of special functions [14], for instance Nisar et al. [9] evaluated unified integrals associated with the Struve function; Suthar et al. [19] evaluated unified integrals associated with the hypergeometric function; Choi et al. [2], Choi et al. [3], Menaria et al. [8], Nisar et al. [9], Suthar and Habenom [20] established certain integrals involving Bessel type functions.

In this study, under certain known integrals, we look at the possibility of having some new integrals that include incomplete Fox-Wright functions as well as family of polynomials. The integral formulas established in the present work are very useful to obtain the transformations of various simpler special functions. The findings from this research are of a generic nature and are extremely beneficial in the fields of engineering, economics, and chemical sciences, digital signals, image processing, finance and ship target recognition by sonar system and radar signals.

For our ends, we begin by looking back on the preceding incomplete Fox-Wright functions (see [18] also), ${}_p\Psi_q^{(\gamma)}$ and ${}_p\Psi_q^{(\Gamma)}$, with both the p numerator and q denominator parameters, introduced by Choi et. al [4]:

$$ {}_p\Psi_q^{(\gamma)}\left[\begin{array}{rc} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; & z \\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; & \end{array}\right] = \sum_{\ell=0}^{\infty}\frac{\gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)}\frac{z^\ell}{\ell!} \tag{1} $$

and

$$ {}_p\Psi_q^{(\Gamma)}\left[\begin{array}{rc} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; & z \\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; & \end{array}\right] = \sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)}\frac{z^\ell}{\ell!}\,, \tag{2} $$

where, $\mathfrak{F}_j,\ \mathfrak{G}_j\ \in \mathbb{R}^+,\ \mathfrak{f}_j,\ \mathfrak{g}_j\ \in \mathbb{C}$ and series converges absolutely $\forall z \in \mathbb{C}$ when $\Delta = 1+\sum_{j=1}^{q}\mathfrak{G}_j - \sum_{j=1}^{p}\mathfrak{F}_j\ > 0$ (see [7,16]).

The incomplete Fox-Wright functions, ${}_p\Psi_q^{(\gamma)}$ and ${}_p\Psi_q^{(\Gamma)}$ fulfill the decomposition formula given below:

$$ {}_p\Psi_q^{(\gamma)}[z] + {}_p\Psi_q^{(\Gamma)}[z] = {}_p\Psi_q[z], \tag{3} $$

where, ${}_p\Psi_q[z]$ is Fox-Wright function [22].

Srivastava's polynomials [15] or broader category of polynomials of index $n(n = 0,1,2,...)$ are described as follows (see [1,13] also):

$$ S_n^m[x] = \sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\,x^s, \tag{4} $$

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where $m \in \mathbb{Z}^+$ and $A_{n,\,s}(n,\, s \geq 0)$ are real or complex numbers arbitrary constant. The notations "$[\,.\,]$" and $(-n)_m$, respectively represent greatest integer function and Pochhammer symbol. Numerous well-known polynomials are provided by Srivastava's polynomials as special cases for appropriately specializing the coefficient $A_{n,\,s}$.

The rest of the paper is organized as follows. In section 2 incomplete Fox-Wright function and Srivastava polynomial are combined, and various integrals like: Oberhettinger type integrals, Lavoie type integral, MacRobert type integral and integral defined by Srivastava and Panda have been established. In section 3, we develop the particular instances of the main findings by specializing the parameters. In section 4, the paper is completed by presenting concluding remark of the paper.

2 Main Results

In this part, we use some existing integral identities which shall be helpful to develop new results involving incomplete Fox-Wright functions, ${}_p\Psi_q^{(\gamma)}$ and ${}_p\Psi_q^{(\Gamma)}$.

2.1 Integral defined by Srivastava and Panda

Srivastava and Panda [17] defined and studied the following integral

$$\int_0^\infty u^{\varpi-1}(u+v)^{-\varsigma}\, du = \frac{\Gamma(\varpi)\Gamma(\varsigma-\varpi)}{\Gamma(\varsigma)} v^{\varpi-\varsigma} \tag{5}$$

with $\Re(\varsigma) > \Re(\varpi) > 0$.

Theorem 1. *If $\Delta,\, \varkappa,\, \varrho,\, \vartheta,\, \Omega > 0$ and $\Re(\varsigma) > \Re(\varpi) > 0$, then*

$$\begin{aligned}
&\int_0^\infty u^{\varpi-1}(u+v)^{-\varsigma}\, S_n^m[u^{\varkappa}(u+v)^{-\varrho}]\, {}_p\Psi_q^{(\Gamma)}[z\, u^{\vartheta}(u+v)^{-\Omega}]du \\
&= v^{\varpi-\varsigma} \sum_{s=0}^{[n/m]} \frac{(-n)_{m\,s}}{s!} A_{n,\,s}\, v^{s(\varkappa-\varrho)} \times \\
&\quad {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{l} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\varpi+s\varkappa, \vartheta), (\varsigma-\varpi+s(\varrho-\varkappa), \Omega-\vartheta), \\ (\varsigma+s\varrho, \Omega), \end{array}\right. \\
&\qquad\qquad \left.\begin{array}{l} (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\,p}\,; \\ (\mathfrak{g}_j, \mathfrak{G}_j)_{1,\,q}\,; \end{array} \; z v^{\vartheta-\Omega} \right].
\end{aligned} \tag{6}$$

Proof. To demonstrate the outcome (6), we begin with L.H.S. Let us consider

$$I = \int_0^\infty u^{\varpi-1}(u+v)^{-\varsigma}\, S_n^m[u^{\varkappa}(u+v)^{-\varrho}]\, {}_p\Psi_q^{(\Gamma)}[z\, u^{\vartheta}(u+v)^{-\Omega}]du. \tag{7}$$

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Using the definitions (2) and (4) in (7), we obtain

$$I=\int_0^\infty u^{\varpi-1}(u+v)^{-\varsigma}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\,u^{s\varkappa}(u+v)^{-s\varrho}$$
$$\times\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)}\frac{z^\ell\,u^{\ell\vartheta}(u+v)^{-\ell\Omega}}{\ell!}du$$
$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)\;\ell!}z^\ell$$
$$\times\int_0^\infty u^{\varpi+s\varkappa+\ell\vartheta-1}(u+v)^{-\varsigma-s\varrho-\ell\Omega}du$$
$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)\;\ell!}z^\ell$$
$$\times\frac{\Gamma(\varpi+s\varkappa+\ell\vartheta)\Gamma(\varsigma+s\varrho+\ell\Omega-\varpi-s\varkappa-\ell\vartheta)}{\Gamma(\varsigma+s\varrho+\ell\Omega)}v^{\varpi-\varsigma+s(\varkappa-\varrho)+\ell(\vartheta-\Omega)},\ \text{(using (5))}$$

$$=v^{\varpi-\varsigma}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\,v^{s(\varkappa-\varrho)}\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)}$$
$$\times\frac{\Gamma(\varpi+s\varkappa+\ell\vartheta)\Gamma(\varsigma-\varpi+s(\varrho-\varkappa)+\ell(\Omega-\vartheta))}{\Gamma(\varsigma+s\varrho+\ell\Omega)}\frac{z^\ell\,v^{\ell(\vartheta-\Omega)}}{\ell!}.$$

Next, using (2) we achieved the desired outcome (6). □

The proof of below theorem is similiar to Theorem 1, so it is claim here without proof.

Theorem 2. *If* $\Delta,\ \varkappa,\ \varrho,\ \vartheta,\ \Omega>0$ *and* $\Re(\varsigma)>\Re(\varpi)>0$*, then*

$$\int_0^\infty u^{\varpi-1}(u+v)^{-\varsigma}\,S_n^m[u^{\varkappa}(u+v)^{-\varrho}]\,{}_p\Psi_q^{(\gamma)}[z\,u^{\vartheta}(u+v)^{-\Omega}]du$$
$$=v^{\varpi-\varsigma}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\,v^{s(\varkappa-\varrho)}\times$$
$${}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{matrix}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma-\varpi+s(\varrho-\varkappa),\Omega-\vartheta)\,,\\ (\varsigma+s\varrho,\Omega),\end{matrix}\right.$$
$$\left.\begin{matrix}(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{matrix}\quad zv^{\vartheta-\Omega}\right].\tag{8}$$

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2.2 Oberhettinger type integral

Oberhettinger [11] gives an integral formula as (see [10] also):

$$\int_0^\infty u^{\varpi-1}\left(u+k+\sqrt{u^2+2ku}\right)^{-\varsigma}du = 2\varsigma k^{-\varsigma}\left(\frac{k}{2}\right)^{\varpi}\frac{\Gamma(2\varpi)\Gamma(\varsigma-\varpi)}{\Gamma(\varpi+\varsigma+1)}, \quad (9)$$

with $\varpi, \varsigma \in \mathbb{C}$ and $0 < (\varpi + \overline{\varpi}) < (\varsigma + \overline{\varsigma})$.

Theorem 3. *If* $\Delta > 0$, $\varkappa > 0, \vartheta > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi+\overline{\varpi}) < (\varsigma+\overline{\varsigma})$, *then*

$$\int_0^\infty u^{\varpi-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\, S_n^m[u^{\varkappa}]\, {}_p\Psi_q^{(\Gamma)}[z\, u^{\vartheta}]du$$

$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\left(\frac{k}{2}\right)^{s\varkappa}\times$$

$${}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{matrix}(\mathfrak{f}_1,\mathfrak{F}_1,x),(2\varpi+2s\varkappa,2\vartheta),(\varsigma-\varpi-s\varkappa,-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (1+\varpi+\varsigma+s\varkappa,\vartheta),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{matrix}\; z\left(\tfrac{k}{2}\right)^{\vartheta}\right]. \quad (10)$$

Proof. To demonstrate the outcome (10), we begin with L.H.S. Let us consider

$$I = \int_0^\infty u^{\varpi-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\, S_n^m[u^{\varkappa}]\, {}_p\Psi_q^{(\Gamma)}[z\, u^{\vartheta}]du. \quad (11)$$

Using the definitions (2) and (4) in (11), after simplification we obtain

$$I = \sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)\ \ell!}z^{\ell}$$

$$\times\int_0^\infty u^{\varpi+s\varkappa+\ell\vartheta-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}du$$

$$= \sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\sum_{\ell=0}^{\infty}\frac{\Gamma(\mathfrak{f}_1+\mathfrak{F}_1\ell,x)\prod_{j=2}^{p}\Gamma(\mathfrak{f}_j+\mathfrak{F}_j\ell)}{\prod_{j=1}^{q}\Gamma(\mathfrak{g}_j+\mathfrak{G}_j\ell)\ \ell!}z^{\ell}$$

$$\times\, 2\varsigma k^{-\varsigma}\left(\frac{k}{2}\right)^{\varpi+s\varkappa+\ell\vartheta}\frac{\Gamma(2\varpi+2s\varkappa+2\ell\vartheta)\Gamma(\varsigma-\varpi-s\varkappa-\ell\vartheta)}{\Gamma(1+\varpi+\varsigma+s\varkappa+\ell\vartheta)},\ \text{(using (9))}.$$

Next, using (2) we achieved the desired outcome (10). □

The proof of below theorem is similiar to Theorem 3, so it is claim here without proof.

Theorem 4. *If* $\Delta > 0$, $\varkappa > 0, \vartheta > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi + \overline{\varpi}) < (\varsigma + \overline{\varsigma})$*, then*

$$\int_0^\infty u^{\varpi-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\, S_n^m[u^\varkappa]\, {}_p\Psi_q^{(\gamma)}[z\,u^\vartheta]du$$
$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi} \sum_{s=0}^{[n/m]} \frac{(-n)_{m\,s}}{s!} A_{n,\,s} \left(\frac{k}{2}\right)^{s\varkappa} \times$$
$${}_{p+2}\Psi_{q+1}^{(\gamma)} \left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), (2\varpi + 2s\varkappa, 2\vartheta), (\varsigma - \varpi - s\varkappa, -\vartheta), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\,p}\,; \\ (1+\varpi+\varsigma+s\varkappa, \vartheta), (\mathfrak{g}_j, \mathfrak{G}_j)_{1,\,q}\,; \end{matrix} \quad z\left(\tfrac{k}{2}\right)^{\vartheta} \right]. \tag{12}$$

2.3 Lavoie type integral

Lavoie [5] gives an integral formula as:

$$\int_0^\infty u^{\varpi-1}\left(1-\frac{u}{3}\right)^{2\varpi-1}(1-u)^{2\varsigma-1}\left(1-\frac{u}{4}\right)^{\varsigma-1}du = \left(\frac{2}{3}\right)^{2\varpi} \frac{\Gamma(\varpi)\Gamma(\varsigma)}{\Gamma(\varpi+\varsigma)}, \tag{13}$$

with $\varpi, \varsigma \in \mathbb{C}$ and $\Re(\varpi), \Re(\varsigma) > 0$.

Theorem 5. *If* $\Delta > 0$, $\varkappa > 0, \varrho > 0, \vartheta > 0, \mathit{\Omega} > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $\Re(\varpi), \Re(\varsigma) > 0$*, then*

$$\int_0^\infty u^{\varpi-1}\left(1-\frac{u}{3}\right)^{2\varpi-1}(1-u)^{2\varsigma-1}\left(1-\frac{u}{4}\right)^{\varsigma-1}$$
$$\times S_n^m\left[u^\varkappa\left(1-\frac{u}{3}\right)^{2\varkappa}(1-u)^{2\varrho}\left(1-\frac{u}{4}\right)^{\varrho}\right]$$
$$\times\ {}_p\Psi_q^{(\Gamma)}\left[z\,u^\vartheta\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\mathit{\Omega}}\left(1-\frac{u}{4}\right)^{\mathit{\Omega}}\right]du$$
$$= \left(\frac{2}{3}\right)^{2\varpi} \sum_{s=0}^{[n/m]} \frac{(-n)_{m\,s}}{s!} A_{n,\,s} \left(\frac{2}{3}\right)^{2s\varkappa}$$
$$\times\ {}_{p+2}\Psi_{q+1}^{(\Gamma)} \left[\begin{matrix} (\mathfrak{f}_1, \mathfrak{F}_1, x), (\varpi + s\varkappa, \vartheta), (\varsigma + s\varrho, \mathit{\Omega}), (\mathfrak{f}_j, \mathfrak{F}_j)_{2,\,p}\,; \\ (\varpi+\varsigma+s(\varkappa+\varrho), \vartheta+\mathit{\Omega}), (\mathfrak{g}_j, \mathfrak{G}_j)_{1,\,q}\,; \end{matrix} \quad z\,\left(\tfrac{2}{3}\right)^{2\vartheta} \right]. \tag{14}$$

Theorem 6. *If* $\Delta > 0$, $\varkappa > 0, \varrho > 0, \vartheta > 0, \mathit{\Omega} > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $\Re(\varpi), \Re(\varsigma) >$

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0, *then*

$$\int_0^\infty u^{\varpi-1}\left(1-\frac{u}{3}\right)^{2\varpi-1}(1-u)^{2\varsigma-1}\left(1-\frac{u}{4}\right)^{\varsigma-1} \times S_n^m\left[u^{\varkappa}\left(1-\frac{u}{3}\right)^{2\varkappa}(1-u)^{2\varrho}\left(1-\frac{u}{4}\right)^{\varrho}\right] \times {}_p\Psi_q^{(\gamma)}\left[z\,u^{\vartheta}\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\Omega}\left(1-\frac{u}{4}\right)^{\Omega}\right]du$$
$$=\left(\frac{2}{3}\right)^{2\varpi}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\left(\frac{2}{3}\right)^{2s\varkappa} \times {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{array}\ z\left(\tfrac{2}{3}\right)^{2\vartheta}\right]. \tag{15}$$

The proof of above theorems are immediate consequences of definitions (1), (2), (4) and (13), hence they are given without proof here.

2.4 MacRobert type integral

MacRobert [6] gives an integral formula as:

$$\int_0^1 u^{\varpi-1}(1-u)^{\varsigma-1}[au+b(1-u)]^{-\varpi-\varsigma}du=\frac{1}{a^{\varpi}b^{\varsigma}}\frac{\Gamma(\varpi)\Gamma(\varsigma)}{\Gamma(\varpi+\varsigma)} \tag{16}$$

with $\Re(\varpi)$, $\Re(\varsigma)>0$ and $|u|\leq 1$.

Theorem 7. *If* $\Delta>0$, $\varkappa>0,\varrho>0,\vartheta>0,\Omega>0$, $\Re(\varpi)$, $\Re(\varsigma)>0$ *and* $|u|\leq 1$, *then*

$$\int_0^1 u^{\varpi-1}(1-u)^{\varsigma-1}[au+b(1-u)]^{-\varpi-\varsigma} \times S_n^m\left[u^{\varkappa}(1-u)^{\varrho}[au+b(1-u)]^{-\varkappa-\varrho}\right] \times {}_p\Psi_q^{(\Gamma)}\left[z\,u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right]du$$
$$=\frac{1}{a^{\varpi}\,b^{\varsigma}}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\frac{1}{a^{s\varkappa}\,b^{s\varrho}} \times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{array}\ \frac{z}{a^{\vartheta}\,b^{\Omega}}\right]. \tag{17}$$

Theorem 8. *If* $\Delta>0$, $\varkappa>0,\varrho>0,\vartheta>0,\Omega>0$, $\Re(\varpi)$, $\Re(\varsigma)>0$ *and* $|u|\leq 1$,

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then

$$\int_0^1 u^{\varpi-1}(1-u)^{\varsigma-1}[au+b(1-u)]^{-\varpi-\varsigma} \times S_n^m\left[u^{\varkappa}(1-u)^{\varrho}[au+b(1-u)]^{-\varkappa-\varrho}\right] \times {}_p\Psi_q^{(\gamma)}\left[z\,u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right]du$$
$$= \frac{1}{a^{\varpi}\,b^{\varsigma}}\sum_{s=0}^{[n/m]}\frac{(-n)_{m\,s}}{s!}A_{n,\,s}\,\frac{1}{a^{s\varkappa}\,b^{s\varrho}} \times {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{array}\ \frac{z}{a^{\vartheta}\,b^{\Omega}}\right]. \quad (18)$$

The proof of above theorems are immediate consequences of definitions (1), (2), (4) and (16), hence they are given without proof here.

3 Particular Cases

By appropriately specializing the coefficient $A_{n,\,s}$, specific special cases of derived findings can be developed to identify many spectrums of the existing polynomials. Only two special cases are given here and the remaining we kept for interested readers. If we put $m = 2$ and $A_{n,\,s} = (-1)^s$ in the broad category of polynomials (i.e., $S_n^2[x] = x^{n/2}\mathrm{H}_n\left(\frac{1}{2\sqrt{x}}\right)$, where $\mathrm{H}_n(x)$ is Hermite polynomial [1, 15]) of above theorems, then we obtain the following respective corollaries.

Corollary 1. *If* $\Delta,\ \varkappa,\ \varrho,\ \vartheta,\ \Omega > 0$ *and* $\Re(\varsigma) > \Re(\varpi) > 0$*, then*

$$\int_0^{\infty} u^{\varpi+\frac{n}{2}\varkappa-1}(u+v)^{-\varsigma-\frac{n}{2}\varrho}\,\mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}(u+v)^{-\varrho}}}\right)\,{}_p\Psi_q^{(\Gamma)}[z\,u^{\vartheta}(u+v)^{-\Omega}]du$$
$$= v^{\varpi-\varsigma}\,n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\,v^{s(\varkappa-\varrho)}\times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma-\varpi+s(\varrho-\varkappa),\Omega-\vartheta)\,,\\ (\varsigma+s\varrho,\Omega),\\ (\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,;\\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,;\end{array}\ z v^{\vartheta-\Omega}\right]. \quad (19)$$

Corollary 2. *If* $\Delta > 0,\ \varkappa,\ \varrho,\ \vartheta,\ \Omega > 0$ *and* $\Re(\varsigma) > \Re(\varpi) > 0$, *then*

$$\int_0^\infty u^{\varpi+\frac{n}{2}\varkappa-1}(u+v)^{-\varsigma-\frac{n}{2}\varrho}\,\mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}(u+v)^{-\varrho}}}\right)\,{}_p\Psi_q^{(\gamma)}[z\,u^{\vartheta}(u+v)^{-\Omega}]du$$
$$= v^{\varpi-\varsigma}\,n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\,v^{s(\varkappa-\varrho)}\times$$
$${}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma-\varpi+s(\varrho-\varkappa),\Omega-\vartheta),\\(\varsigma+s\varrho,\Omega),\end{array}\right.$$
$$\left.\begin{array}{cc}(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; & zv^{\vartheta-\Omega}\\(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; & \end{array}\right]. \tag{20}$$

Corollary 3. *If* $\Delta > 0,\ \varkappa > 0, \vartheta > 0,\ \varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi+\overline{\varpi}) < (\varsigma+\overline{\varsigma})$, *then*

$$\int_0^\infty u^{\varpi+\frac{n}{2}\varkappa-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\,\mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}}}\right)\,{}_p\Psi_q^{(\Gamma)}[z\,u^{\vartheta}]du$$
$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi}\,n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\left(\frac{k}{2}\right)^{s\varkappa}\times$$
$${}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{cc}(\mathfrak{f}_1,\mathfrak{F}_1,x),(2\varpi+2s\varkappa,2\vartheta),(\varsigma-\varpi-s\varkappa,-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; & z\left(\frac{k}{2}\right)^{\vartheta}\\(1+\varpi+\varsigma+s\varkappa,\vartheta),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; & \end{array}\right]. \tag{21}$$

Corollary 4. *If* $\Delta > 0,\ \varkappa > 0, \vartheta > 0,\ \varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi+\overline{\varpi}) < (\varsigma+\overline{\varsigma})$, *then*

$$\int_0^\infty u^{\varpi+\frac{n}{2}\varkappa-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\,\mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}}}\right)\,{}_p\Psi_q^{(\gamma)}[z\,u^{\vartheta}]du$$
$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi}\,n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\left(\frac{k}{2}\right)^{s\varkappa}\times$$
$${}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{cc}(\mathfrak{f}_1,\mathfrak{F}_1,x),(2\varpi+2s\varkappa,2\vartheta),(\varsigma-\varpi-s\varkappa,-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; & z\left(\frac{k}{2}\right)^{\vartheta}\\(1+\varpi+\varsigma+s\varkappa,\vartheta),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; & \end{array}\right]. \tag{22}$$

Corollary 5. *If* $\Delta > 0,\ \varkappa > 0, \varrho > 0, \vartheta > 0, \Omega > 0,\ \varpi, \varsigma \in \mathbb{C}$ *and* $\Re(\varpi), \Re(\varsigma) >$

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0, *then*

$$\int_0^\infty u^{\varpi+\frac{n}{2}\varkappa-1}\left(1-\frac{u}{3}\right)^{2(\varpi+\frac{n}{2}\varkappa)-1}(1-u)^{2(\varsigma+\frac{n}{2}\varrho)-1}\left(1-\frac{u}{4}\right)^{\varsigma+\frac{n}{2}\varrho-1} \times \mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}\left(1-\frac{u}{3}\right)^{2\varkappa}(1-u)^{2\varrho}\left(1-\frac{u}{4}\right)^{\varrho}}}\right) \times {}_p\Psi_q^{(\Gamma)}\left[z\,u^{\vartheta}\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\Omega}\left(1-\frac{u}{4}\right)^{\Omega}\right]du$$
$$=\left(\frac{2}{3}\right)^{2\varpi} n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\left(\frac{2}{3}\right)^{2s\varkappa} \times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p};\\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q};\end{array}\ z\left(\tfrac{2}{3}\right)^{2\vartheta}\right]. \tag{23}$$

Corollary 6. *If* $\Delta>0,\ \varkappa>0,\varrho>0,\vartheta>0,\Omega>0,\ \varpi,\varsigma\in\mathbb{C}$ *and* $\Re(\varpi),\Re(\varsigma)>0$, *then*

$$\int_0^\infty u^{\varpi+\frac{n}{2}\varkappa-1}\left(1-\frac{u}{3}\right)^{2(\varpi+\frac{n}{2}\varkappa)-1}(1-u)^{2(\varsigma+\frac{n}{2}\varrho)-1}\left(1-\frac{u}{4}\right)^{\varsigma+\frac{n}{2}\varrho-1} \times \mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}\left(1-\frac{u}{3}\right)^{2\varkappa}(1-u)^{2\varrho}\left(1-\frac{u}{4}\right)^{\varrho}}}\right) \times {}_p\Psi_q^{(\gamma)}\left[z\,u^{\vartheta}\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\Omega}\left(1-\frac{u}{4}\right)^{\Omega}\right]du$$
$$=\left(\frac{2}{3}\right)^{2\varpi} n!\sum_{s=0}^{[n/2]}\frac{(-1)^s}{s!(n-2s)!}\left(\frac{2}{3}\right)^{2s\varkappa} \times {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c}(\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p};\\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q};\end{array}\ z\left(\tfrac{2}{3}\right)^{2\vartheta}\right]. \tag{24}$$

Corollary 7. *If* $\Delta>0,\ \varkappa>0,\varrho>0,\vartheta>0,\Omega>0,\ \Re(\varpi),\ \Re(\varsigma)>0$ *and*

$|u| \leq 1$, *then*

$$\begin{aligned}
&\int_0^1 u^{\varpi+\frac{n}{2}\varkappa-1}(1-u)^{\varsigma+\frac{n}{2}\varrho-1}[au+b(1-u)]^{-\varpi-\varsigma-\frac{n}{2}(\varkappa+\varrho)} \\
&\qquad \times \mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}(1-u)^{\varrho}[au+b(1-u)]^{-\varkappa-\varrho}}}\right) \\
&\qquad\qquad \times {}_p\Psi_q^{(\Gamma)}\left[z\, u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right] du \\
&= \frac{n!}{a^{\varpi}\, b^{\varsigma}} \sum_{s=0}^{[n/2]} \frac{(-1)^s}{s!(n-2s)!} \frac{1}{a^{s\varkappa}\, b^{s\varrho}} \\
&\times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; \\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; \end{array} \frac{z}{a^{\vartheta}\, b^{\Omega}}\right]. \qquad (25)
\end{aligned}$$

Corollary 8. *If* $\Delta > 0$, $\varkappa > 0, \varrho > 0, \vartheta > 0, \Omega > 0$, $\Re(\varpi)$, $\Re(\varsigma) > 0$ *and* $|u| \leq 1$, *then*

$$\begin{aligned}
&\int_0^1 u^{\varpi+\frac{n}{2}\varkappa-1}(1-u)^{\varsigma+\frac{n}{2}\varrho-1}[au+b(1-u)]^{-\varpi-\varsigma-\frac{n}{2}(\varkappa+\varrho)} \\
&\qquad \times \mathrm{H}_n\left(\frac{1}{2\sqrt{u^{\varkappa}(1-u)^{\varrho}[au+b(1-u)]^{-\varkappa-\varrho}}}\right) \\
&\qquad\qquad \times {}_p\Psi_q^{(\gamma)}\left[z\, u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right] du \\
&= \frac{n!}{a^{\varpi}\, b^{\varsigma}} \sum_{s=0}^{[n/2]} \frac{(-1)^s}{s!(n-2s)!} \frac{1}{a^{s\varkappa}\, b^{s\varrho}} \\
&\times {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+s\varkappa,\vartheta),(\varsigma+s\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; \\ (\varpi+\varsigma+s(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; \end{array} \frac{z}{a^{\vartheta}\, b^{\Omega}}\right]. \qquad (26)
\end{aligned}$$

Again, by setting $m = 1$ and $A_{n,s} = \dfrac{s!}{(-n)_{m\,s}}$ for $s = r$ and $A_{n,s} = 0$ for $s \neq r$ in the general class of polynomials (i.e., $S_n^1[x] = x^r$) of above theorems, then we obtain the following respective corollaries.

Corollary 9. *If* $\Delta > 0$, $\varkappa$, ϱ, ϑ, $\Omega > 0$ *and* $\Re(\varsigma) > \Re(\varpi) > 0$, *then*

$$\begin{aligned}
&\int_0^{\infty} u^{\varpi+r\varkappa-1}(u+v)^{-\varsigma-r\varrho}\, {}_p\Psi_q^{(\Gamma)}[z\, u^{\vartheta}(u+v)^{-\Omega}]du \\
&= v^{\varpi-\varsigma+r(\varkappa-\varrho)} \times {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta), \\ (\varsigma+r\varrho,\Omega), \end{array}\right. \\
&\qquad\qquad \left.\begin{array}{c} (\varsigma-\varpi+r(\varrho-\varkappa),\Omega-\vartheta)\,,(\mathfrak{f}_j,\mathfrak{F}_j)_{2,p}\,; \\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,q}\,; \end{array} z v^{\vartheta-\Omega}\right]. \qquad (27)
\end{aligned}$$

Corollary 10. *If* $\Delta > 0$, $\varkappa$, ϱ, ϑ, $\Omega > 0$ *and* $\Re(\varsigma) > \Re(\varpi) > 0$, *then*

$$\int_0^\infty u^{\varpi+r\varkappa-1}(u+v)^{-\varsigma-r\varrho}\,{}_p\Psi_q^{(\gamma)}[z\,u^\vartheta(u+v)^{-\Omega}]du$$
$$= v^{\varpi-\varsigma+r(\varkappa-\varrho)} \times {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta), \\ (\varsigma+r\varrho,\Omega), \\ (\varsigma-\varpi+r(\varrho-\varkappa),\Omega-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array} zv^{\vartheta-\Omega}\right]. \quad (28)$$

Corollary 11. *If* $\Delta > 0$, $\varkappa > 0, \vartheta > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi + \overline{\varpi}) < (\varsigma + \overline{\varsigma})$, *then*

$$\int_0^\infty u^{\varpi+r\varkappa-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\,{}_p\Psi_q^{(\Gamma)}[z\,u^\vartheta]du$$
$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi+r\varkappa}\times$$
$${}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(2\varpi+2r\varkappa,2\vartheta),(\varsigma-\varpi-r\varkappa,-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (1+\varpi+\varsigma+r\varkappa,\vartheta),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array} z\left(\tfrac{k}{2}\right)^\vartheta\right]. \quad (29)$$

Corollary 12. *If* $\Delta > 0$, $\varkappa > 0, \vartheta > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $0 < (\varpi + \overline{\varpi}) < (\varsigma + \overline{\varsigma})$, *then*

$$\int_0^\infty u^{\varpi+r\varkappa-1}(u+k+\sqrt{u^2+2ku})^{-\varsigma}\,{}_p\Psi_q^{(\gamma)}[z\,u^\vartheta]du$$
$$= 2\varsigma(k)^{-\varsigma}(k/2)^{\varpi+r\varkappa}\times$$
$${}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(2\varpi+2r\varkappa,2\vartheta),(\varsigma-\varpi-r\varkappa,-\vartheta),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (1+\varpi+\varsigma+r\varkappa,\vartheta),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array} z\left(\tfrac{k}{2}\right)^\vartheta\right]. \quad (30)$$

Corollary 13. *If* $\Delta > 0$, $\varkappa > 0, \varrho > 0, \vartheta > 0, \Omega > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and* $\Re(\varpi), \Re(\varsigma) > 0$, *then*

$$\int_0^\infty u^{\varpi+r\varkappa-1}\left(1-\frac{u}{3}\right)^{2(\varpi+r\varkappa)-1}(1-u)^{2(\varsigma+r\varrho)-1}\left(1-\frac{u}{4}\right)^{\varsigma+r\varrho-1}$$
$$\times\,{}_p\Psi_q^{(\Gamma)}\left[z\,u^\vartheta\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\Omega}\left(1-\frac{u}{4}\right)^{\Omega}\right]du = \left(\frac{2}{3}\right)^{2(\varpi+r\varkappa)}$$
$$\times\,{}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{c} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta),(\varsigma+r\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (\varpi+\varsigma+r(\varkappa+\varrho),\vartheta+\Omega),(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array} z\left(\tfrac{2}{3}\right)^{2\vartheta}\right]. \quad (31)$$

Corollary 14. *If* $\Delta > 0$, $\varkappa > 0, \varrho > 0, \vartheta > 0, \Omega > 0$, $\varpi, \varsigma \in \mathbb{C}$ *and*

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$\Re(\varpi), \Re(\varsigma) > 0$, *then*

$$\int_0^\infty u^{\varpi+r\varkappa-1}\left(1-\frac{u}{3}\right)^{2(\varpi+r\varkappa)-1}(1-u)^{2(\varsigma+r\varrho)-1}\left(1-\frac{u}{4}\right)^{\varsigma+r\varrho-1}$$
$$\times\ {}_p\Psi_q^{(\gamma)}\left[z\,u^{\vartheta}\left(1-\frac{u}{3}\right)^{2\vartheta}(1-u)^{2\Omega}\left(1-\frac{u}{4}\right)^{\Omega}\right]du=\left(\frac{2}{3}\right)^{2(\varpi+r\varkappa)}$$
$$\times\ {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{r} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta),(\varsigma+r\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (\varpi+\varsigma+r(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array}\ z\left(\tfrac{2}{3}\right)^{2\vartheta}\right]. \tag{32}$$

Corollary 15. *If* $\Delta>0,\ \varkappa>0, \varrho>0, \vartheta>0, \Omega>0,\ \Re(\varpi),\ \Re(\varsigma)>0$ *and* $|u|\le 1$, *then*

$$\int_0^1 u^{\varpi+r\varkappa-1}(1-u)^{\varsigma+r\varrho-1}[au+b(1-u)]^{-\varpi-\varsigma-r(\varkappa+\varrho)}$$
$$\times\ {}_p\Psi_q^{(\Gamma)}\left[z\,u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right]du=\frac{1}{a^{\varpi+r\varkappa}\,b^{\varsigma+r\varrho}}$$
$$\times\ {}_{p+2}\Psi_{q+1}^{(\Gamma)}\left[\begin{array}{r} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta),(\varsigma+r\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (\varpi+\varsigma+r(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array}\ \frac{z}{a^{\vartheta}\,b^{\Omega}}\right]. \tag{33}$$

Corollary 16. *If* $\Delta>0,\ \varkappa>0, \varrho>0, \vartheta>0, \Omega>0,\ \Re(\varpi),\ \Re(\varsigma)>0$ *and* $|u|\le 1$, *then*

$$\int_0^1 u^{\varpi+r\varkappa-1}(1-u)^{\varsigma+r\varrho-1}[au+b(1-u)]^{-\varpi-\varsigma-r(\varkappa+\varrho)}$$
$$\times\ {}_p\Psi_q^{(\gamma)}\left[z\,u^{\vartheta}(1-u)^{\Omega}[au+b(1-u)]^{-\vartheta-\Omega}\right]du=\frac{1}{a^{\varpi+r\varkappa}\,b^{\varsigma+r\varrho}}$$
$$\times\ {}_{p+2}\Psi_{q+1}^{(\gamma)}\left[\begin{array}{r} (\mathfrak{f}_1,\mathfrak{F}_1,x),(\varpi+r\varkappa,\vartheta),(\varsigma+r\varrho,\Omega),(\mathfrak{f}_j,\mathfrak{F}_j)_{2,\,p}\,; \\ (\varpi+\varsigma+r(\varkappa+\varrho),\vartheta+\Omega)\,,(\mathfrak{g}_j,\mathfrak{G}_j)_{1,\,q}\,; \end{array}\ \frac{z}{a^{\vartheta}\,b^{\Omega}}\right]. \tag{34}$$

4 Concluding Remarks

The integral formulas concerning combination of a general polynomial system and incomplete Fox-Wright functions are investigated and the outcomes are described in terms of those other incomplete Fox-Wright functions. When $x = 0$, the incomplete Fox-Wright function mentioned by (2) reduces to the Fox-Wright function ${}_p\Psi_q(\mathrm{t})$, whose particular cases are known to the number of special functions arising in the mathematical, physical and engineering sciences. In addition, a huge number of recognized polynomials may be obtained as a specific case of general class of polynomials by correctly specialization the factor $A_{n,\,s}$. We conclude by stating that the findings mentioned here appear to be of broad importance and can lead to multiple integrals for a particular class of hypergeometric polynomials as well as other special functions, which we left for interested readers.

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