Vertex Coloring of Graph Using Incidence Matrix

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ABSTRACT
Graph coloring is one of the potential area of research in Graph theory. The vertex coloring problem is one of the fundamental problem on graphs which often appears in various scheduling problems like file transfer problem on computer networks. Various algorithms for vertex coloring, edge coloring, total coloring etc., are described by various researchers. In this paper, a simple approach is proposed to color all the vertices of a graph with the minimum number of colors. This approach helps us to find chromatic number of a graph using incidence matrix.

Key words: Vertex coloring, Chromatic number, Incidence matrix.

1. INTRODUCTION

A graph is an abstract structure which consists of vertices and edges, each edge joins two vertices called ends of the edge. It can be used to represent various combinatorial or topological structures that can be modelled as objects and connections between those objects. A graph structure is very suitable for representing relationships between objects in the abstract, and a large number of combinatorial problems can be modelled as problems on the graph structure [11].

In Graph theory, coloring is an important area which has been extensively studied. Coloring theory started with the problem of coloring the countries of a map in such a way that no two countries that have a common border receive the same color. If we denote the countries by points in the plane and connect each pair of points that correspond to countries with a common border by a curve, we obtain a planar graph. The celebrated Four-Color Problem asks if every planar graph can be colored with four colors. It seems to have been mentioned for the first time in writing in an 1852 letter from A. De Morgan to W.R. Hamilton. Nobody thought at that time that it was the beginning of a new theory. The first proof was given by Kempe in 1879[12].The fundamental parameter in the theory of graph coloring is the chromatic number $\chi(G)$ of a graph G which is defined to be the minimum number of colors required to color the vertices of G in such a way that no two adjacent vertices receive the same color. If $\chi(G) = k$, we say that G is k-chromatic. The edge-coloring problem is to color all edges of a given graph with the minimum number of colors so that no two adjacent edges are assigned the same color[10].

2. Preliminaries

Some basic definitions and their remarks are presented in this section to understand this approach in a better way.

2.1 Definition

Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called the proper coloringor simply coloringof a graph.

A graph in which every vertex has been assigned a color according to a proper coloring is called a proper colored graph. A graph G that requires k different colors for its proper coloring, and no less, is called a k – chromatic graph, and the number k is called the chromatic number of G and is denoted by $\chi(G)$.

2.2 Definition

Let G be a graph with n vertices and m edges. The incidence matrix A(G) is defined by $A(G) = [a_{ij}]$ where,

$[a_{ij}] = 1$, if v_i is incident with e_j,
$\quad = 0$, if v_i is not incident with e_j

Note:

- A graph consisting of only isolated vertices is 1 – chromatic.
- A graph with one or more edges (not a self – loop) is at least 2 – chromatic.
- A complete graph of n vertices is n – chromatic.
- A graph consisting of simply one circuit with n≥3 vertices is 2 – chromatic if n is even and 3 – chromatic if n is odd.

3. A simple approach for graph coloring

Several authors developed algorithms for vertex coloring, edge coloring, total coloring etc., using different approaches. In this section a new algorithm for vertex coloring using incidence matrix is presented in detail.

Algorithm

Step 1: Construct an Incidence matrix for the given graph.

Step 2: Find the sum of the elements in each row of the matrix. Select the row that has maximum value.

Step 3: If the maximum value is unique orthere is a tie in the maximum value, select anyone arbitrarily then go to step 4.

Step 4: Assign new color to the vertex corresponding to the row of the identified maximum value and delete the row then go to step 5.

Step 5: Look for the columns with single ones in the obtainedmodified matrix and mark the row associated with single ones then strike off those columns.

Step 6: Select the vertices associated with the unmarked rows in the reduced matrix obtained from step 5.

Case (a)

If there is no unmarked rows then go to step 2.

Case (b)

If there is only one unmarked row assign the same color to the associated vertex and delete the corresponding row and columns associated with ones in that row.Further, if there is no column in the reduced matrix assign new color to the remaining vertices and stop the process. Otherwise go to step 2.

Case (c)

If there are more than one unmarked row in the reduced matrix then check the degrees of the vertices associated with each unmarked row. Assign same color to the vertex which has maximum degree and delete the row then go to step5.

Illustration:1

Consider a graph with 6 vertices and 9 edges as shown in the figure 1. Find proper coloring of a graph using the above algorithm.

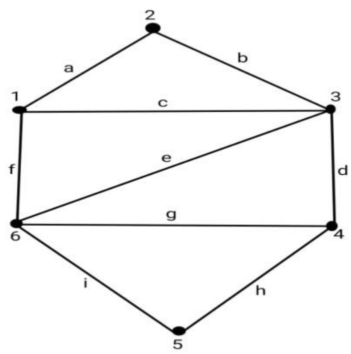

Figure 1. (6,9) graph

Solution

As per the first and second step of the algorithm construct an incidence matrix and compute the sum of the elements in each row of the corresponding matrix is shown in the table 1.

Table 1. Incidence matrix of (6,9) graph

	a	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**deg**
1	1	0	1	0	0	1	0	0	0	**3**
2	1	1	0	0	0	0	0	0	0	**2**
3	0	1	1	1	1	0	0	0	0	**4**
4	0	0	0	1	0	0	1	1	0	**3**
5	0	0	0	0	0	0	0	1	1	**2**
6	0	0	0	0	1	1	1	0	1	**4**

Table 1 shows that, the maximum value is 4.Bystep 3, there is tie in the maximum value and the associated vertices are 3 and 6 respectively. Select anyone arbitrarily. Let us choose third row and the corresponding vertex is 3.By step 4, assign new color (say Red) to the vertex 3 and delete that row.By step 5, the columns 'b', 'c', 'd'and 'e'have single ones associated with the vertices 1,2,4 and 6. Mark those rows andstrike off the columns b, c, d, e.By step 6, select the vertex associated with unmarked row i.e.) vertex 5. By case (b) of step 6, there is only one unmarked row and assign the same color (say Red) to the vertex 5. Delete the corresponding row and columns with single ones say 'h' and 'i'. Again, by step 2, the reduced incidence matrix and the sum of the elements of each row of the uncolored vertices are given in table 2.

Table 2

	a	**f**	**g**	**deg**
1	1	1	0	**2**
2	1	0	0	**1**
4	0	0	1	**1**
6	0	1	1	**2**

Table 2 shows that, the maximum value is 2. Bystep 3, there is tie in the maximum value and the associated vertices are 1 and 6 respectively. Select anyone arbitrarily. Let us choose first row and the corresponding vertex is 1.By step 4, assign new color (say Pink) to the vertex 1 and delete that row.By step 5, the columns 'a' and 'f'have single ones associated with the vertices 2 and 6. Strike off the columns and neglect the marked vertices.By step 6, select the unmarked vertex 4. By case (b) of step 6, there is only one unmarked row in the reduced matrix. Assign same color (say Pink) to the vertex 4 and delete the corresponding row and columns with single ones say 'g'.
The remaining vertices 2 and 6 are distinct and there is no columns in the reduced matrix. By step 4, assign new color (say Green) to vertices 2 and 6. The resulting graph is shown in figure 2.

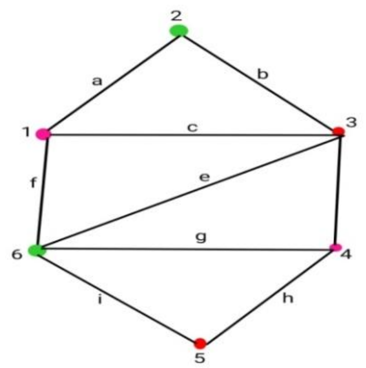

Figure 2

By the proposed method, the vertices of a given graph is colored with minimum three colors and its chromatic number is 3.

CONCLUSION

In this paper we have presented a simple approach for finding chromatic number of a graph using incidence matrix.The illustration discussed here can clearly indicate the perfection of the simple approach for proper coloring of given graphs. Further a computer-based algorithm can be developed in future by using any computer languages which will make more easier to color any larger size of graphs.

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