

Hilbert Mean Labeling Of Some Special Graphs

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ABSTRACT

Let $G = (V(G), E(G))$ be a graph with p vertices and q edges. A hilbert mean labeling is an injective function $\mu: V(G) \rightarrow \{0, 1, 2, \dots, H_q\}$ where H_q is the q^{th} hilbert number and $H_q = 4(q - 1) + 1, q \geq 1$ that

induces a bijection $\mu^*: E(G) \rightarrow \{H_1, H_2, \dots, H_q\}$ defined by $\mu^*(uv) = \begin{cases} \frac{\mu(u) + \mu(v) + 1}{2} & \text{if } \mu(u) + \mu(v) \text{ is odd} \\ \frac{\mu(u) + \mu(v)}{2} & \text{if } \mu(u) + \mu(v) \text{ is even} \end{cases}$

for all $uv \in E(G)$. A graph which admits such labeling is called a hilbert mean graph. In this paper, the hilbert mean labeling of some special graphs are studied.

Keywords: Hilbert numbers, Hilbert mean labeling, Hilbert mean graph.

1. INTRODUCTION

The paper deals with finite, undirected graphs that lack loops or multiple edges. Let $G = (V, E)$ represent a graph with p vertices and q edges, where terms follow Harary's [5] definitions. Unspecified terms adhere to [1] for number theoretic terminology. A graph labeling involves assigning integers to the vertices, edges or both under specific conditions. Vertex, edge or total labeling is determined by the domain of the mapping. Gallian [2] maintains a dynamic survey of graph labeling regularly updated and published by the Electronic Journal of Combinatorics. The notation of mean labeling for graphs was introduced by S. Somasundaram and R. Ponraj [7]. The concept of hilbert mean labeling was introduced in [8]. Also, we have referred [3],[4],[6] and [9] for my results.

2. Preliminaries

Definition 2.1: The n^{th} hilbert number H_n is given by the formula $4(n - 1) + 1$ for $n \geq 1$. The first few hilbert numbers are 1,5,9,13,17, 21, 25, 29, 33, 37,41,45,49,53,57 etc.

Definition 2.2: Let G be a graph with p vertices and q edges. A hilbert mean labeling is an injective function $\mu: V(G) \rightarrow \{0, 1, 2, \dots, H_q\}$ where H_q is the q^{th} hilbert number and $H_q = 4(q - 1) + 1, q \geq 1$ that

induces a bijection $\mu^*: E(G) \rightarrow \{H_1, H_2, \dots, H_q\}$ defined by $\mu^*(uv) = \begin{cases} \frac{\mu(u) + \mu(v) + 1}{2} & \text{if } \mu(u) + \mu(v) \text{ is odd} \\ \frac{\mu(u) + \mu(v)}{2} & \text{if } \mu(u) + \mu(v) \text{ is even} \end{cases}$

for all $uv \in E(G)$. A graph which admits such labeling is called a hilbert mean graph.

3. Main Results

Theorem 3.1: $M(P_m)$ is a hilbert mean graph where m is odd and $m \geq 3$.

Proof: Let $G = M(P_m), V(G) = \{w_{i_1}, v_{i_1} : 1 \leq i_1 \leq m\}$ and

$E(G) = \{w_{i_1} v_{i_1} : 1 \leq i_1 \leq m, w_{i_1} w_{i_1+1}, v_{i_1} v_{i_1+1} : 1 \leq i_1 \leq m - 1\}$.

We observe that G has $2m$ vertices and $3m - 2$ edges.

Define $\mu: V(G) \rightarrow \{0, 1, 2, \dots, H_{3m-2}\}$ as follows.

For $1 \leq i_1 \leq m, \mu(w_{i_1}) = \begin{cases} 4(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 4(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$

$$\mu(v_{i_1}) = \begin{cases} 4(2m + i_1 - 3) + 1 & \text{if } i_1 \text{ is odd} \\ 4(2m + i_1 - 2) & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(G) \rightarrow \{H_1, H_2, \dots, H_{3m-2}\}$ is defined as follows.

$$\begin{aligned} \mu^*(w_{i_1}w_{i_1+1}) &= H_{i_1} & 1 \leq i_1 \leq m-1 \\ \mu^*(w_{i_1}v_{i_1}) &= H_{(m+i_1-1)} & 1 \leq i_1 \leq m \\ \mu^*(v_{i_1}v_{i_1+1}) &= H_{(2m+i_1-1)} & 1 \leq i_1 \leq m-1 \end{aligned}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3m-2}$.

Hence $M(P_m)$ is a hilbert mean graph where m is odd and $m \geq 3$.

Example 3.2: The hilbert mean labeling of $M(P_3)$ is given in figure 1.

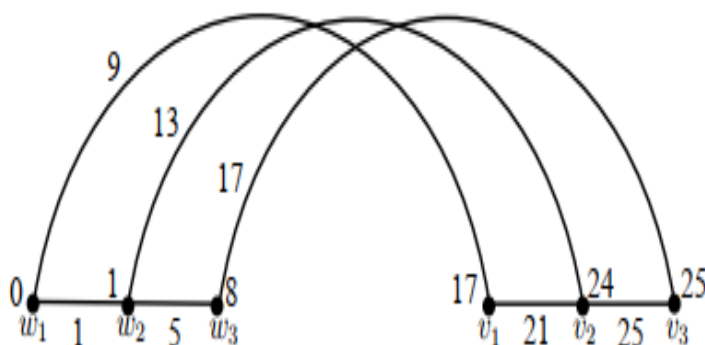


Figure 1: Hilbert mean labeling of $M(P_3)$

Theorem 3.3: $VD(P_m)$ is a hilbert mean graph where m is even and $m \geq 4$.

Proof: Let $G = VD(P_m)$, $V(G) = \{w_{i_1} : 1 \leq i_1 \leq m-1, w'_{m-1}\}$ and

$$E(G) = \{w_{i_1}w_{i_1+1} : 1 \leq i_1 \leq m-1, w_{m-2}w'_{m-1}, w_mw'_{m-1}\}.$$

We observe that G has $m+1$ vertices and $m+1$ edges. Also, $N(w_{m-1}) = N(w'_{m-1})$.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{m+1}\}$ as follows.

$$\text{For } 1 \leq i_1 \leq m-2, \mu(w_{i_1}) = \begin{cases} 4(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 4(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(w_{m-1}) = H_{m+1} - 1, \mu(w_m) = H_{m+1}, \mu(w'_{m-1}) = H_{m-1}$$

Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(G) \rightarrow \{H_1, H_2, \dots, H_{m+1}\}$ is defined as follows.

$$\mu^*(w_{i_1}w_{i_1+1}) = H_{i_1} \quad 1 \leq i_1 \leq m-3$$

$$\mu^*(w_{m-2}w_{m-1}) = H_{m-1}, \mu^*(w_{m-1}w_m) = H_{m+1}$$

$$\mu^*(w_mw'_{m-1}) = H_m, \mu^*(w_{m-2}w'_{m-1}) = H_{m-2}$$

Thus, we get the induced edge labels as H_1, H_2, \dots, H_{m+1} .

Hence $VD(P_m)$ is a hilbert mean graph where m is even and $m \geq 4$.

Example 3.4: The hilbert mean labeling of $VD(P_4)$ is given in figure 2.

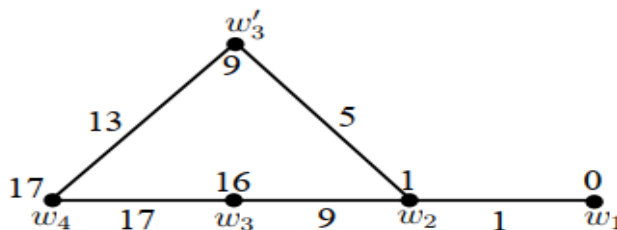


Figure 2: Hilbert mean labeling of $VD(P_4)$

Theorem 3.5: $G = P_m(QS_1)$ where m is even is a hilbert mean graph.

Proof: Let $V(G) = \{w_{i_1} : 1 \leq i_1 \leq m, x_{i_1} : 1 \leq i_1 \leq 2m, y_{i_1} : 1 \leq i_1 \leq m\}$ and

$E(G) = \{w_{i_1}w_{i_1+1} : 1 \leq i_1 \leq m-1, y_{i_1}x_{2i_1-1}, y_{i_1}x_{2i_1}, w_{i_1}x_{2i_1-1}, w_{i_1}x_{2i_1} : 1 \leq i_1 \leq m\}$.

We observe that G has $4m$ vertices and $5m-1$ edges.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{5m-1}\}$ as follows.

$$\text{For } 1 \leq i_1 \leq m, \mu(w_{i_1}) = \begin{cases} 20(i_1 - 1) + 16 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 17 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{2i_1-1}) = \begin{cases} 20(i_1 - 1) + 1 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 25 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{2i_1}) = \begin{cases} 20(i_1 - 1) + 9 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 33 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(y_{i_1}) = \begin{cases} 20(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 32 & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling is defined as follows.

$$\mu^*(w_{i_1}w_{i_1+1}) = H_{5(i_1-1)+5} \quad 1 \leq i_1 \leq m-1$$

For $1 \leq i_1 \leq m$,

$$\mu^*(w_{i_1}x_{2i_1-1}) = \begin{cases} H_{5(i_1-1)+3} & \text{if } i_1 \text{ is odd} \\ H_{5(i_1-2)+6} & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu^*(w_{2i_1}x_{2i_1}) = \begin{cases} H_{5(i_1-1)+4} & \text{if } i_1 \text{ is odd} \\ H_{5(i_1-2)+7} & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu^*(y_{i_1}x_{2i_1-1}) = \begin{cases} H_{5(i_1-1)+1} & \text{if } i_1 \text{ is odd} \\ H_{5(i_1-2)+8} & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu^*(y_{2i_1}x_{2i_1}) = \begin{cases} H_{5(i_1-1)+2} & \text{if } i_1 \text{ is odd} \\ H_{5(i_1-2)+9} & \text{if } i_1 \text{ is even} \end{cases}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{5m-1}$.

Hence $G = P_m(QS_1)$ where m is even is a hilbert mean graph.

Theorem 3.6: $G = P_2(QS_m)$ where $m \geq 1$ is a hilbert mean graph.

Proof: Let $G = P_2(QS_m), V(G) = \{y_{i_1} : 1 \leq i_1 \leq 2m+2, x_{i_1} : 1 \leq i_1 \leq 4m\}$ and

$E(G) = \{x_{2i_1-1}y_{i_1} : 1 \leq i_1 \leq m, x_{2i_1}y_{i_1} : 1 \leq i_1 \leq m, x_{2i_1-1}y_{i_1+1} : 1 \leq i_1 \leq m, x_{2i_1}y_{i_1+1} : 1 \leq i_1 \leq m, x_{2m+i_1+1}y_{m+i_1+1} : 1 \leq i_1 \leq m, x_{2m+i_1}y_{m+i_1+2} : 1 \leq i_1 \leq m, x_{2m+i_1+1}y_{m+i_1+2} : 1 \leq i_1 \leq m, y_{m+1}y_{m+2}\}$

We observe that G has $6m+2$ vertices and $8m+1$ edges.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{8m+1}\}$ as follows.

$$\mu(y_{i_1}) = 16(i_1 - 1), \quad 1 \leq i_1 \leq m+1$$

$$\mu(y_{m+i_1+1}) = 16(m + i_1 - 1) + 1, \quad 1 \leq i_1 \leq m+1$$

$$\text{For } 1 \leq i_1 \leq 2m, \mu(x_{i_1}) = \begin{cases} 8(i_1 - 1) + 1 & \text{if } i_1 \text{ is odd} \\ 8(i_1 - 2) + 9 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{2m+i_1}) = \begin{cases} 8(2m + i_1) + 1 & \text{if } i_1 \text{ is odd} \\ 8(2m + i_1) & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling is defined as follows.

For $1 \leq i_1 \leq m$,

$$\mu^*(x_{2i_1-1}y_{i_1}) = H_{4(i_1-1)+1}, \quad \mu^*(x_{2i_1}y_{i_1}) = H_{4(i_1-1)+2}$$

$$\mu^*(x_{2i_1-1}y_{i_1+1}) = H_{4(i_1-1)+3}, \quad \mu^*(x_{2i_1}y_{i_1+1}) = H_{4(i_1-1)+4}$$

$$\mu^*(x_{2m+i_1}y_{m+i_1+1}) = H_{4(m+i_1-1)+2}, \quad \mu^*(x_{2m+i_1+1}y_{m+i_1+1}) = H_{4(m+i_1-1)+3}$$

$$\mu^*(x_{2m+i_1}y_{m+i_1+2}) = H_{4(m+i_1-1)+4}, \quad \mu^*(x_{2m+i_1+1}y_{m+i_1+2}) = H_{4(m+i_1-1)+1}$$

$$\mu^*(y_{m+1}y_{m+2}) = H_{4m+1}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{8m+1}$.

Hence $G = P_2(QS_m)$ where $m \geq 1$ is a hilbert mean graph.

Example 3.7: The hilbert mean labeling of $P_2(QS_2)$ is given in figure 3.

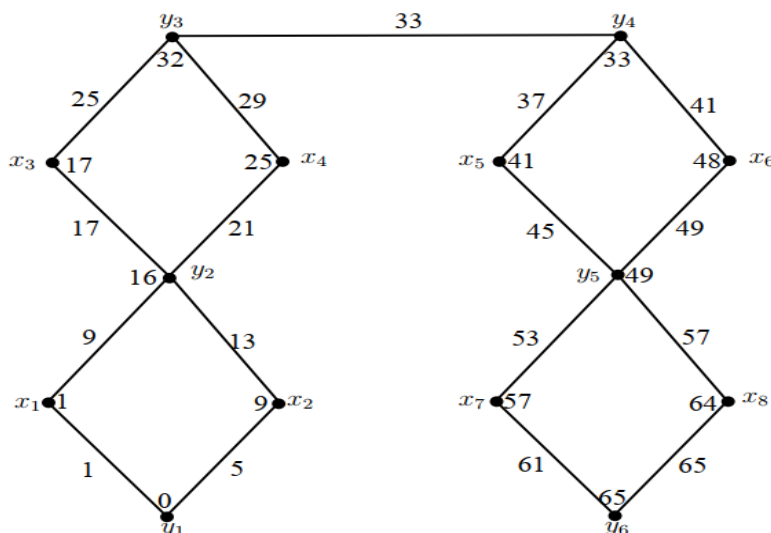


Figure 3: Hilbert mean labeling of $P_2(Q_{S_2})$

Theorem 3.8: Alternate Quadrilateral Snake graph $G = A(C_4^n)$ is a hilbert mean graph where n is even.

Proof: Let $G = A(C_4^n), V(G) = \{w_{i_1}, x_{i_1} : 1 \leq i_1 \leq n\}$ and

$$E(G) = \{x_{i_1}w_{i_1}, x_{2i_1}w_{2i_1-1}, x_{i_1}w_{i_1+1}, x_{2i_1}w_{2i_1} : 1 \leq i_1 \leq n\}$$

We observe that G has $2n$ vertices and $2n + 1$ edges. Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{2n+1}\}$ as follows.

$$\text{For } 1 \leq i_1 \leq n, \mu(w_{2i_1-1}) = \begin{cases} 20(i_1 - 1) + 1 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 25 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(w_{2i_1}) = \begin{cases} 20(i_1 - 1) + 9 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 32 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{2i_1}) = \begin{cases} 20(i_1 - 1) + 16 & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 33 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{2i_1-1}) = \begin{cases} 20(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 20(i_1 - 2) + 17 & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(G) \rightarrow \{H_1, H_2, \dots, H_{2n+1}\}$ is defined as follows.

$$\mu^*(x_{i_1}w_{i_1}) = H_{5(i_1-1)+1} \quad 1 \leq i_1 \leq n$$

$$\mu^*(x_{2i_1}w_{2i_1-1}) = H_{5(i_1-1)+3} \quad 1 \leq i_1 \leq n$$

$$\mu^*(x_{2i_1-1}w_{2i_1}) = H_{5(i_1-1)+2} \quad 1 \leq i_1 \leq n$$

$$\mu^*(x_{2i_1}w_{2i_1}) = H_{5(i_1-1)+4} \quad 1 \leq i_1 \leq n$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{2n+1}$.

Hence Alternate Quadrilateral Snake graph $G = A(C_4^n)$ is a hilbert mean graph where n is even.

Example 3.9: The hilbert mean labelling of $A(C_4^2)$ is given in figure 4.

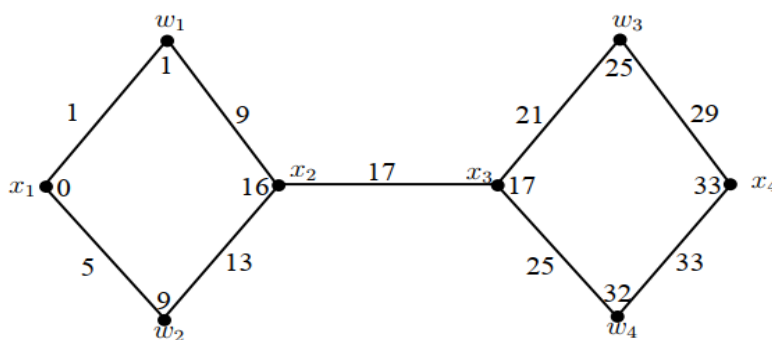


Figure 4: Hilbert mean labelling of $A(C_4^2)$

Theorem 3.10: The Bull graph is a Hilbert mean graph.

Proof: Let us denote Bull graph by BG . Let $V(BG) = \{w_1, w_2, \dots, w_5\}$ and $E(BG) = \{w_{i_1}w_{i_1+1} : 1 \leq i_1 \leq 4, w_2w_{n-1}\}$. We observe that BG has 5 vertices and 5 edges. Define $\mu : V(BG) \rightarrow \{0, 1, 2, \dots, H_5\}$ as follows.

$$\mu(w_1) = H_1 - 1, \quad \mu(w_2) = H_1$$

$$\text{For } 3 \leq i_1 \leq 5, \mu(w_{i_1}) = \begin{cases} H_{i_1} & \text{if } i_1 \text{ is odd} \\ H_{i_1+1} - 1 & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(BG) \rightarrow \{H_1, H_2, \dots, H_5\}$ is defined as follows.

$$\mu^*(w_{i_1}w_{i_1+1}) = H_{i_1}, i_1 = 1, 2 \quad \mu^*(w_{i_1+2}w_{i_1+3}) = H_{i_1+3}, i_1 = 1, 2 \quad \mu^*(w_2w_4) = H_3$$

Thus, we get the induced edge labels as H_1, H_2, \dots, H_5 .

Hence the Bull graph is a hilbert mean graph.

Example 3.11: The hilbert mean labeling of BG is given in figure 5.

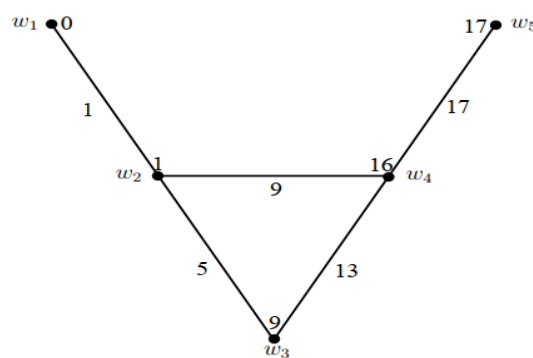


Figure 5: Hilbert mean labeling of BG

Theorem 3.12: The splitting graph $Spl(P_m)$ is a hilbert mean graph, where $m \geq 4$ and m is even.

Proof: Let $G = Spl(P_m), V(G) = \{w_{i_1}, x_{i_1} : 1 \leq i_1 \leq m\}$ and

$$E(G) = \{x_{i_1}x_{i_1+1}, w_{i_1}x_{i_1+1}, x_{i_1}w_{i_1+1} : 1 \leq i_1 \leq m-1\}$$

We observe that G has $2m$ vertices and $3m - 3$ edges.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{3m-3}\}$ as follows.

$$\text{For } 1 \leq i_1 \leq m, \mu(w_{i_1}) = \begin{cases} 12(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 12(i_1 - 2) + 9 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu(x_{i_1}) = \begin{cases} 12(i_1 - 1) + 8 & \text{if } i_1 \text{ is odd} \\ 12(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$$

Clearly, μ is an injective function and the induced bijective edge labeling $\mu^* : E(G) \rightarrow \{H_1, H_2, \dots, H_{3m-3}\}$ is defined as follows.

$$\text{For } 1 \leq i_1 \leq m-1, \mu^*(x_{i_1}w_{i_1+1}) = \begin{cases} H_{3(i_1-1)+3} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+4} & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu^*(w_{i_1}x_{i_1+1}) = \begin{cases} H_{3(i_1-1)+1} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+6} & \text{if } i_1 \text{ is even} \end{cases}$$

$$\mu^*(x_{i_1}x_{i_1+1}) = \begin{cases} H_{3(i_1-1)+2} & \text{if } i_1 \text{ is odd} \\ H_{3(i_1-2)+5} & \text{if } i_1 \text{ is even} \end{cases}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3m-3}$.

Hence the splitting graph $Spl(P_m)$ is a hilbert mean graph, where $m \geq 4$ and m is even.

Theorem 3.13: An alternate trianglular snake graph $ATS(P_n)$ is a hilbert mean graph, where $n \geq 4$ and n is even.

Proof: Let $G = ATS(P_n), V(G) = \{x_{i_1} : 1 \leq i_1 \leq n, y_{i_1} : 1 \leq i_1 \leq \frac{n-2}{2}\}$ and

$$E(G) = \left\{ x_{i_1}x_{i_1+1} : 1 \leq i_1 \leq n-1, x_{2i_1}y_{i_1} \cup x_{2i_1+1}y_{i_1} : 1 \leq i_1 \leq \frac{n-2}{2} \right\}$$

We observe that G has $\frac{3n-2}{2}$ vertices and $2n - 3$ edges.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{2n-3}\}$ as follows.

For $1 \leq i_1 \leq n$, $\mu(x_{i_1}) = \begin{cases} 8(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 8(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$

$$\mu(y_{i_1}) = 16(i_1 - 1) + 9 \quad 1 \leq i_1 \leq \frac{n-2}{2}$$

Clearly, μ is an injective function and the induced bijective edge labeling μ^* is defined as follows.

$$\mu^*(x_{i_1}x_{i_1+1}) = H_{2(i_1-1)+1} \quad 1 \leq i_1 \leq n-1$$

$$\mu^*(x_{2i_1}y_{i_1}) = H_{4(i_1-1)+2} \quad 1 \leq i_1 \leq \frac{n-2}{2}$$

$$\mu^*(x_{2i_1+1}y_{i_1}) = H_{4(i_1-1)+4} \quad 1 \leq i_1 \leq \frac{n-2}{2}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{2n-3}$.

Hence the alternate triangular snake graph $ATS(P_n)$ is a hilbert mean graph, where $n \geq 4$ and n is even.

Example 3.14: The hilbert mean labeling of $ATS(P_6)$ is given in figure 6.

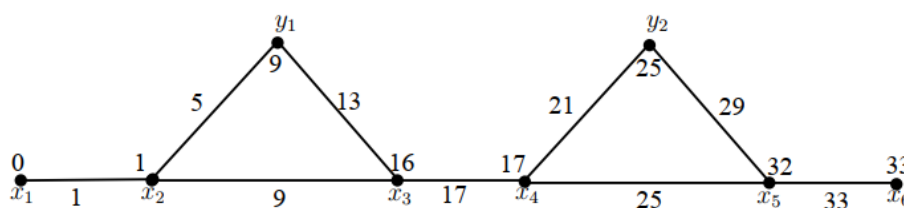


Figure 6: Hilbert mean labeling of $ATS(P_6)$

Theorem 3.15: A double alternate triangular snake graph $DATS(P_n)$ is a hilbert mean graph, where $n \geq 4$ and n is even.

Proof: Let $G = DATS(P_n), V(G) = \{x_{i_1} : 1 \leq i_1 \leq n, y_{i_1} : 1 \leq i_1 \leq \frac{n-2}{2}, z_{i_1} : 1 \leq i_1 \leq \frac{n-2}{2}\}$ and $E(G) = \{x_{i_1}x_{i_1+1} : 1 \leq i_1 \leq n-1 \cup x_{2i_1}y_{i_1} \cup x_{2i_1+1}y_{i_1} \cup x_{2i_1}z_{i_1} \cup x_{2i_1+1}z_{i_1} : 1 \leq i_1 \leq \frac{n-2}{2}\}$. We observe that G has $2n-2$ vertices and $3n-5$ edges.

Define $\mu : V(G) \rightarrow \{0, 1, 2, \dots, H_{3n-5}\}$ as follows.

$$\text{For } 1 \leq i_1 \leq n, \mu(x_{i_1}) = \begin{cases} 12(i_1 - 1) & \text{if } i_1 \text{ is odd} \\ 12(i_1 - 2) + 1 & \text{if } i_1 \text{ is even} \end{cases}$$

$$\text{For } 1 \leq i_1 \leq \frac{n-2}{2}, \mu(y_{i_1}) = 24(i_1 - 1) + 9, \mu(z_{i_1}) = 24(i_1 - 1) + 17$$

Clearly, μ is an injective function and the induced bijective edge labeling μ^* is defined as follows.

$$\mu^*(x_{i_1}x_{i_1+1}) = H_{3(i_1-1)+1} \quad 1 \leq i_1 \leq n-1$$

$$\text{For } 1 \leq i_1 \leq \frac{n-2}{2}, \mu^*(x_{2i_1}y_{i_1}) = H_{6(i_1-1)+2}, \mu^*(x_{2i_1+1}y_{i_1}) = H_{6(i_1-1)+5}$$

$$\mu^*(x_{2i_1}z_{i_1}) = H_{6(i_1-1)+3}, \mu^*(x_{2i_1+1}z_{i_1}) = H_{6(i_1-1)+6}$$

Thus, we get the induced edge labels as $H_1, H_2, \dots, H_{3n-5}$.

Hence the double alternate triangular snake graph $DATS(P_n)$ is a hilbert mean graph, where $n \geq 4$ and n is even.

Example 3.16: The hilbert mean labeling of $DATS(P_6)$ is given in figure 7.

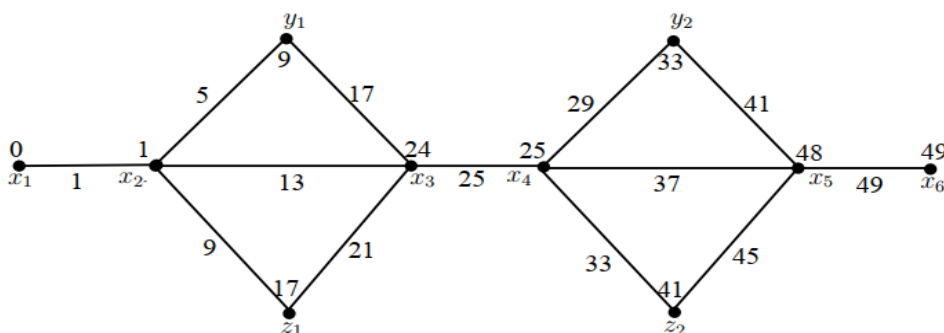


Figure 7: Hilbert mean labeling of $DATS(P_6)$

4. CONCLUSION

In this paper, we have investigated the hilbert mean labelling of certain special graphs. This work contributes several new results to the theory of graph labeling.

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