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A COMPARATIVE STUDY OF THREE FORMS OF ENTROPY ON TRADE VALUES BETWEEN KOREA AND FOUR COUNTRIES

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Abstract. Recently Wood-Jang[21] studied some applications of the Choquet integral in the trading relationship that Korea shares with selected trading partners. In this study, we consider the fuzzy entropy and the Shannon entropy, in addition we also define the Choquet entropy on a fuzzy set and develop four fuzzy sets which are related to that of the Choquet expected utility $CEU(u(a))$ for the trade values of utility u from an act a on S. Using this data set, we calculate three forms of entropy on four fuzzy sets as in the Choquet expcetes utility for the trade values that exist between Korea and four trading partner countries. Furthermore, we provide comparisons with three forms of entropy on four fuzzy sets which are representative of the four trading partner countries analyzed in this study.

1. Introduction

Many researchers have studied the Choquet integrals with respect to a fuzzy measure of fuzzy sets or interval-valued fuzzy sets and their applications in [2,4-13,19,20,26]. There are several examples of such analysis, these include student evaluations, similarity measures, the examination of the Choquet expected utility, and various other forms of inequalities. Recently, by using Choquet integrals with respect to a fuzzy measure, Wood-Jang[20,21] studied applications of them . These include some applications of the Choquet integral by firstly examining the imprecise market premium functions, and then more recently the trade relationship that Korea shares with selected trading partners. By using fuzzy sets and Choquet integrals in [17], studies utilized the concept of Choquet integral expected utility and its related areas(see[12,13,19,21,26]). Note that Biswas [2] investigated a student's evaluation on the space of fuzzy sets which include data information from the students' respective classes.

Our first motivation was to build on our previous efforts by considering three forms of fuzzy entropy in [1,3,14,15,18,24,25], the Shannon entropy in [3,21,24,25], and the Choquet entropy which we define in this study. From this point of view, we provide a comparative study of three

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forms of entropy on the level of trade that exists between Korea and four trading partners using data obtained from the WTO [23]. Our second motivation for conducting this study, was to provide a unique analysis of international trade flows using the Shannon entropy, fuzzy entropy, and Choquet entropy techniques.

In this study, we consider the fuzzy entropy and the Shannon entropy, in addition we provide a definition of the Choquet entropy on a fuzzy set and also develop four fuzzy sets which are related to the Choquet expected utility $CEU(u(a))$ for the trade values of utility u from an act a on S for the specified 2-digit HS product codes $(01-05)$ for animal product exports between Korea and selected trading partners for years 2010-2013 using date obtained from the WTO regional trade database[23]. Using this data set, we calculate three forms of entropy on four fuzzy sets as in the Choquet expected utility for the trade values that exist between Korea and four trading partner countries (New Zealand, USA, India, Turkey). Furthermore, we provide comparisons for three forms of entropy on four fuzzy sets for the four trading partner countries.

2. Three forms of entropy on fuzzy sets

Let X be a finite set of states of nature and $F(X)$ be the set of all fuzzy sets $A = \{(x, m_A(x)) \mid x \in X,\ m_A \longrightarrow [0,1] \text{ is a function}\}$. Recall that m_A is called a membership function of A.

Definition 2.1. ([2,4-13,19,20,26])

(1) A real-valued function μ on X is called a fuzzy measure if it satisfies

$$\begin{array}{ll} \text{(i)} & \mu(\emptyset)=0,\ \mu(X)=1, \\ \text{(ii)} & A \subset B \Rightarrow \mu(A) \leq \mu(B), \end{array} \tag{1}$$

where A, B are subsets of X.

(2) The Choquet integrals with respect to a fuzzy measure μ of $A \in F(X)$ is defined by

$$(C)\int m_A d\mu = \int_0^1 \mu(\{x \in X | m_A(x) \geq \alpha\})d\alpha. \tag{2}$$

Remark 2.1. *Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite set. It is well known that the discrete Choquet integral with respect to a fuzzy measure μ is followings.*

$$(C)\int m_A d\mu = \sum_{i=1}^{n} m_A(x_{(i)}) \left[\mu(E^{(i)}) - \mu(E^{(i+1)})\right], \tag{3}$$

where $(\cdot)$ indicates a permutation on $\{1, 2, \cdots, n\}$ such that

$$m_A(x_{(1)}) \leq m_A(x_{(2)} \leq \cdots \leq m_A(x_{(n)}), \tag{4}$$

$E^{(i)} = \{x \in X | m_A(x) \geq m_A(x^{(i)}\}$ *for $i = 1, 2, \cdots, n$ and let $E^{n+1} = \emptyset$* (see [4-7,9,10,11,13]).

By using the Choquet integral, we consider the Choquet expected utility $CEU(u(a))$ of utility $u(a)$ from an act a as follows. Note that in economics, the utility function is an important concept that measures preferences over a set of goods and services. Utility is measured in units called utils, which represent the welfare or satisfaction of a consumer from consuming a certain number of goods. Here, we assume that an act is a function from S to X, where S is a finite set of states of nature.

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Definition 2.2. ([5]) Let $u : X \longrightarrow [0,1]$ be a utility and a be an act from S to X. The full version of the Choquet expected utility is mentioned above so we can used the CEU abbreviation here. with respect to a fuzzy measure μ of utility u from act a is defined by

$$CEU(u(a)) = (C)\int u(a(s))d\mu(s). \tag{5}$$

Now, we introduce three forms of entropy on a fuzzy set as follows. Firstly, the Shannon entropy which was first developed by Shannon [24]. The mathematical communication theory has been utilized to measure the fuzziness in a fuzzy set or system [25]. According to Shannon, the information source is a person or a device that produces messages, using the average minimum amount of information.

Definition 2.3. ([3,22,24,25]) Let $A \in F(X)$ and $X = \{x_1, x_2, \cdots, x_n\}$ be finite set. The Shannon entropy on A is defined by

$$E_S(A) = -\sum_{i=1}^{n} m_A(x_i)\log m_A(x_i). \tag{6}$$

Secondly, the fuzzy entropy on a fuzzy set is used to express the mathematical values of the fuzziness of fuzzy sets. The concept of entropy, the basic subject of information theory and telecommunication, is a measure of fuzziness in fuzzy sets. Luca-Termini [18] note that Fuzzy entropy $D(A)$ can be represented by the Shannon function as follows

$$D(A) = k\sum_{i=1}^{n} s(m_A(x_i)),$$

where $s(x) = -x\log x - (1-x)\log(1-x)$ is the Shannon function. When we put $X = \{x_1, x_2, \cdots, x_n\}$, we consider the followin fuzzy entropy $E_F(A)$ which is the fuzzy entropy $D(A)$ with $k = -\frac{1}{n}$.

Definition 2.4. ([1,3,14,15,18,24,25]) Let $A \in F(X)$ and $X = \{x_1, x_2, \cdots, x_n\}$ be finite set. The fuzzy entropy on A is defined by

$$E_F(A) = -\frac{1}{n}\sum_{i=1}^{n}[m_A(x_i)\log m_A(x_i) + (1-m_A(x_i))\log(1-m_A(x_i))]. \tag{7}$$

Thirdly, we define the Choquet entropy on a fuzzy set and compare the Choquet entropy to another two forms of entropy, this helps to demonstrate the role of the Choquet entropy through the trading relationship that exists between Korea and four of its trading partners in the next section.

Definition 2.5. Let $A \in F(X)$ and X be a set. The fuzzy entropy on A is defined by

$$E_C(A) = 1 - (C)\int m_A(x)d\mu(x). \tag{8}$$

Note that if $X = \{x_1, x_2, \cdots, x_n\}$ is finite set, then we get

$$E_C(A) = 1 - \sum_{i=1}^{n} m_A(x_{(i)})[\mu(A_{(i)}) - \mu(A_{(i+1)})], \tag{9}$$

where $(\cdot)$ indicates a permutation on $\{1, 2, \cdots, n\}$ such that

$$m_A(x_{(1)}) \le m_A(x_{(2)}) \le \cdots \le m_A(x_{(n)}) \tag{10}$$

and $A_{(i)} = i, 2, \cdots, n$ and $A_{(n+1)} = \emptyset$.

3. Constructions of four fuzzy sets

In this section, we create four fuzzy sets, the USA-fuzzy set U, the NZ-fuzzy set N, IN-fuzzy set I, TR-fuzzy set T for the CEU of the trade values that exist between Korea and four countries. Now, we consider the CEU of a utility on a set of trade values (in USD) that represent the trading relationship that Korea shares with selected trading partners(i.e. Korea-USA, Korea-New Zealand, Korea-India, and Korea-Turkey). In [21], we examined these respective trading relationships by incorporating a clearly defined set of Harmonized System (HS) product code categories (i.e. HS Codes $i = 1, 2, 3, 4, 5$) for each individual year that is under review (i.e. $2010, 2011, 2012, 2013$). We note that the product code definitions have been provided by the UN Comtrade's online database and the relevant categories are defined as follows(see [21]):

1. Live animals; animal products.
2. Meat and edible meat offal.
3. Fish and crustaceans, mollusks and other aquatic invertebrates.
4. Dairy produce; birds' eggs; natural honey; edible products of animal origin, not elsewhere specified or included.
5. Products of animal origin, not elsewhere specified or included.

Denote that HSPC=HS Product Code, s=Year, $a(s)$=Trade Value, $u(a(s))$=the utility of $a(s)$, $CEU(u(a(s))$=the Choquet Expected Utility of $u(a)$ from a. By using the trade values in tables A_1 through to A_4 in the Appendix, we calculate the Choquet expected utility $CEU(u(a))$ for the set of trade values (in USD) that represent Korea's trading relationship with a particular country for years 2010, 2011, 2012, and 2013. Let $s_1 = 2010, s_2 = 2011, s_3 = 2012, s_4 = 2013$. Let $X = \{1, 2, 3, 4, 5\}$. Note that $(\cdot)$ indicates a permutation on $\{1, 2, 3, 4, 5\}$, such that

$$CEU(u(a_{(1)})) \leq CEU(u(a_{(2)}) \leq \cdots \leq CEU(u(a_{(5)}). \tag{11}$$

Then we denote that $a_{(i)} = a(s_{(i)})$ for $i = 1, 2, 3, 4, 5$ satisfy the equation (11). We define a fuzzy measure μ on X as follows.

$$\begin{aligned}&\mu(E^{(4)}) = \mu_1(\{a_{(4)}\}) = 0.1, \ \ \mu(E^{(3)}) = \mu_1(\{a_{(3)}, a_{(4)}\}) = 0.3,\\ &\mu(E^{(2)}) = \mu_1(\{a_{(2)}, a_{(3)}, a_{(4)}\}) = 0.6, \ \ \mu(E^{(1)}) = \mu_1(\{a_{(4)}, a_{(3)}, a_{(2)}, a_{(1)}\}) = 1,\end{aligned} \tag{12}$$

and if $a(s)$ is the trade value of s and $u(a) = \sqrt{\frac{a}{100141401}}$, then by using Definition 2.3, we obtain the following $CEU(u(a))$ as follows:

$$\begin{aligned} CEU(u(a)) &= \sum_{i=1}^{4} u(a(s^{(i)})) \left(\mu(E^{(i)}) - (\mu(E^{(i+1)})\right) \\ &= 0.4u(a(s^{(1)})) + 0.3u(a(s^{(2)})) + 0.2u(a(s^{(3)})) + 0.1u(a(s^{(4)})). \end{aligned} \tag{13}$$

By using (5), we obtained the four tables $A_1 \sim A_4$ (see [17]). If we take $m_Y(i) = CEU(u(a(s_i)))$, then we develop four fuzzy sets $Y : \{1, 2, 3, 4, 5\} \to [0, 1]$ by $Y = \{(i, m_X(i)) | i = 1, 2, 3, 4, 5\}$. Here, Y is one of USA-fuzzy set U, NZ-fuzzy set N, IN-fuzzy set I, and TR-fuzzy set T defined by

$$U = \{(1, 0.05664),\ (2, 0.04483),\ (3, 0.93879),\ (4, 0.20821),\ (5, 0.04858)\} \tag{14}$$

$$N = \{(1, 0.00533),\ (2, 0.00000),\ (3, 0.78873),\ (4, 0.15976),\ (5, 0.01557)\} \tag{15}$$

$$I = \{(1, 0.00154),\ (2, 0.00000),\ (3, 0.04570),\ (4, 0.00000),\ (5, 0.00000)\} \tag{16}$$

$$T = \{(1, 0.00264),\ (2, 0.00887),\ (3, 0.00368),\ (4, 0.00470),\ (5, 0.00000)\} \tag{17}$$

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4. Calculate three forms of entropy on four fuzzy sets

In this section, we calculate three forms of entropy on four fuzzy sets U, N, I, T. Note that $0log0 = 1$. Firstly, from (6) and (14)-(17), we get Shannon entroys $E_S(U), E_S(N), E_S(T)$ on four fuzzy sets as follows.

$$\begin{aligned} E_S(U) = &-(0.05664 \log 0.05664 + 0.04483 \log 0.04483 + 0.93879 \log 0.93879 \\ &+ 0.20821 \log 0.20821 + 0.04858 \log 0.04858) = 0.83476, \end{aligned} \tag{18}$$

$$\begin{aligned} E_S(N) = &-(0.00533 \log 0.00533 + 0.00000 \log 0.00000 + 0.78873 \log 0.78878 \\ &+ 0.15978 \log 0.15978 + 0.01557 \log 0.01557) = 0.57291, \end{aligned} \tag{19}$$

$$\begin{aligned} E_S(I) = &-(0.00154 \log 0.00154 + 0.00000 \log 0.00000 + 0.04570 \log 0.04570 \\ &+ 0.00000 \log 0.00000 + 0.00000 \log 0.00000) = 0.15099, \end{aligned} \tag{20}$$

$$\begin{aligned} E_S(T) = &-(0.00264 \log 0.00264 + 0.00887 \log 0.00887 + 0.00368 \log 0.00368 \\ &+ 0.00470 \log 0.00470 + 0.00000 \log 0.00000) = 0.10340. \end{aligned} \tag{21}$$

Secondly, from (7) and (14)-(17), we get fuzzy entropys $E_F(U), E_F(N), E_F(I), E_F(T)$ on four fuzzy sets as follows.

$$\begin{aligned} E_F(U) = &-\frac{1}{5}(0.05664 \log 0.05664 + (1 - 0.05664) \log(1 - 0.05664) \\ &+ 0.04483 \log 0.04483 + (1 - 0.04483) \log(1 - 0.04483) \\ &+ 0.93879 \log 0.93879 + (1 - 0.93879) \log(1 - 0.93879) \\ &+ 0.20821 \log 0.20821 + (1 - 0.20821) \log(1 - 0.20821) \\ &+ 0.04858 \log 0.04858 + (1 - 0.04858) \log(1 - 0.04858)) = 0.26736, \end{aligned} \tag{22}$$

$$\begin{aligned} E_F(N) = &-\frac{1}{5}(0.00533 \log 0.00533 + (1 - 0.00533) \log(1 - 0.00533) \\ &+ 0.00000 \log 0.00000 + (1 - 0.00000) \log(1 - 0.00000) \\ &+ 0.78873 \log 0.78878 + (1 - 0.78873) \log(1 - 0.78878) \\ &+ 0.15978 \log 0.15978 + (1 - 0.15978) \log(1 - 0.15978) \\ &+ 0.01557 \log 0.01557 + (1 - 0.01557) \log(1 - 0.01557)) = 0.21368, \end{aligned} \tag{23}$$

$$\begin{aligned} E_F(I) = &-\frac{1}{5}(0.00154 \log 0.00154 + (1 - 0.00154) \log(1 - 0.00154) \\ &+ 0.00000 \log 0.00000 + (1 - 0.00000) \log(1 - 0.00000) \\ &+ 0.04570 \log 0.04570 + (1 - 0.04570) \log(1 - 0.04570) \\ &+ 0.00000 \log 0.00000 + (1 - 0.00000) \log(1 - 0.00000) \\ &+ 0.00000 \log 0.00000 + (1 - 0.00000) \log(1 - 0.00000)) = 0.03834, \end{aligned} \tag{24}$$

$$\begin{aligned} E_F(T) = &-\frac{1}{5}(0.00264 \log 0.00264 + (1 - 0.00264) \log(1 - 0.00264) \\ &+ 0.00887 \log 0.00887 + (1 - 0.00887) \log(1 - 0.00887) \\ &+ 0.00368 \log 0.00368 + (1 - 0.00368) \log(1 - 0.00368) \\ &+ 0.00470 \log 0.00470 + (1 - 0.00470) \log(1 - 0.00470) \\ &+ 0.00000 \log 0.00000 + (1 - 0.00000) \log(1 - 0.00000)) = 0.01447. \end{aligned} \tag{25}$$

Thirdly, from (9) and (14)-(17), we get Choquet entropys $E_C(U), E_C(N), E_C(I), E_C(T)$ on four fuzzy sets as follows.

$$\begin{aligned} E_C(U) = 1 - (&0.04483 \times 0.1 + 0.04858 \times 0.1 + 0.05664 \times 0.6 \\ &+ 0.20821 \times 0.1 + 0.93879 \times 0.1) = 0.536119, \end{aligned} \tag{26}$$

$$\begin{aligned} E_C(N) = 1 - (&0.00000 \times 0.1 + 0.00533 \times 0.1 + 0.01557 \times 0.6 \\ &+ 0.15976 \times 0.1 + 0.78873 \times 0.1) = 0.895276, \end{aligned} \tag{27}$$

$$\begin{aligned} E_C(I) = 1 - (&0.00154 \log 0.00154 + 0.00000 \log 0.00000 + 0.04570 \log 0.04570 \\ &+ 0.00000 \log 0.00000 + 0.00000 \log 0.00000) = 0.15099, \end{aligned} \tag{28}$$

$$\begin{aligned} E_C(T) = 1 - (&0.00264 \log 0.00264 + 0.00887 \log 0.00887 + 0.00368 \log 0.00368 \\ &+ 0.00470 \log 0.00470 + 0.00000 \log 0.00000) = 0.10340. \end{aligned} \tag{29}$$

Through investigations (18)-(29), we can compare the three forms of entropy on the four fuzzy sets U, N, I, T as follows.

(1) In this study, we found that the Shannon entropy had a range $[0, 1]$ and the fuzzy entropy had a range $[0, 0.5]$ and the Choquet entropy had a range $[0.5, 1]$.

(2) The Shannon entropy E_S and the fuzzy entropy E_F have the same order of four countries in the trading relationship that Korea shares with selected trading partner as follows.

$$E_S(U) \geq E_S(N) \geq E_S(I) \geq E_S(T), \tag{30}$$

and

$$E_F(U) \geq E_F(N) \geq E_F(I) \geq E_F(T). \tag{31}$$

(3) The Choquet entropy has the following order of the trading relationship that Korea shares with selected trading partner.

$$E_C(U) \leq E_F(N) \leq E_F(I) \leq E_F(T). \tag{32}$$

From (2) and (3), we observe the order in which Choquet entropy was investigated is the opposite of the order in the other two forms.

5. Conclusions

According to the data analyzed in experiments (27) - (29), we can see that the Shannon entropy and the Fuzzy entropy contain the same order of results. However, the results for the Choquet entropy were very different to that of the Fuzzy and Shannon entropies, as the opposite order of results being obtained. This finding also suggests that the results for the Choquet entropy exhibit a much higher level of ambiguity across the four countries (New Zealand, India, the US, and Turkey) analyzed, particular those countries that have a smaller trading relationship with Korea.

From an economic perspective, the Shannon and Fuzzy entropies have provided scholars with a means of better understanding the scope and magnitude of the potential relationship that exists between particular entities, in this case the trading relationship between Korea and four trading partners. In an era where the development of stronger bilateral economic ties through trade, such an analysis provides a unique but timely portrayal.

Furthermore, the ambiguities present in the Choquet entropy findings highlight the important need to carry out additional research. Such efforts would help to establish a clearer understanding on the types of trading relationships present between Korea and the four countries selected for this study.

References

[1] S. Al-sharhan, F. Karray, W. Gveaieb, O. Bosir, *Fuzzy entropy; A brief surbey*, *10th IEEE International conference an fuzzy systems*, Melboarne, Australia, 1135-1139.

[2] R. Biswas, *An apllication of fuzzy sets in students' evaluation*, *Fuzzy Sets Syst.* **74** (1995) 187-194.

[3] P. Burillo, *Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets*, *Fuzzy Sets Syst.* **78** (1996) 305-316.

[4] G. Choquet, *Theory of capacities*, *Ann. Inst. Fourier* **5** (1953) 131-295.

[5] I. Gilboa, D. Schmeidler, *Maximin expected utility with non-unique prior*, *J. Math. Econ.* **18** (1989) 141-153.

[6] L.C. Jang, B.M. Kil, Y.K. Kim, J.S. Kwon, *Some properties of Choquet integrals of set-valued functions*, *Fuzzy Sets Syst.*, **91** (1997) 61-67.

[7] L.C. Jang, J.S. Kwon, *On the representation of Choquet integrals of set-valued functions and null sets*, *Fuzzy Sets Syst.*, **112** (2000) 233-239.

[8] L.C. Jang, *A note on the interval-valued fuzzy integral by means of an interval-representable pseudo-multiplication and their convergence properties*, *Fuzzy Sets Syst.* **137** (2003) 11-26.

[9] L.C. Jang, *The application of interval-valued Choquet integrals in multicriteria decision aid*, *J. Appl. Math. computing*, **20(1-2)** (2006) 549-556.

[10] L.C. Jang, *A note on Jensen type inequality for Choquet integrals*, *Int. J. Fuzzy logic and Intell. syst.*, **9(2)** (2009) 71-75.

[11] L.C. Jang, *On properties of the Choquet integral of interval-valued functions*, *Journal of Applied Mathematics*, **2011)** (2011) Article ID 492149, 10 pages.

[12] L.C. Jang, J. Wood, *The application of the Choquet integral expected utility in international trade*, *Advan. Stud. Contem. Math.*, **27(2)** (2017) 159-173.

[13] L.C. Jang, *A note on convergence properties of interval-valued capacity functionals and Choquet integrals*, *Information Sciences* **183** (2012) 151-158.

[14] B. Kasko, *Fuzzy entropy and conditioning*, *Information Sciences* **40** (1986) 165-174.

[15] A.D. Luca, S. Termini, *A definition of a nonprobabilistic entropy in setting of fuzzy sets theory*, *Information Control* **20** (1972) 301-312.

[16] L. Mangelsdorff, M. Weber, *Testing Choquet expected utility J. Econ. Behavior and Organization* **25** (1994) 437-457.

[17] T. Murofushi, M. Sugeno, *Some quantities represented by Choquet integral Fuzzy Sets Syst.* **56** (1993) 229-235.

[18] E. Pasha, R. Farnoosh, A. Fatemi, *Fuzzy entropy as cost function in image processing Proceedings of the 2nd IMT-GT Regionsl Conference on Mathematics, Statistics and Applications* Universiti Sains Malaysia, Penang, June 13-15, 2006.

[19] D. Schmeidler, *Subjective probability and expected utility without additivity*, *Econometrica* **57(3)** (1989) 571-587.

[20] J. Wood, L.C. Jang, *A note on Choquet integrals and imprecise market premium functionals*, *Proc. Jangjeon Math.Soc.*, **18(4)** (2015) 601-608.

[21] J. Wood, L.C. Jang, *A study on the Choquet integral with respect to a capacity and its applications*, *Global J. Pure Applied Math.*, **12(2)** (2016) 1593-1599.

[22] C.E. Shannon, *Amathematical theory of communication*, *The Best System Technical Journal*, **27** (1948) 379-423.

[23] WTO (2016). WTO Regional Trade Database.

[24] L. Xuechang, *Entropy, distance maesure and similarity measure of fuzzy sets and their relations*, *Fuzzy Sets Syst.*, **52** (1992) 201-227.

[25] W. Zeng, H. Li, *Relationship btween similarity maesure and entropy of interval-valued fuzzy sets* , *Fuzzy Sets Syst.*, **57** (2006) 1477-1484.

[26] D. Zhang, *Subjectiv ambiguity, expected utility and Choquet expected utility*, *Econ. Theory* **20** (2002) 159-181.

6. Appendix

Table A1: The CEU for animal product exports between Korea and the USA for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU_{(i,USA)}(u(a))$
1	s_1	$286892 = a(s^{(1)})$	0.05352	0.05664
	s_2	$330299 = a(s^{(2)})$	0.05743	
	s_3	$358496 = a(s^{(3)})$	0.05983	
	s_4	$364918 = a(s^{(4)})$	0.06037	
2	s_1	$997539 = a(s^{(4)})$	0.09981	0.04483
	s_2	$376805 = a(s^{(3)})$	0.06034	
	s_3	$30005 = a(s^{(1)})$	0.01731	
	s_4	$272884 = a(s^{(2)})$	0.05220	
3	s_1	$74866073 = a(s^{(1)})$	0.86464	0.93879
	s_2	$95654573 = a(s^{(2)})$	0.97734	
	s_3	$100141401 = a(s^{(4)})$	1.00000	
	s_4	$99871717 = a(s^{(3)})$	0.99865	
4	s_1	$3722326 = a(s^{(1)})$	0.19280	0.20821
	s_2	$4323214 = a(s^{(2)})$	0.20778	
	s_3	$5016833 = a(s^{(4)})$	0.22382	
	s_4	$4910771 = a(s^{(3)})$	0.22145	
5	s_1	$235669 = a(s^{(2)})$	0.04851	0.04858
	s_2	$359747 = a(s^{(3)})$	0.05994	
	s_3	$101795 = a(s^{(1)})$	0.05994	
	s_4	$863858 = a(s^{(4)})$	0.09088	

Table A2: The CEU for animal product exports between Korea and New Zealand for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU_{(i,NZ)}(u(a))$
1	s_1	$6650 = a(s^{(4)})$	0.00815	0.00533
	s_2	$4497 = a(s^{(3)})$	0.00670	
	s_3	$1589 = a(s^{(1)})$	0.00398	
	s_4	$2779 = a(s^{(2)})$	0.00527	
2	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
3	s_1	$70759196 = a(s^{(2)})$	0.84059	0.78873
	s_2	$91263506 = a(s^{(4)})$	0.95464	
	s_3	$70763937 = a(s^{(3)})$	0.84062	
	s_4	$46632301 = a(s^{(1)})$	0.68240	
4	s_1	$165773 = a(s^{(3)})$	0.04069	0.15976
	s_2	$113751 = a(s^{(1)})$	0.03370	
	s_3	$148756 = a(s^{(2)})$	0.03854	
	s_4	$277350 = a(s^{(4)})$	0.05263	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.01557
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$218022 = a(s^{(3)})$	0.04666	
	s_4	$393025 = a(s^{(4)})$	0.00265	

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Table A3: the CEU for Animal product expert between Korea and India for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU(u(a))$
1	s_1	$1050 = a(s^{(3)})$	0.00324	0.00264
	s_2	$1300 = a(s^{(4)})$	0.00360	
	s_3	$450 = a(s^{(1)})$	0.00212	
	s_4	$700 = a(s^{(2)})$	0.00264	
2	s_1	$35432 = a(s^{(3)})$	0.01881	0.00887
	s_2	$50639 = a(s^{(4)})$	0.02249	
	s_3	$2656 = a(s^{(1)})$	0.00515	
	s_4	$8230 = a(s^{(2)})$	0.00907	
3	s_1	$8695 = a(s^{(4)})$	0.009318	0.00368
	s_2	$5247 = a(s^{(3)})$	0.00724	
	s_3	$0 = a(s^{(1)})$	0.00000	
	s_4	$1865 = a(s^{(2)})$	0.00432	
4	s_1	$0 = a(s^{(1)})$	0.00000	0.00470
	s_2	$21614 = a(s^{(3)})$	0.01469	
	s_3	$30938 = a(s^{(4)})$	0.01758	
	s_4	$0 = a(s^{(2)})$	0.00000	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	

Table A4: The CEU for animal product exports between Korea and Turkey for years 2010-2013

HSPC	s	$a(s)$(USD)	$u(a(s))$	$CEU(u(a))$
1	s_1	$0 = a(s^{(1)})$	0.00000	0.00154
	s_2	$6900 = a(s^{(4)})$	0.00830	
	s_3	$150 = a(s^{(2)})$	0.00122	
	S_4	$300 = a(s^{(3)})$	0.00173	
2	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
3	s_1	$0 = a(s^{(1)})$	0.00000	0.04570
	s_2	$672952 = a(s^{(3)})$	0.08198	
	s_3	$2532837 = a(s^{(4)})$	0.15904	
	s_4	$199874 = a(s^{(2)})$	0.04468	
4	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	
5	s_1	$0 = a(s^{(1)})$	0.00000	0.00000
	s_2	$0 = a(s^{(2)})$	0.00000	
	s_3	$0 = a(s^{(3)})$	0.00000	
	s_4	$0 = a(s^{(4)})$	0.00000	