Dynamical Analysis on Two Dose Vaccines in the Presence of Media

Payal Rana[1], Dinkar Jha[1], and Sudipa Chauhan*[1]

[1]Department Of Mathematics, Amity Institute Of Applied Science, Amity University , Sector 125 , Noida , INDIA

Abstract

The Covid-19 outbreak hit us as a serious health crisis with vaccination to be seen as the only way out of it. And media can play the role of an advocate to fight against this epidemic by spreading important awareness regarding various protocols and vaccination strategies. Since breakthrough infections are becoming highly common, a two dose vaccine regime may be the need of the hour even for individuals with a pre-existing illness to build better immunity. Thus in this paper, our aim is to analyse a mathematical model with two dose vaccination strategy in the presence of media and breakthrough infections. An SIV_1V_2R model is formulated and the dynamical analysis is done. The basic reproduction number is obtained and the local stability analysis of both the disease-free and endemic equilibrium point is discussed based on reproduction number. The global stability of the endemic equilibrium point is done by graph theoretic approach. Finally, the numerical validation of the analytic solution is done using MATLAB using the real data of India for some important parameters to address a few vital questions which involves the role of media on the vaccination strategy. And sensitive model parameters effecting the basic reproduction number and endemic equilibrium points are identified using sensitivity analysis techniques like PRCC(partial rank correlation coefficient). Thus, the outcomes demonstrated the trend a two-dose vaccine model can follow and how the effect of media can help bring down the infections. This model provides support that two dose vaccination against COVID-19 with media presence for awareness is highly effective in controlling this epidemic.

Keywords: Vaccine; Covid-19; Global Stability; Parameter Sensitivity

1 Introduction

Since the advancements in medication and technology particularly with the initiation of vaccination, there has been an improvement in the quality of life

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in the age of infectious diseases. After the primary creation of vaccine by a doctor for pox from a live pox virus in 1796, in order to produce vaccines for alternative diseases several scientists and doctors followed his path as well. Many diseases are currently preventable like contagious disease, mumps, rubella, serum hepatitis, and respiratory illness due to the use of such vaccines[1, 2]. Due to the spread of SARS-CoV-2 virus, the COVID-19 outbreak was declared a pandemic and since then the development of COVID-19 vaccines has been the top priority of responsible stakeholders to control the outbreak. Afterwards with the availability of various types of vaccines to the world, the focus shifted to process of vaccination. There is an urgent need for the disbursement of vaccines to the general public so that the vaccines are effective to suppress the infection and in a very timely manner. Therefore, coming up with concerned strategies is crucial to the success of vaccination and control of epidemic as several of the vaccines need over one dose over a lifespan. Specially since the talk has now been shifted on the number of doses(one or two) [3], decisions regarding single or multi dose vaccines is a matter of great importance to avoid confusion among the general public.

1.1 Two-Dose Vaccine

For the ongoing Covid-19 epidemic, the often suggested vaccine strategy may be a two-dose program and even a booster dose in future[4, 5]. The second dose isn't thought of to be a booster vaccine but rather the use of this particular dose is with the aim to produce immunity to those that don't answer the primary dose. For example, more or less two to five of individuals don't develop immunity once the primary dose of the MMR(Measles, Mumps and Rubella) vaccine is given emphasising on the need of multi dose regime. Recently Covid-19 Vaccines have received emergency use authorization developed by Oxford-AstraZeneca, Pfizer-BioNTech and Moderna in different countries. Mass vaccination campaigns and clinical trials can provide high levels of protection against severe and symptomatic disease using 2 dose vaccines [6]. Due to a weak one-dose vaccine immunity in some vaccines, there could be short-term benefits where the virus may still continue to replicate [7]. This could eventually lead to immune escape mutations by the virus and thus a two-dose strategy may be able to mitigate this effect. Even for individuals who already have some existing illness, when co-infected with Covid may show better immunity when administered by two dose of vaccines than one dose vaccine as seen in the case of cancer patients in [8]. Multidose vaccines when compared to single-dose injection may offer a stronger protection against infection of the same vaccines and communication initiatives are needed to spread information about such regimes[9]. The ongoing discussions related to vaccination regimes are often led by media and influences the decision making of individuals.

1.2 Effect of media

The behavior of public with respect to vaccination may be altered due to involvement of media. People who may be infected may not come in contact with others because of the weakening effects due to their illness or due to the suggestions by public health organizations to quarantine to avoid infecting others. Hygienic measures may be taken up by general public to reduce the chance of getting infected and take steps to avoid large public gatherings. An example is of the 1994 outbreak of plague which presented with complex dynamics in a state in India [10]. After the outbreak of the disease many people in order to escape the disease fled the state of Surat and led to the transmission of the disease to other parts of the country. Thus, there is a need of proper discourse of information to the public. The media in particular greatly influences an individual's behavior toward a diseases and may also lead to interventions to control disease spread by governmental health care institutions. Awareness programs by media can make people comprehensive about a disease towards taking precautionary measures like wearing protective masks, social distancing and more importantly vaccination to suppress the chances of infection.

1.3 Empirical Literature and Structure of study

The most recent development in mathematical modelling in the field of biology or epidemic can be seen in [11, 12, 13, 14, 15]. There has been innumerable developments in mathematical modeling and numerical methods and its applications which is able to provide a better understanding and prediction for various types of systems like models depicting the relationship between computer viruses and epidemiology [16, 17, 18, 19, 20]. Mathematical models are able to provide a compatible understanding with the real-world dynamics of infection diseases. In order to exhibit the dynamics of Covid-19, there are many models available in the literatures for systems of nonlinear differential equations, making the models more realistic [21]. We have seen good researches in epidemic or infectious disease models[22, 23]. In [24], a deterministic model for Varicella Zoster Virus dynamics with vaccination is studied. Mathematical model on the outbreaks of influenza and to manage it by vaccination is discussed in [25]. A Dengue Epidemic model is considered amid vaccination in [26] and in [27] dynamic models is discussed with the importance of vaccination. And as of the recent Covid-19 outbreak some models with respect to vaccinations are discussed are discussed in [28, 29]. In [7] one dose regime is recommend if it produces a strong immune response. However, if a single vaccine dose is poor then the manufacturer recommended two-dose regime is suggested for a potential positive long term outcomes. Thus, a two dose vaccine Covid-19 model needs to be studies to understand its impact on the transmission of infection.

There may be some countries like the developing nations that may not be able to sustain a two-dose vaccination program for respiratory illness, and definitely would not be able to get funding for the multi-dose respiratory illness inoculation process. Thus, one needs to address the following questions: is it

doable to form one dose respiratory illness vaccination program that would replace a two-dose respiratory illness vaccination strategy? Is the involvement of media important in increasing vaccination process and reducing infection?. We shall aim to address these questions by developing a multi-dose vaccine model consisting of the susceptible, infected, vaccinated(First and second dose) and recovered (SIV_1V_2R) individuals in Section 2 and investigating the impact of media involvement to dispense information to the public. In section 3 the model dynamics are analyzed for the equilibrium point. We shall establish the local and global(graph theoretic method) for the endemic equilibrium. In Section 5 we will proceed with the numerical simulation where in we shall valid our results and understand the behaviour of our system. Under numerical discuss, we aim to find the sensitivity Indices of endemic equilibrium point to find the relevant parameters and their impact on the populations, followed by uncertainty analysis for the basic reproduction number to find important parameters related to transmission of infection in a two dose regime system. As part of our study, numerical discussion will help quantify the sensitivity index of the various parameters and give an insight to understand the effectiveness of the two dose regime and media to our variables and transmission of infection.
The novelty of our study is to encapsulate a two dose vaccination regime and the role of the media for a Covid-19 system and dynamically analysing thoroughly along with real data numerical validation.

2 The Model

The Model developed in this paper is motivated by the model by Kermack and McKendrick [30] which consists of the Susceptible, Infected and Removed (SIR) epidemiological class. SIR model was one of the revolutionary in its time but in present life with full of advanced technology, SIR model is one of the cornerstones of Mathematical Epidemiology. While assuming constant birth and death rate, SIR model divided the population into three different classes; Susceptible(S),Infected(I) and Recovered (R). The working of the SIR model can bee seen in Fig 1 for better understanding.

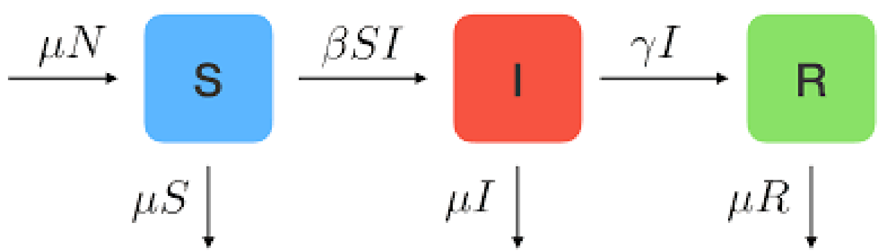

Figure 1: Flowchart of Model

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The differential Equations for the basic **SIR** model is ans follows:

$$\frac{dS}{dt} = \mu N - \beta SI - \mu S$$
$$\frac{dI}{dt} = \beta SI - \gamma I - \mu I$$
$$\frac{dR}{dt} = \gamma I - \mu R.$$

A two-dose regime may be able to provide better immunity to the general public and even to those who have some pre-existing illness [8]. Thus with this as motivation, we have extended the paper by incorporating two new vaccinated classes V_1 and V_2 in reference to the current scenario of covid. The model assumptions are considered as follows:

- Only a fraction of susceptible population get vaccinated due to the rumours regarding vaccinations.
- The interaction between susceptible and infected classes follow Holling type-II functional response.
- The population can still join the susceptible class and be prone to getting infected after two doses. These kind of infections are termed as 'Breakthrough' infections [31, 32] and exist for all types of vaccines prescribed against SARS-COVID-2. Breakthrough infections can be attributed to occurrence of severe variants (such as the delta variant), low immune response to vaccination and traveling to places that are seeing significant surge in cases. But the infections are mild in nature and may not lead to hospitalisation.
- The natural recovery is also not permanent and they can still get reinfected as defined by Indian Council Of Medical Research (ICMR). ICMR defines this reinfection as occurrence of two positive tests at a gap of at least 102 days with one interim negative test [31] .

Therefore, in reference to the assumptions, the extended model SIV_1V_2R is given by,

$$\begin{aligned}
\frac{dS}{dt} &= (1-p)\mu N - \mu S - \frac{b_1 IS}{(1+\alpha I)} + \psi R \\
\frac{dI}{dt} &= \frac{b_1 IS}{(1+\alpha I)} - \mu I - \gamma I \\
\frac{dV_1}{dt} &= p\mu N - \mu V_1 - p_1 V_1 \\
\frac{dV_2}{dt} &= p_1 V_1 - \mu V_2 - p_2 V_2 \\
\frac{dR}{dt} &= \gamma I + p_2 V_2 - \mu R - \psi R.
\end{aligned} \tag{1}$$

The system is bounded in the region $\{S, I, V_1, V_2, R; S + I + V_1 + V_2 + R = N\}$.

Table 1: Table for Variables and Parameters

Variables and Parameters	**Interpretation**
S	Susceptible Individual Density
I	Infected Individual Density
R	Recovered Individual Density
V_1	Vaccinated Individual Density After 1st Dose
V_2	Vaccinated Individual Density After 2nd Dose
N	Total Population Density
μ	Birth and Death rate
p	Rate of First Dose of Vaccine
p_1	Rate of Second Dose of Vaccine
p_2	Rate at which Vaccinated Individuals get Recovered
b_1	Rate of Infection
γ	Rate at which Infected Individuals Recover/ Natural Recovery Rate
ψ	Rate at which Recovered Individuals get Susceptible Again
α	Effect of Media

The description of the parameters and variables can be seen in Table 1 for our system. We can see the mechanism graphically in the schematic diagram in Fig 2 for the proposed model.

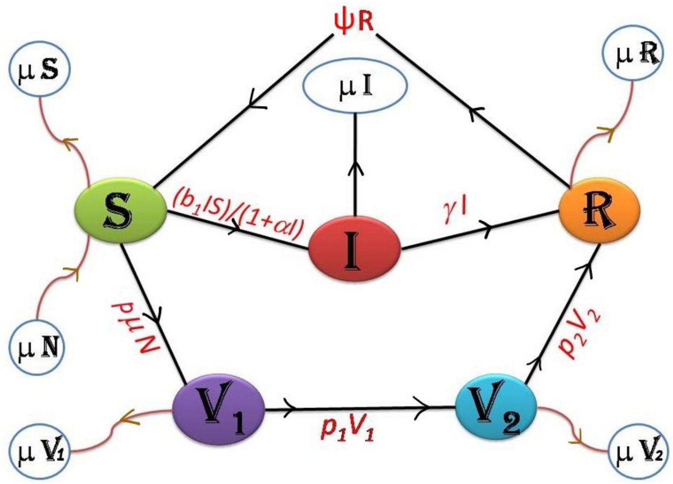

Figure 2: Schematic diagram of the SIV_1V_2R model. Susceptible individuals S can either move to the infected I or the vaccinated class V_1. At the rate of p, the susceptible who get vaccinated with the first dose will join vaccinated class V_1 . After recovering naturally the infected individuals join the recovered class R at the rate of γ. At the rate of p_1 individuals who receive the second dose of vaccination after getting the first dose will move towards the vaccinated class V_2. Individuals move towards the recovered class R at the rate of p_2 after receiving both the doses. Individuals from the recovered class R join back to the class of susceptible S at the rate of ψ owing to breakthrough infections/reinfections even after after getting vaccinated with both the doses or recovering from the infection naturally.

2.0.1 Questions Addressed by the Research Work

Our research takes a dig at the following unaddressed issues:

- Exploring the calibre of infected class response to complete vaccination. Can vaccination act as a lodestar to reduce infection ?
- The relation of media and pandemic. What is the role of media in infected and susceptible classes ?
- What is the degree of correlation, if there exist any, between the parametric values and the endemic equilibrium?.

3 Model Dynamics

3.1 Basic Reproduction Number

The basic reproduction number particularly for the study infectious diseased is considered a central concept and is the spectral radius of the next generation method. For the dynamic analysis of any general disease transmission model, the basic reproduction number R_0 [33, 34] is a crucial element. The trend or behaviour of R_0 can give significant implications for upcoming outbreaks. It gives the number of secondary infections arising due to a single infection. In the system, if $R_0 > 1$ then the disease will continue and if $R_0 \leq 1$ then the disease will die out. To explore the impact of vaccination we will be dealing with $R_0 > 1$ when the infection is present in the system. R_0 can be written as,

$$R_0 = \frac{b_1 N}{(\mu + \gamma)}.$$

3.2 Existence of equilibrium points

Equilibrium points are the steady state solutions where the system may approach in the long run. Our analysis will be around the equilibrium points and its stability as it can help to study the system behaviour in long run in reference to multi-dose vaccination. Thus, we shall first obtain the equilibrium points and the conditions for their existence. Our main focus will be on disease-free equilibrium point and endemic equilibrium point [35]. The system (1) posses a disease free or boundary equilibrium point $E^0(S^0, 0, V_1^0, V_2^0, R^0)$ given by,

$$S^0 = \frac{(1-p)N(\mu+\psi)(\mu+p_1)(\mu+p_2) + pp_1p_2\psi N}{(\mu+\psi)(\mu+p_1)(\mu+p_2)}, \quad V_1^0 = \frac{p\mu N}{(\mu+p_1)}$$

$$V_2^0 = \frac{pp_1\mu N}{(\mu+p_1)(\mu+p_2)}, \quad R^0 = \frac{pp_1p_2\mu N}{(\mu+\psi)(\mu+p_1)(\mu+p_2)}.$$

Endemic or interior equilibrium point $E^*(S^*, I^*, V_1^*, V_2^*, R^*)$ for the system (1) is given by

$$S^* = \frac{(\mu+\gamma)(1+\alpha I^*)}{b_1} = \frac{(1+\alpha I^*)N}{R_0},$$

$$V_1^* = \frac{p\mu N}{(\mu+p_1)}, \quad V_2^* = \frac{pp_1\mu N}{(\mu+p_1)(\mu+p_2)},$$

$$I^* = \frac{\mu(\mu+\psi)(\mu+p_1)(\mu+p_2)[(1-p)Nb_1 - (\mu+\gamma)] + \psi b_1 pp_1p_2\mu N}{[(\mu+\psi)(\mu+\gamma)(\alpha\mu+b_1) - \psi b_1\gamma](\mu+p_1)(\mu+p_2)},$$

$$R^* = \frac{\gamma\mu(\mu+\psi)(\mu+p_1)(\mu+p_2)[(1-p)Nb_1 - (\mu+\gamma)] + \psi b_1 pp_1p_2\mu N\gamma + pp_1p_2\mu N[(\mu+\gamma)(\mu+\psi)(\alpha\mu+b_1-\psi b_1\gamma)]}{(\mu+\psi)(\mu+p_1)(\mu+p_2)[(\mu+\psi)(\mu+\gamma)(\alpha\mu+b_1) - \psi b_1\gamma]},$$

where the equilibria exists if $R_0 > \frac{1}{(1-p)}$ and $(\alpha\mu + b_1) > \psi b_1 \gamma$.

We will now be analysing the stability of boundary and interior equilibrium points for the system (1).

3.3 Local Stability analysis

We shall prove the local stability for the model about the disease free and endemic equilibrium point to visualize the conditions under which the epidemic system can be stabilized [36]. General Jacobian matrix for our system is given by,

$$J = \begin{vmatrix} (-\mu - \frac{b_1 I}{1+\alpha I}) & -\frac{b_1 S}{(1+\alpha I)^2} & 0 & 0 & \psi \\ \frac{b_1 I}{(1+\alpha I)} & (\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma) & 0 & 0 & 0 \\ 0 & 0 & (-\mu - p_1) & 0 & 0 \\ 0 & 0 & p_1 & (-\mu - p_2) & 0 \\ 0 & \gamma & 0 & p_2 & (-\mu - \psi). \end{vmatrix}$$

General characteristic equation pertaining to the jacobian matrix above is given by,

$$\left[\left(\mu + \frac{b_1 I}{1+\alpha I} + \lambda\right)\left(\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma - \lambda\right)(\mu + p_1 + \lambda)(\mu + p_2 + \lambda)(\mu + \psi + \lambda)\right],$$
$$\left[\left(-\frac{b_1 I}{1+\alpha I}\right)\left(\frac{b_1 S}{(1+\alpha I)^2}\right)(\mu + p_1 + \lambda)(\mu + p_2 + \lambda)(\mu + \psi + \lambda)\right]\left[\left(\frac{b_1 I \gamma \psi}{1+\alpha I}\right)(\mu + p_1 + \lambda)(\mu + p_2 + \lambda)\right] = 0.$$

Characteristic equation pertaining to the boundary equilibrium point E^0 is given by,

$$(\mu + \lambda)(b_1 S^0 - \mu - \gamma - \lambda)(\mu + p_1 + \lambda)(\mu + p_2 + \lambda)(\mu + \psi + \lambda) = 0.$$

Eigen values corresponding to boundary equilibrium point E^0 are $\lambda_1 = -\mu, \lambda_2 = b_1 S^0 - (\mu + \gamma) < 0$ if $b_1 S^0 < (\mu + \gamma)$ or $R_0 < \frac{N}{S^0}, \lambda_3 = -(\mu + p_1), \lambda_4 = -(\mu + p_2), \lambda_5 = -(\mu + \psi)$. Consequently, E^0 is stable if $R_0 < \frac{N}{S^0}$. Next, the characteristic equation pertaining to interior equilibrium point $E^*(S^*, I^*, V_1^*, V_2^*, R^*)$ is given as follows:

$$\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5 = 0,$$

where
$a_1 = 5\mu + \psi + p_1 + p_2 + \gamma + \frac{b_1 I}{1+\alpha I} - \frac{b_1 S}{(1+\alpha I)^2}$,
$a_2 = -(\mu + \frac{b_1 I}{1+\alpha I})(\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma) + \frac{b_1^2 SI}{(1+\alpha I)^3} + (3\mu + \psi + p_1 + p_2)(2\mu + \gamma + \frac{b_1 I}{1+\alpha I} - \frac{b_1 S}{(1+\alpha I)^2}) + (\mu + \psi)(2\mu + p_1 + p_2) + (\mu + p_1)(\mu + p_2)$,
$a_3 = (3\mu + \psi + p_1 + p_2)\{\frac{b_1^2 SI}{(1+\alpha I)^3} - (\mu + \frac{b_1 I}{1+\alpha I})(\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma)\} + [(2\mu + p_1 + p_2)(\mu + \psi) + (\mu + p_1)(\mu + p_2)][2\mu + \gamma + \frac{b_1 I}{1+\alpha I} - \frac{b_1 S}{(1+\alpha I)^2}] + (\mu + \psi)(\mu + p_1)(\mu + p_2) - \frac{b_1 I \psi \gamma}{1+\alpha I}$,
$a_4 = \{(2\mu + p_1 + p_2)(\mu + \psi) + (\mu + p_1)(\mu + p_2)\}\{\frac{b_1^2 SI}{(1+\alpha I)^3} - (\mu + \frac{b_1 I}{1+\alpha I})(\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma)\} + (\mu + \psi)(\mu + p_1)(\mu + p_2)[2\mu + \gamma + \frac{b_1 I}{1+\alpha I} - \frac{b_1 S}{(1+\alpha I)^2}] - (2\mu + p_1 + p_2)\frac{b_1 I \psi \gamma}{1+\alpha I}$,
$a_5 = (\mu + \psi)(\mu + p_1)(\mu + p_2)[\frac{b_1^2 SI}{(1+\alpha I)^3} - (\mu + \frac{b_1 I}{1+\alpha I})(\frac{b_1 S}{(1+\alpha I)^2} - \mu - \gamma)] - (\mu + p_1)(\mu + p_2)\frac{b_1 I \psi \gamma}{1+\alpha I}$.
Thus, the endemic equiliria is locally stable according to Routh- Hurwitz criteria

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if $a_i's > 0, i = 1,2,3,4,5$ under the following conditions: $I^*(1+\alpha I^*) > S^*$, $R_0 > \frac{N(1+\alpha I^*)^2}{S^*}$, $\frac{b_1^2 S^* I^*}{(1+\alpha I^*)^3} > (\mu + \frac{b_1 I^*}{1+\alpha I^*})(\frac{b_1 S^*}{(1+\alpha I^*)^2} - \mu - \gamma)$, $\frac{I^*}{1+\alpha I^*} < \frac{(\mu+\psi)(\mu+p_1)(\mu+p_2)}{b_1\psi\gamma}$, $(\mu+\psi)(\mu+p_1)(\mu+p_2)[2\mu+\gamma+\frac{b_1 I^*}{1+\alpha I^*} - \frac{b_1 S^*}{(1+\alpha I^*)^2}] > (2\mu+p_1+p_2)\frac{b_1 I^*\psi\gamma}{1+\alpha I^*}$ and $(\mu+\psi)[\frac{b_1^2 S^* I^*}{(1+\alpha I^*)^3} - (\mu + \frac{b_1 I^*}{1+\alpha I^*})(\frac{b_1 S^*}{(1+\alpha I^*)^2} - \mu - \gamma)] > \frac{b_1 I^*\psi\gamma}{1+\alpha I^*}$ along with $a_1a_2a_3 > a_3^2 + a_1^2 a_4$ and $(a_1a_4 - a_5)(a_1a_2a_3 - a_3^2 - a_1^2a_4) > a_5(a_1a_2 - a_3)^2 + a_5^2 a_1$.

4 Global stability

To establish global stability we shall use the graph-theoretic method as in [37, 38, 39]. We shall construct the lyapunov function through a directed graph with the help of the terminologies from [37]. To construct the lyapunov function we shall use a directed graph . A directed graph has a set of ordered pair say (i,j) and vertices where (i,j) is known as arc to terminal vertex j from initial vertex i. For the terminal vertex j, $d^-(j)$ is the in-degree of j which denotes the number of arcs in the digraph. And for initial vertex is i, $d^+(i)$ is the out-degree of vertex i which denotes the number of arcs in the digraph. Let us consider a weighted directed graph say $\chi(J)$ over a $q \times q$ weighted matrix J where the weights(a_{ij}) of each arc if they exist are $a_{ij} > 0$, and if otherwise then $a_{ij} = 0$. we consider c_i as the co-factor of l_{ij} of the Laplacian of $\chi(J)$ which is given by:

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \sum_{k\neq i} a_{ij} & i = j. \end{cases}$$

If there is a strongly connected path i.e directed to and fro path for the arcs in $\chi(J)$ then $c_i > 0 \forall i = 1,2...,q$. We shall also use Theorem 3.3 and Theorem 3.4 from [37, 39], which will help us in the construction of lyapunov function. The theorems states:

- **Theorem3.3 of [37]**: if $a_{ij} > 0$ and $d^-(i) = 1$, for some i,j, then

$$c_i a_{ij} = \sum_{k=1}^{q} c_j a_{jk}.$$

- **Theorem 3.4 of [37]**: if $a_{ij} > 0$ and $d^+(j) = 1$, for some i,j, then

$$c_i a_{ij} = \sum_{k=1}^{q} c_k a_{ki.}$$

We shall also use **Theorem 3.5** of [37] as below:

Theorem 4.1. *Let us consider an open set $L \subset R^m$ and a function $f : L \to R^m$ for a system*

$$\dot{z} = f(z) \tag{2}$$

and assuming:
a) $\exists\ M_i : L \to R$, $H_{ij} : L \to R$ and $a_{ij} \geq 0$ such that

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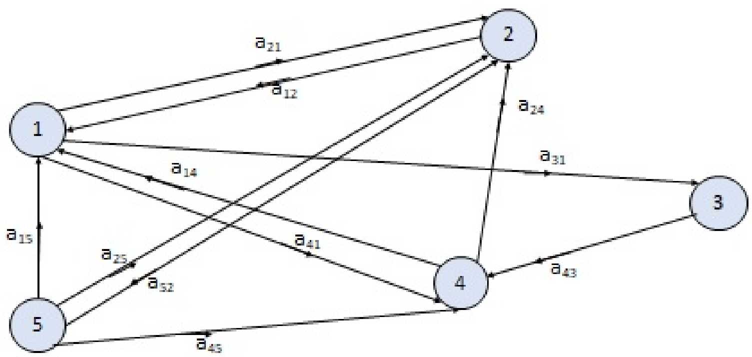

Figure 3: Directed graph for $\alpha = 1$ for the associated weights

$$M_i' = M_i'|_2 \leq \textstyle\sum_{j=1}^{q} a_{ij} H_{ij}(z), \text{ with } z \in L,\ i = 1, \ldots, q.$$

b) For $J = [a_{ij}]$, of (H, J) each directed cycle B_c satisfies:

$$\textstyle\sum_{(i,j)\in\epsilon(B_c)} H_{ij}(z) \leq 0\ ,\ z \in L$$

where $\epsilon(B_c)$ is set of arcs in B_c.
Then for $c_i \geq 0, i = 1, \ldots, q$ the function is:

$$M(z) = \sum_{i=1}^{q} c_i M_i(z)$$

satisfies $M'|_2 \leq 0$, that is, $M(z)$ is a Lyapunov function for 2.

4.1 Lyapunov Function Construction

Construction: $M_1 = \frac{(S-S^*)^2}{2}$, $M_2 = I - I^* - I^* \ln \frac{I}{I^*}$, $M_3 = \frac{(V_1-V_1^*)^2}{2}$, $M_4 = \frac{(V_2-V_2^*)^2}{2}$ and $M_5 = \frac{(R-R^*)^2}{2}$. Now by differentiation;
$M_1' = (S - S^*)S' \leq ((1-p)\mu N + \mu S^*)(S + V_2) + \frac{b_1 S^* IS}{(1+\alpha I)} + \psi RS = a_{14}H_{12} + a_{12}H_{12} + a_{15}H_{15}$ where $a_{14} = (1-p)\mu N + \mu S^*, a_{12} = b_1 S^*, a_{15} = \psi$.
$M_2' = (\frac{I-I^*}{I})I' \leq \frac{b_1 IS}{(1+\alpha I)} + (\mu + \gamma)I^*(I + V_2 + 1) = a_{21}H_{21} + a_{24}H_{24}$ where $a_{21} = b_1, a_{24} = (\mu + \gamma)I^*$.
$M_3' = (V_1 - V_1^*)V_1' \leq (p\mu N + \mu V_1^* + p_1 V_1^*)(V_1 + S) = a_{31}G_{31}$ where $a_{31} = (p\mu N + \mu V_1^* + p_1 V_1^*)$.
$M_4' = (V_2 - V_2^*)V_2' \leq p_1 V_1 V_2 + (\mu + p_2)(V_2 + S) = a_{43}G_{43} + a_{41}G_{41}$ where $a_{43} = p_1, a_{41} = (\mu + p_2)$.
$M_5' = (R - R^*)R' \leq \gamma IR + p_2 V_2 R + (\mu + \psi)(R + I) = a_{25}G_{25} + a_{45}G_{45} + a_{52}G_{52}$ where $a_{25} = \gamma, a_{45} = p_2, a_{52} = (\mu + \psi)$. We get an associated weighted directed graph as shown in Fig 3. Then by Theorem3.5[37] $\exists c_i' s, 1 \leq i \leq 5$ such that $M = \sum_{i=1}^{q} c_i M_i$ is a lyapunov function. Using Theorem 3.3 and 3.4 we get the relation between c_i.

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For $a_{31} > 0$ and $d^{-}(3) = 1$, we get $c_3 a_{31} = c_4 a_{43}$ and for $a_{52} > 0$ and $d^{-}(5) = 1$ we get $c_5 a_{52} = c_1 a_{15} + c_2 a_{25} + c_4 a_{45}$. Hence, $c_1 = c_4 = c_2 = 1$, $c_3 = \frac{a_{43}}{a_{31}}$ and $c_5 = \frac{a_{15}+a_{25}+a_{45}}{a_{52}}$. Thus, the lyapunov function is $M = M_1 + M_2 + \frac{p_1}{(p\mu N+\mu V_1^*+p_1 V_1^*)} M_3 + M_4 + \frac{p_2+\gamma+\psi}{(\mu+\psi)} M_5$. And for M':
$M' = (S - S^*)S' + (\frac{I-I^*}{I})I' + \frac{p_1}{(p\mu N+\mu V_1^*+p_1 V_1^*)}(V_1 - V_1^*)V_1' + (V_2 - V_2^*)V_2' + \frac{p_2+\gamma+\psi}{(\mu+\psi)}(R - R^*)R'$.
If we consider the set $U = \{x \in R_+^5 : M' = 0\}$ then we see that $S = S^*$, $V_1 = V_1^*$, $V_2 = V_2^*$, $I = I^*$ and $R = R^*$. Hence, we get the unique equilibrium point $(S^*, I^*, V_1^*, V_2^*, R^*)$. Therefore we say that the equilibrium point is globally stable using LaSalle's Invariance principle .

5 Numerical Simulation

In this section, we will discuss a numerical example in support of the analytic results of our system. We would try to encapsulate the sensitivity analysis of endemic equilibrium, global sensitivity analysis of basic reproduction number along with validation of our analytic solution for media effect. So, on computing

Parameters	**Values/Units**	**Source**
μ	0.0035342	Assumed
p	0.004545	https://www.mygov.in/covid-19
p_1	0.0001	https://www.mygov.in/covid-19
p_2	0.00909	https://www.mygov.in/covid-19
b_1	0.62	Assumed
γ	0.0476	Assumed
ψ	0.0011	https://www.mygov.in/covid-19
α	0.5	Assumed
N	140	Assumed

Table 2: Parameters and Values for $\text{SIV}_1\text{V}_2\text{R}$ model

the values using the parameters (mentioned in Table 2 where some values taken from the mygov.in site on 15 june,2021.) in the system of equations and we'll get unique positive equilibrium at (SIV_1V_2R) resting at (0.3348, 6.0267, 16.8756, 1.5341 ,25.3862) as seen in Fig 4, giving a glimpse about the behaviour or the condition of epidemic in future. Now, if we focus on the effect of vaccination on other classes like Susceptible and Infected, then we get to know by graph that:

- From Fig 5a, graph to analyse the relation between susceptible and Vaccinated class(First dose). Here we can see that the susceptible individuals are constant with increase in the vaccination process but on further increasing the vaccination, the susceptible individuals are exponentially increasing. As more and more people get vaccinated then through word of mouth and more confidence build on the idea of vaccination, more susceptible people will be willing to get vaccinated.

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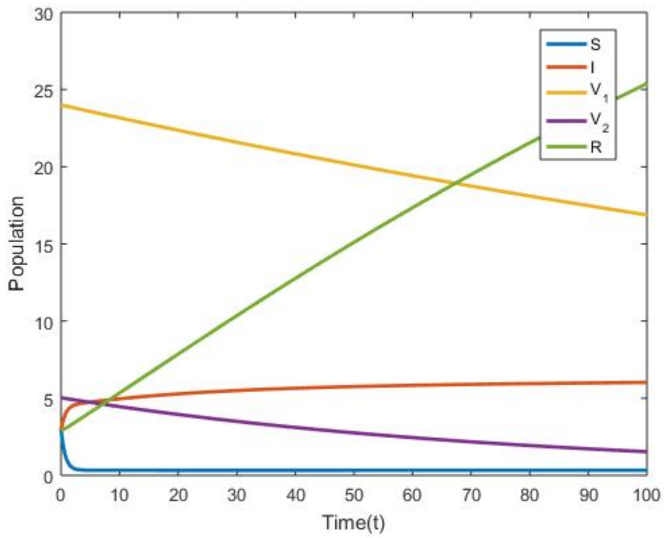


Figure 4: $\mathrm{SIV_1V_2R}$ Dynamic Graph

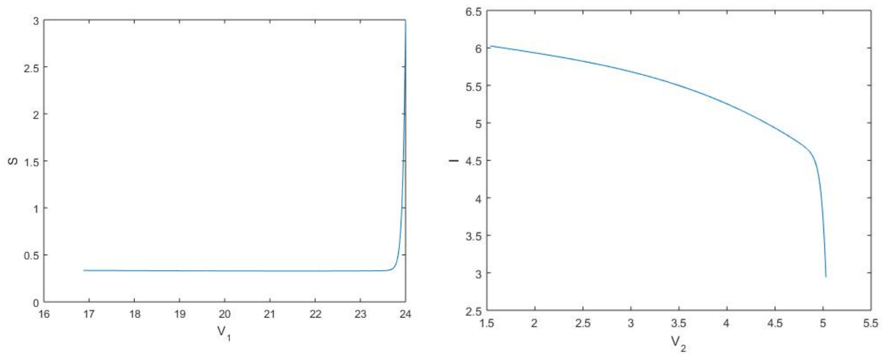
- Now by analysing the effect of complete(two dose) vaccination over infection rate (from Fig 5b), we get to know that the infectants goes on declining as individuals are getting vaccinated. This implies that two dose vaccination regime will help suppress transmission of infection and suppress the outbreak.

5.1 Sensitivity Indices of endemic equilibrium point

We shall now discuss the sensitivity analysis of equilibrium point with respect to the estimated parameters of our system. We aim to investigate the degree to which a parameter may affect the concerned variable through an affirmative relationship or a negative relationship through this process of parameter sensitivity analysis. We get the proportion that a relative change in a parameter brings to the relative change in a variable through the sensitivity index obtained. *Definition*[40, 41]: For the variable v that depends differentiably on a parameter h, we define the normalised forward sensitivity index Ω of a variable as:

$$\Omega_h^v = \frac{\partial v}{\partial h} \times \frac{h}{v}. \tag{3}$$

Thus, using the above method we get the sensitivity indices for each variable with respect to the parameters for endemic equilibrium as given in Table 3 and shown in Fig 6. For interpretation, if the index is positive it means that an increase in parameter will lead to the increase in the variable with the index value/magnitude. And a negative index implies that a increase in the parameter will lead to decrease in variable by the index value.



(a) Relation between Susceptible and Vaccinated classes

(b) Relation between Vaccinated and Infected classes

Figure 5: Relations with respect to Vaccinated class

-	S^*	I^*	V_1^*	V_2^*	R^*
p	-0.039	$-.0046$	1.1240	1.0837	-0.0096
α	-0.8494	0.0036	0	0	0.0042
p_1	$1.0794 * 10^{-4}$	$1.2664 * 10^{-4}$	-0.2222	0.7498	$6.6345 * 10^{-4}$
p_2	$3.9757 * 10^{-5}$	$4.7074 * 10^{-5}$	0	-0.7214	$2.4647 * 10^{-4}$
γ	0.1506	-0.9124	0	0	0.1019
μ	0.7197	0.7636	-0.7854	-1.0332	0.1196
ψ	0.1591	0.1867	0	0	-0.0219
N	0.8817	1.0461	1.1336	1.1475	1.2177
b_1	-0.9964	0.0042	0	0	0.0049

Table 3: Sensitivity indices, $\Omega_{h_j}^{v_i} = \frac{\partial v_i}{\partial h_j} * \frac{h_j}{v_i}$, of the state variables at the endemic equilibrium, v_i, to the parameters, h_j

5.2 Uncertainty analysis of R_0

For our model we shall also determine uncertainty analysis for R_0 by LHS method to get more validation for the relation between R_0 and its parameters. PRCC(partial rank correlation coefficient)[42] is one technique which will help us quantify the uncertainty for any model. We consider the four parameters from R_0 and have chosen normal distribution for them as in Fig 7. We find the PRCC values using Matlab with the following pdfs and shown in:
$b_1 \sim \quad Normal(0.62, 0.01)$,
$N \sim \quad Normal(140, 0.2)$,
$\gamma \sim \quad Normal(0.0476, 0.01)$,
$\mu \sim \quad Normal(0.0035342, 0.01)$.

We get the PRCC values for our input parameters which can be seen in Fig8. We get the following indexes for the parameters: $b_1 = 0.21, N = 0.26, \mu = -0.96$

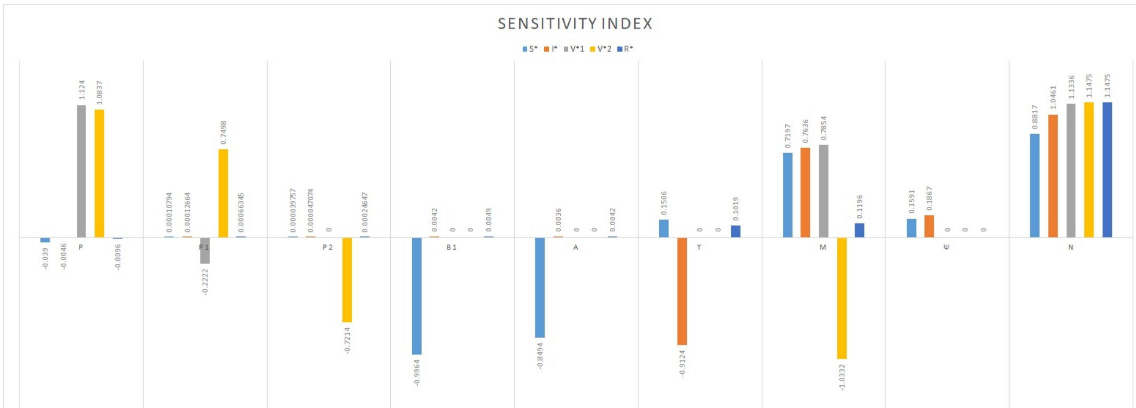


Figure 6: Bar graph for Sensitivity Index of $S^*, I^*, V_1^*, V_2^*, R^*$ with respect to the parameters

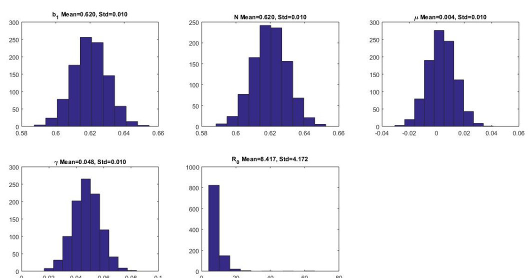


Figure 7: Distribution of parameters for R_0

and $\gamma = -0.96$. The graphs shows that R_0 is positively correlated to b_1 and N. The effect of the parameter μ and γ will bring about an opposite change in the transmission of infection as it is negatively correlated. Since the value μ and γ parameters are close to 1, it indicates a strong correlation to change in R_0. Thus, these two parameters are strongly negatively correlated to R_0.

We shall now check the contour plot for R_0 with respect to some combination of important parameters as in Fig 9. In Fig 9a we can see that the b_1 has a direct response to R_0, as the value of b_1 increases the gradation of color approaches to the highest color which is yellow. In a similar manner in Fig 9b we see that N too has direct response as we can see the color gradation approach yellow as N increases. And μ has a high indirect response to R_0, as the value of μ increases the gradation of color approaches to the lowest color which is dark blue and R_0 decreases.

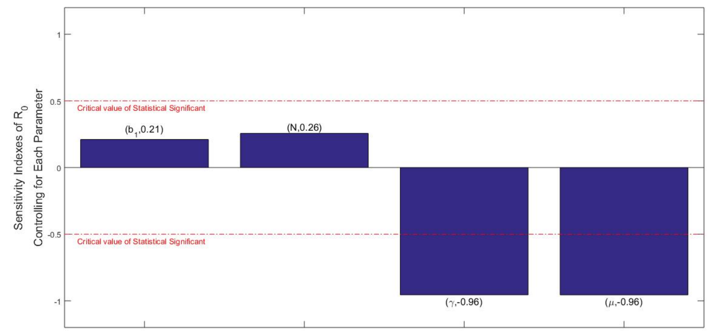
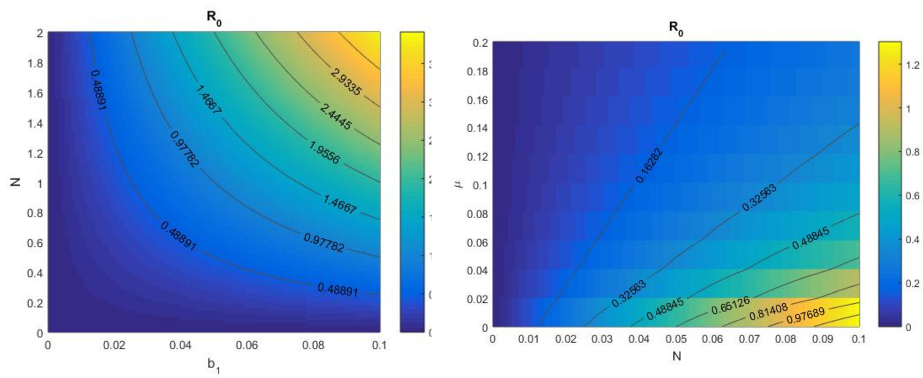


Figure 8: PRCC: input variables



(a)

(b)

Figure 9: Contour plots of ρ^0.

5.3 Media Effect (α) on Susceptible and Infected class

Media Effect helps in spreading awareness among people to stay safe from epidemic and has an ideal impact on the epidemic dynamics which can be seen through graph line when α is either at 0.5 or at 0 in the following:

- In Fig 10a, both values of α effect on susceptible population are shown. As the value of α increases, S also increases as now they will be less vulnerable to catching infection and avoiding decrease in S population. Thus, media will have a positive impact on S individuals as they get awareness of the implications of catching infections, advantages of vaccinations or protocols to follow to avoid risk of getting infected.

- We see in Fig 10b, that media causes a decreasing effect in infected class. Its shows that media effect plays an effective role in decreasing infection

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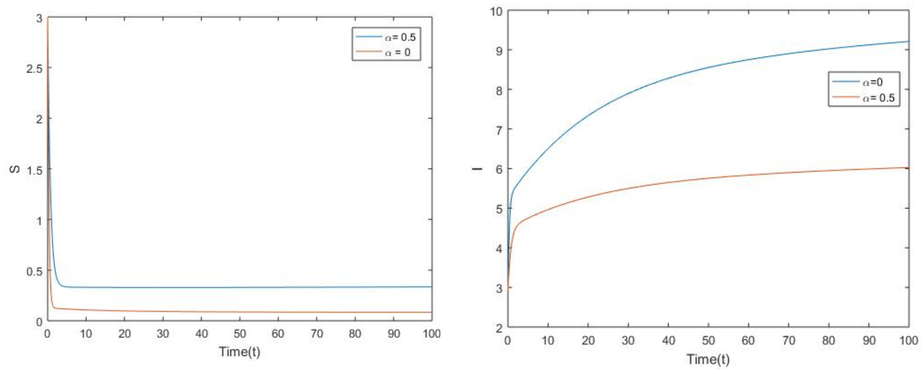
and spreading awareness among population about the epidemic outbreak.



(a) Media effect on Susceptible class

(b) Media effect on Infected class

Figure 10: Media Effect

6 Conclusion

In this paper our aim is to completely analyze the model and be able to become answerable to all those question we aimed to address at the start of our study. As novelty we have studied a two dose vaccination regime and the role of the media for a Covid-19 system and dynamically analysed our system thoroughly along with real data numerical validation. We have looked up on the analysis of model and studied their steady states like Disease Free Equilibrium(DFE) and Endemic Equilibrium(EE). We also found their local stability by finding their eigen values and using Routh-Hurwitz Stability Criteria. In context to the endemic equilibria, global stability analysis of the system has been performed using the graph-theoretic method. For numerical analysis we have taken some real data from Government site of India as an example of Covid-19 to make graph on them thus to analyse the situation created. The incorporation of two dose vaccination regime in the system brings about a desirable outcome for reducing infection. Later we used sensitivity analysis technique to identify sensitive model parameters effecting the R_0 and endemic equilibrium point. It was able to provide us with the degree of positive or negative correlation with which each variable is bound to the parameters present in the model. We have performed Latin hypercube sampling method for our model to determine uncertainty analysis for R_0 to understand the role certain parameters play in the transmission of infection. We also showed the effective role media plays in spreading awareness among population and help reduce infection. Also it is understood that α (the effect of media) will bring down the infections and increase the susceptible population by offering subsequent protection by awareness.

Therefore, on analysing we have seen that without media effect there is no such great awareness among people because media is one of the important ways that makes a country united or be aware of the various protocols and regimes related to an outbreak. Also, a two dose vaccination regime may be the need of the hour to vanquish such an infectious diseases. *As the cases of breakthrough infections and co-infections of already existing ailments increase, so the application of a two dose vaccine regime along with media influence can help provide better immunity and suppress infections as seen in our results.* Thus, a two dose vaccination regime and the role of the media is of paramount importance in policy formation and execution for this deadly epidemic.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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